Landau Theoretical Minimum and related problems: Quantum Mechanics

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1 Introduction.

This collection of problems consists of three parts. The first part is the problems that were given for the Landau Theoretical Minimum exam on Quantum Mechanics. The second part is the problems that were given as the test the students had to pass to be accepted in Lev Gor'kov's theory group in Landau Institute. The third part is the problems that were given as the test the students had to pass to be accepted in Ter-Martirosyan's theory group in the Institute for Theoretical and Experimental Physics.

These problems were collected by the generations of students over the years and passed from one generation to another as a hand written notebook. Each year students copied this notebook from the previous year students (by hand) and added more problems. The notebook which was used here is the one written by Alexander G. Abanov.

2 Landau Theoretical Minimum.

Problem 1. ID:TM-QM-L-3

An oscillator of mass m and frequency ω is in a ground state. Suddenly the frequency changes to ω'. Find the probability of transition to an excited state.

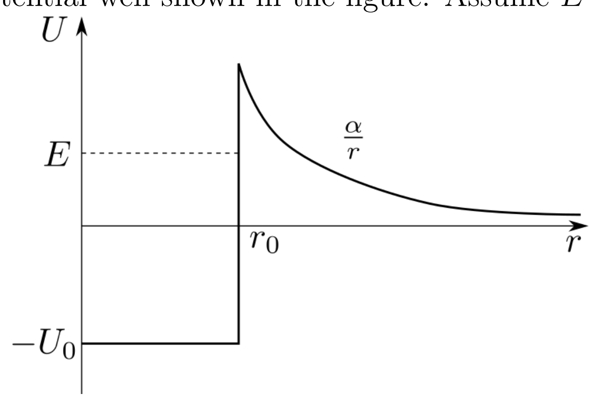
Problem 2. ID:TM-QM-L-6

Find

1. Born scattering amplitude for a slow particle on a potential which decays as λ/r^3 at infinity.
2. Scattering cross-section.

Problem 3. ID:TM-QM-L-9

Find the probability for a particle with the kinetic energy $E = \gamma p^2$ to escape from a potential well shown in the figure. Assume $E \ll \alpha/r_0$.



Problem 4. ID:TM-QM-L-12

Find the ground state energy of positronium which is placed in between two parallel hard walls. The distance between the walls is $a \ll \frac{\hbar}{me^2}$.

Problem 5. ID:TM-QM-L-15

Two non-interacting neutrons are in a potential $U = \frac{1}{2}m\omega^2 r^2$ in magnetic field H. What should be H for given ω in order for the ground state to have spin 0?

Problem 6. ID:TM-QM-L-15

Two electrons are inside a sphere of radius $R \gg \frac{\hbar}{me^2}$. Find the ground state energy and the energies of the rotating states.

Problem 7. ID:TM-QM-L-18

$N \gg 1$ electrons are in a potential $U = \alpha r^3$. Find the ground state energy neglecting the Coulomb interaction. Estimate the energy of the Coulomb interaction.

Problem 8. ID:TM-QM-L-21

Find how the transport cross-section ($\sigma_{tr} = \int d\sigma(1 - \cos\theta)$) of an electron scattered off a screened Coulomb potential ($U = \frac{\alpha}{r} e^{-\gamma r}$) depends on energy E, for large $E \gg \hbar^2 \frac{\gamma^2}{m}; \frac{m\alpha^2}{\hbar^2}$.

Problem 9. ID:TM-QM-L-24

For a hydrogen atom find the transition probability from the ground state to the first excited state after a weak electric field is suddenly turned on.

Problem 10. ID:TM-QM-L-27

Two spin-1/2 particles with magnetic moment $\hat{\vec{M}} = g\hat{\vec{S}}$ are placed at some distance. The particles interact as magnetic dipoles. What is the most favorable spin configuration of the particles?

Problem 11. ID:TM-QM-L-30

Find the density of states $\frac{\partial n}{\partial E}$ for $E \to 0$ for a particle in a potential well given by

$$U(x) = U_0 \times \begin{cases} 0, & |x| > a \\ -(1 + \cos(\pi x/a)), & |x| < a \end{cases},$$

where $U_0 \gg \frac{\hbar^2}{ma^2}$.

Problem 12. ID:TM-QM-L-33

Find the scattering cross-section for scattering of a slow particle ($kr_0 \ll 1$) off a potential $U(\vec{r}) = \lambda\delta(r - r_0)$. Find the conditions for existence of virtual and real levels.

Problem 13. ID:TM-QM-L-36

Two neutrons are in the harmonic potential $U = \alpha r^2/2$. Their spins are interacting according to $\lambda\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2$. Find what conditions on λ and α must be satisfied so that the total spin in the ground state is 1.

Problem 14. ID:TM-QM-L-39

Two electrons are in the cylinder $R \gg \frac{\hbar^2}{me^2} \gg h$. Find the energy of the ground state.

Problem 15. ID:TM-QM-L-42

Find with exponential accuracy the reflection coefficient for a particle on quasi-classical hump $U = \frac{\alpha}{1+x^2/a^2}$ for $E_{\text{particle}} \gg \alpha$.

Problem 16. ID:TM-QM-L-45

Find the transport cross-section ($\sigma_{tr} = \int d\sigma(1 - \cos\theta)$) of a slow particle on a potential decaying as α/r^2, for $\alpha \ll \hbar^2/m$.

Problem 17. ID:TM-QM-L-48

A weak electric field $\vec{E}(t) = \vec{E}_0 \frac{\tau^2}{t^2+\tau^2}$ acts on a linear oscillator. Initially the oscillator was in the ground state. Find the probability that the oscillator will be found in an excited state after long time.

Problem 18. ID:TM-QM-L-51

Two particles are in a bound state with total momentum $l = 1$. Find the state with what projection on z-axis is the most energetically favorable if the particles interact as magnetic dipoles with magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ directed along z.

Problem 19. ID:TM-QM-L-54

In the potential

$$U(x) = \begin{cases} \alpha x, & x > 0 \\ \infty, & x < 0 \end{cases}$$

find how average kinetic energy depends on the level number n for $n \gg 1$.

Problem 20. ID:TM-QM-L-57

Find the scattering cross-section for scattering of a fast particle ($kr_0 \gg 1$) off a potential $U(\vec{r}) = \lambda\delta(r - r_0)$. Consider both cases $\lambda \gg \frac{\hbar^2}{m}k$ and $\lambda \ll \frac{\hbar^2}{m}k$.

Problem 21. ID:TM-QM-L-60

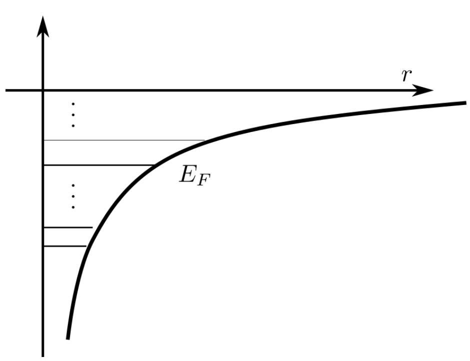
An oscillator of mass m and frequency ω is in the ground state. Suddenly its mass changes to m'. Find the probability for oscillator to be in excited state.

Problem 22. ID:TM-QM-L-63

N non interacting spin-1/2 fermions are sequentially filling up the levels in a potential α/r.

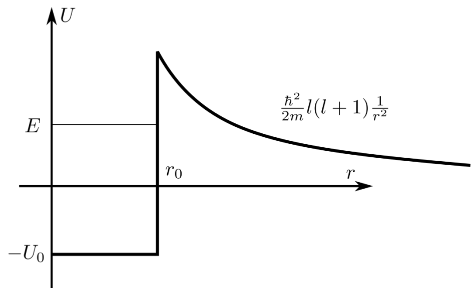
1. Find the Fermi level.

2. Find the total energy.



Problem 23. ID:TM-QM-L-66

Find the escape probability of a particle with the angular momentum l from a centrally symmetric potential well. The effective potntial has the form show in the figure. The energy of the state $E \ll \frac{\hbar^2}{2m}l(l+1)\frac{1}{r_0^2}$.



Problem 24. ID:TM-QM-L-69

Find the energy of the ground state in a Coulomb potential $-e^2/r$, the center of which is somewhere in between two absolutely rigid walls distance at a from each other, $a \ll \frac{\hbar^2}{me^2}$.

Problem 25. ID:TM-QM-L-70

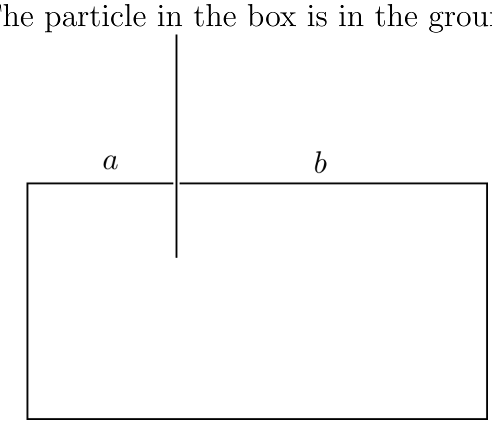
Repeated problem.

Problem 26. ID:TM-QM-L-72

Group: $1, -1, i, -i, j, -j, k, -k$; $i^2 = j^2 = k^2 = -1$; $-ji = ij = k$; $jk = -kj = i$; $ki = -ik = j$; $(-1)^2 = 1$. Find all irreducible representations.

Problem 27. ID:TM-QM-L-75

Find the work which needs to be done to slowly insert a partition in a box, as shown. The particle in the box is in the ground state.



Problem 28. ID:TM-QM-L-78

Incomprehensible.

Problem 29. ID:TM-QM-L-79

Repeated problem.

Problem 30. ID:TM-QM-L-80

Repeated problem.

Problem 31. ID:TM-QM-L-81

$1D$ potential box of size a is split in two parts by a rigid partition which can move. In the left part of the box there is a particle of mass m_L in the right part there is a particle of mass m_R. Both particles are in the ground state. Find the position of the partition.

Problem 32. ID:TM-QM-L-84

Find the number of quasi-classical states $N(E_0)$ for a particle in $3D$ potential $U = -\frac{U_0 a^2}{r^2+a^2}$ for energies $E < E_0 < 0$.

Problem 33. ID:TM-QM-L-87

Find the splitting of the levels with $n = 2$ in a hydrogen atom in a nucleus potential $U = -\frac{e^2}{r+a\cos^2\theta}$, $a \ll r_B$.

Problem 34. ID:TM-QM-L-90

Find the Born scattering amplitude for a slow particle on the potential

$$U = \frac{U_0 a^4}{(a^2+\rho^2)(a^2+z^2)}, \qquad \rho^2 = x^2 + y^2.$$

Problem 35. ID:TM-QM-L-93

Find energy levels and wave functions for a particle of charge e and mass m in a field with potentials $A_x = -Hy$, $A_y = A_z = 0$, $\phi = -Ey$.

Problem 36. ID:TM-QM-L-96

Find energy levels in a field with $U = x^2(1 + ay^2)$, $a \ll 1$.

Problem 37. ID:TM-QM-L-99

Find the low laying energy levels of anisotropic $2D$ oscillator $U(\rho, \phi) = \frac{1}{2}\left(\alpha + \beta\cos^4\phi\right)\rho^2$, $\alpha \gg \beta$.

Problem 38. ID:TM-QM-L-102

Find the Born scattering amplitude for a slow particle on the potential

$$U = e^{-\kappa\rho}\frac{U_0 a}{a+|z|}, \qquad \rho^2 = x^2 + y^2.$$

Problem 39. ID:TM-QM-L-105

A particle is in the potential $U(r) = \epsilon\left[(r_0/r)^{10} - (r_0/r)^6\right]$. What is the minimal ϵ at which the first localized state appears? Us Taylor expansion around the minimum.

Problem 40. ID:TM-QM-L-108

A particle is in the potential $U(r) = \epsilon\left[(r_0/r)^{10} - (r_0/r)^6\right]$. Where $\epsilon \gg \frac{\hbar^2}{mr_0^2}$. Find the energies of low energy states.

Problem 41. ID:TM-QM-L-111

Find at which wave-numbers k the reflection coefficient for a rectangular potential $U(x) = \left\{ \begin{array}{ll} 0, & |x| > a \\ U_0, & |x| < a \end{array} \right.$.

Problem 42. ID:TM-QM-L-114

In the potential of two δ-functional wells $U = -\kappa\left[\delta(x+a/2) + \delta(x-a/2)\right]$ consider two cases $\Psi = \Psi_e$ – even function and $\Psi = \Psi_o$ – odd function. Draw a graph of $E(\kappa)$ for both cases. Find $E_e(\kappa) - E_o(\kappa) = f(\kappa)$ for $\kappa, a \to \infty$.

Problem 43. ID:TM-QM-L-117

An electron ha a constant orbital moment $L = 1$. Its projection on some axis is $m = 1, 0, -1$. Find J_m. Compute

$$\bar{L} = \int \Psi^* \hat{L} \Psi d^3 r = \int \Psi^* \hat{\vec{r}} \times \hat{\vec{p}} \Psi d^3 r = -i\hbar \int \Psi^* \hat{\vec{r}} \times \frac{\partial}{\partial \vec{r}} \Psi d^3 r.$$

Problem 44. ID:TM-QM-L-120

A particle has angular momentum $L = 1$. Its projection on some axis is $m = 0$. Another axis is at the angle θ to the first one. Find the probability that the projection of the momentum on the second axis is $1, 0, -1$. Repeat for the cases $m = 1, -1$.

Problem 45. ID:TM-QM-L-123

A shallow potential well is the one for which $\kappa = \frac{\hbar^2}{U_0 m a^2} \ll 1$. Find the solution for arbitrary shallow well in $1D$. Proof the uniqueness also for $2D$ and $3D$ wells.

Problem 46. ID:TM-QM-L-126

At which $\kappa = \frac{\hbar^2}{U_0 m a^2}$ in a rectangular potential well $U(x) = \left\{ \begin{array}{ll} 0, & |x| > a \\ -U_0, & |x| < a \end{array} \right.$ the first anti-symmetric state ($\Psi(x) = -\Psi(-x)$) appears.

Problem 47. ID:TM-QM-L-129

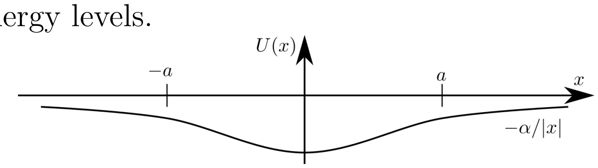
How an energy level in a shallow well behaves as a function of $\kappa = \frac{\hbar^2}{U_0 m a^2} \ll 1$.

Problem 48. ID:TM-QM-L-132

An electron is inside a bubble which it cannot escape. The coefficient of surface tension of the bubble is σ. What is the radius R of the bubble?

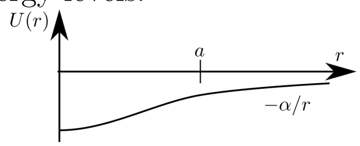
Problem 49. ID:TM-QM-L-135

A shallow $1D$ potential well with Coulomb tails $U(|x| \to \infty) \sim -\alpha/|x|$, $\alpha \ll 1$. Find energy levels.



Problem 50. ID:TM-QM-L-138

A shallow $2D$ potential well with Coulomb tails $U(r \to \infty) \sim -\alpha/r$, $\alpha \ll 1$. Find energy levels.



Problem 51. ID:TM-QM-L-141

The density of an ideal gas of particles of mass m is such, that the average distance between the particles is a. At which temperatures T we consider this gas as classical?

Problem 52. ID:TM-QM-L-144

A $1D$ potential well is given by $U(x) = -\alpha\delta(x)$. Prove, that the well is always shallow.

Problem 53. ID:TM-QM-L-147

$2D$ potential well is given by $U(r) = -\alpha/r^n$.

1. α is given. At which n the particle will fall to the center?
2. $n = 2$. At which α the particle will fall to the center? (Use the variational principle. The particle falls if $E_n \to -\infty$.)

Problem 54. ID:TM-QM-L-150

1. A small perturbation γx^5 is applied to a quantum oscillator. Find the first order perturbative correction to the energy of the ground state.
2. The same for the perturbation γx^6.

Problem 55. ID:TM-QM-L-153

Find the correction to the ground state energy δE_0^1 in the potential $U(r) = \epsilon + \alpha(r - r_0)^2 + \beta(r - r_0)^3 + \gamma(r - r_0)^4$.

Problem 56. ID:TM-QM-L-156

1. Two oscillators m_1, ω_1 and m_2, ω_2 are interacting via a perturbation $\gamma x_1 x_2$. Find δ_0 to the energy of the ground state.
2. The same for $\gamma x_1^2 x_2^2$.

Problem 57. ID:TM-QM-L-159

Find the low lying energy levels in the potential $U(x) = -\frac{U_0}{\cosh^2(\alpha x)}$, ($U_0 \gg \frac{\hbar^2\alpha^2}{2m}$.)

Problem 58. ID:TM-QM-L-162

In the Coulomb potential $-\frac{e^2}{r}$, find the corrections to the ground state energy due to perturbation caused by the finite size of the nucleolus.

1. $-\frac{e^2}{\sqrt{r^2+a^2}}$.
2. $-\frac{e^2}{r+a}$.
3. $-\frac{e^2}{\sqrt[3]{r^3+a^3}}$.

Problem 59. ID:TM-QM-L-165

In the Coulomb potential $-\frac{e^2}{r}$, in $2D$, find the corrections to the ground state energy due to perturbation caused by the finite size of the nucleolus.

1. $-\frac{e^2}{\sqrt{r^2+a^2}}$.
2. $-\frac{e^2}{r+a}$.

Problem 60. ID:TM-QM-L-168

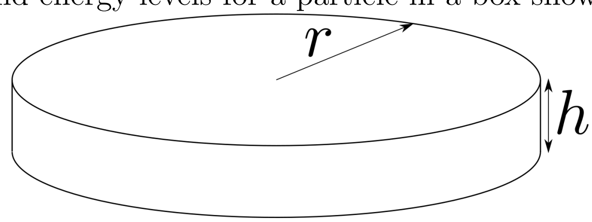
An electric field $\vec{\mathcal{E}}$ is applied to the a hydrogen atom. Find the energy splitting of the four-degenerate level. ($\frac{\mathcal{E}a}{E_1-E_2} \ll 1$, $a-$ atom's size.)

Problem 61. ID:TM-QM-L-171

A nucleolus of a hydrogen atom is fixed at an arbitrary point between two parallel infinite walls. The distance between the walls is d. Find electron's energy levels if $d \ll \hbar^2/m$.

Problem 62. ID:TM-QM-L-174

Find energy levels for a particle in a box shown on the figure, if $h \ll r$.



Problem 63. ID:TM-QM-L-177

Incomprehensible.

Problem 64. ID:TM-QM-L-180

External electric field $\vec{\mathcal{E}}$ is applied to a hydrogen atom. Find the splitting of the level $n = 2$, $l = 1$ up to a linear order in $\mathcal{E}$.

Problem 65. ID:TM-QM-L-183

A weak periodic potential $U(x) = U_0 \cos(kx)$ is applied to a free particle $\Psi = e^{ipx/\hbar}$, $E = p^2/2m$. Find the energy levels.

Problem 66. ID:TM-QM-L-186

A hydrogen atom is electrically neutral. Find its polaraizability.

1. Using the perturbation method.
2. Using variational method.
3. Compare the two answers.

Problem 67. ID:TM-QM-L-189

Landau, vol 3, Section 112, problems 2 and 3.

Problem 68. ID:TM-QM-L-192

A $3D$ potential is such that there is only one localized state and its energy is close to 0. Find polarizability of an electron in the ground state of this potential.

Problem 69. ID:TM-QM-L-195

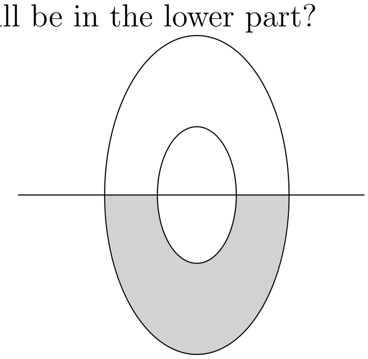
Find the low laying levels for an electron in a non-uniform magnetic field given in cylindrical coordinates by $H = \alpha\rho$ ($A_z = A_\phi = 0$, $A_\rho = \alpha\rho z$) assume that α is large.

Problem 70. ID:TM-QM-L-198

Find energy levels of a $3D$ oscillator in magnetic field. You are given m, ω, $\Omega = \frac{EH}{mc}$, $M = 0$.

Problem 71. ID:TM-QM-L-201

A particle is in between to ellipsoids. What are the conditions at which the particle will be in the lower part?



Problem 72. ID:TM-QM-L-204

Incomprehensible.

Problem 73. ID:TM-QM-L-207

Find the normalization of the quasiclassical wave function.

Problem 74. ID:TM-QM-L-210

Estimate the accuracy of the normalization of the quasiclassical wave function.

Problem 75. ID:TM-QM-L-213

Repeated problem.

Problem 76. ID:TM-QM-L-216

Repeated problem.

Problem 77. ID:TM-QM-L-219

A ball is bouncing elastically on a horizontal floor. Find $E(n)$, compute $\frac{\partial E}{\partial n}$ for $n \to \infty$.

Problem 78. ID:TM-QM-L-222

You are given two wave functions Ψ_{n_1} and Ψ_{n_2}, such that $n_1, n_2 \gg 1$, and $\frac{n_1 - n_2}{n_1} \ll 1$. Find a matrix element $\langle n_1 | A(x) | n_2 \rangle$.

Problem 79. ID:TM-QM-L-225

A weak electric field is applied to a hydrogen atom (potential $-\frac{e^2}{r} - \mathcal{E}z$). Find the probability of ionization.

Problem 80. ID:TM-QM-L-228

A shallow $3D$ potential well does not have localized states $\frac{\hbar^2}{2ma^2U_0} \gg 1$. A cylinder is placed over the well. Will a localized state appear?

Problem 81. ID:TM-QM-L-231

Find the probability of reflection of a particle from a potential $U(x) = \frac{\alpha}{x^2+a^2}$, for $E \gg \alpha/a^2$.

Problem 82. ID:TM-QM-L-234

In the potential $U = \kappa \sum_n \delta(x - an)$ the wave function of a particle has a form $\Psi = \sum_n e^{ikx}\psi(x - an)$. Find $E(k, a)$. Find the behavior of $\frac{\partial^2 E}{\partial k^2} = \frac{1}{m^*}$ at the zone boundary.

Problem 83. ID:TM-QM-L-237

1. In the potential $U = U_0 cos(\kappa x)$ find $E(k, \kappa)$. Find the behavior of $\frac{\partial^2 E}{\partial k^2} = \frac{1}{m^*}$ at the zone boundary.

2. Find $E(k, a)$ for infinite number of potential wells placed periodically with the period a, symmetrically with respect to the coordinate origin.

Problem 84. ID:TM-QM-L-240

A molecular ion M_2^+, where the distance between the atoms $R \gg r_0$ — the atom's size. Find the level splitting $E_g - E_n$.

Problem 85. ID:TM-QM-L-243

A number of electrons in an atom is large, $N \gg 1$, the potential $U = -1/r$. Find the dependence of the total energy on the number of particles.

Problem 86. ID:TM-QM-L-243

How N fermions will be distributed between two oscillators $\Delta_1 + \frac{1}{2}\omega_1^2$ and $\Delta_2 + \frac{1}{2}\omega_2^2$ in the ground state?

Problem 87. ID:TM-QM-L-246

Find the atom polarizability in the Thomas-Fermi model. Potential $U(\vec{r}) = -\frac{Z}{r} + \int \frac{n(\vec{r}')d^3r'}{|\vec{r}-\vec{r}'|} - \mathcal{E}z$.

Problem 88. ID:TM-QM-L-249

Find probability of atom's ionization by a weak electric field.

Problem 89. ID:TM-QM-L-252

In the Thomas-Fermi model estimate electrons angular momentum $L(Z)$, total energy $E(Z)$ (Z is the nucleolus's charge). At what distances the electrons of the "last shell" are? At what distances the quasiclassics is applicable?

Problem 90. ID:TM-QM-L-255

Find how two atoms interact at large distances $R \gg r_B$. For two hydrogen atoms estimate the constant in that interaction using

1. perturbation theory;
2. variational principle.

Problem 91. ID:TM-QM-L-258

Find how two atoms interact at small distances $R \ll r_B$. For two hydrogen atoms estimate the constant in that interaction using

1. perturbation theory;
2. variational principle.

Problem 92. ID:TM-QM-L-261

Find (estimate) oscillation and rotation levels for a two atom molecule.

Problem 93. ID:TM-QM-L-264

Find how two atoms interact at large distances $R \gg r_B$ if one atom is in s and the other in p state. For two hydrogen atoms estimate the constant in that interaction using

1. perturbation theory;
2. variational principle.

Problem 94. ID:TM-QM-L-267

Two fermions are interacting according to $U = \alpha e^{-\gamma^2(\vec{r}_1 - \vec{r}_2)^2}$. They are outside of the Fermi sphere $|\vec{p}_1| > p_0$, $|\vec{p}_2| > p_0$. Show that on small distances $r < 1/\gamma \ll 1/p_0$ (so the interaction must be such that $\gamma \gg p_0$) the particles pair.

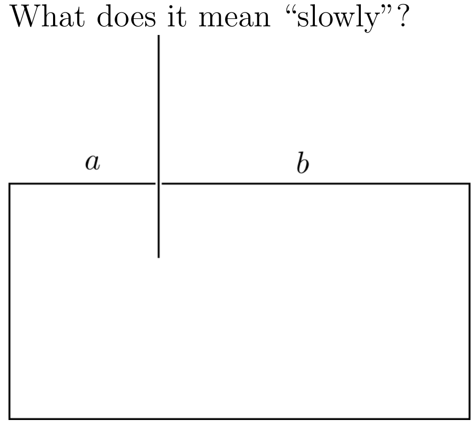
1. For the state with $l = 0$.
2. For the state with $l = 1$.

The energy of the particles is $\frac{p_1^2}{2m} + \frac{p_2^2}{2m}$. Find E and the wave function.

Problem 95. ID:TM-QM-L-270

Find the work which needs to be done to slowly insert a partition in a box, as shown. The particle in the box is in the ground state.

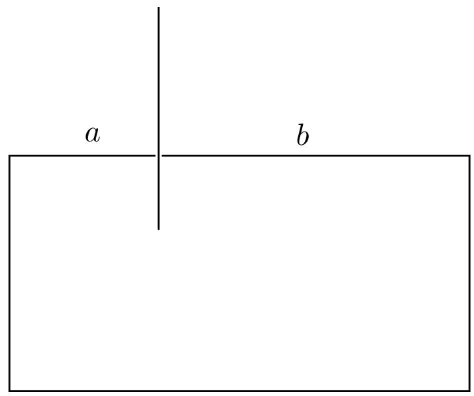
What does it mean "slowly"?



Problem 96. ID:TM-QM-L-270

The partition is inserted in the box fast. Initially, the particle in the box is in the ground state.

What is the probability that particle is in one particular part of the box?



Problem 97. ID:TM-QM-L-273

Take the second order of the S-matrix and find the second order of the probability $p_{\vec{k}}$ for a periodic potential.

Problem 98. ID:TM-QM-L-276

Missed Problem.

Problem 99. ID:TM-QM-L-279

A particle in the time dependent oscillatory potential $U = \frac{kx^2}{2}\Theta(-t)$. At what time it's wave function will spread out to a function of the form e^{iqx}. Find $\Psi(x,t)$.

Problem 100. ID:TM-QM-L-282

Find the scattering cross-section for a particle on Coulomb potential $U = -\alpha/r$ in the first Born approximation.

Problem 101. ID:TM-QM-L-285

Find the full scattering cross-section for a particle on potential $U = \alpha/r^3$.

Problem 102. ID:TM-QM-L-288

The potential $U = 1/r^n$; the scattering amplitude is $f(\theta)$. Find $f(\pi/2) - f(0)$ up to a number.

Problem 103. ID:TM-QM-L-291

For a fast particle, $qa \gg 1$, estimate $\sigma(E)$.

Problem 104. ID:TM-QM-L-294

The potential $U = \frac{1}{r}e^{-\gamma r}$. Find σ for large $E \gg \frac{\gamma^2\hbar^2}{2m}$.

Problem 105. ID:TM-QM-L-297

A Potential

$$U = \begin{cases} \infty, & x < 0 \text{ and } x > 0 \\ 0, & 0 < x < a \end{cases}$$

suddenly switches to

$$U = \begin{cases} \infty, & x < 0 \\ 0, & 0 < x < a \text{ and } x > a + b \\ U_0, & a < x < a + b \end{cases}.$$

What time it will take for the particle to leave the well?

Problem 106. ID:TM-QM-L-300

In $1D$ potential $U = \begin{cases} \infty, & x < 0 \text{ and } x > 0 \\ 0, & 0 < x < a \end{cases}$ in point $x_0 \in (0, a)$ slowly grows a partition $U = \kappa(t)\delta(x - x_0)$ so that $\kappa \to \infty$. Find out and describe what will happen.

Problem 107. ID:TM-QM-L-303

A fast particle ($E \gg U_B$) is flying by a hydrogen atom. The impact parameter $\rho \gg \rho_B$. The atom initial in the ground state. Find the probability for the atom to transition to the first excited state.

Problem 108. ID:TM-QM-L-306

Two identical $3D$ potential wells are at distance $\kappa \gg a$ (where a is the size of the well) from each other. There are virtual levels in the wells. At what κ the first bound state will appear?

Problem 109. ID:TM-QM-L-309

Two non-identical $3D$ potential wells are at distance $\kappa \gg a$ (where a is the size of the well) from each other. There are virtual levels in the wells. At what κ the first bound state will appear?

Problem 110. ID:TM-QM-L-312

Three identical $3D$ potential wells are at random distances $\kappa_i \gg a$ (where a is the size of the well) from each other. There are virtual levels in the wells. At what κ_is the first bound state will appear?

Problem 111. ID:TM-QM-L-315

Two fermions are in a bound state with $l = 3$. Find how the spin operator acts of the wave function of these two particles.

Problem 112. ID:TM-QM-L-318

There are two fermions in a $3D$ box. The fermions are interacting such that $U \ll \frac{\hbar^2}{ma^2}$. What is the total spin in the ground state?

Problem 113. ID:TM-QM-L-321

infinite number of $3D$ potential wells are uniformly distributed over all space. For each the scattering length $\alpha < 0$ and $|\alpha| \gg a$, where a is the size of the potential. At what density of the wells the first bound state will appear? Find this state.

Problem 114. ID:TM-QM-L-324

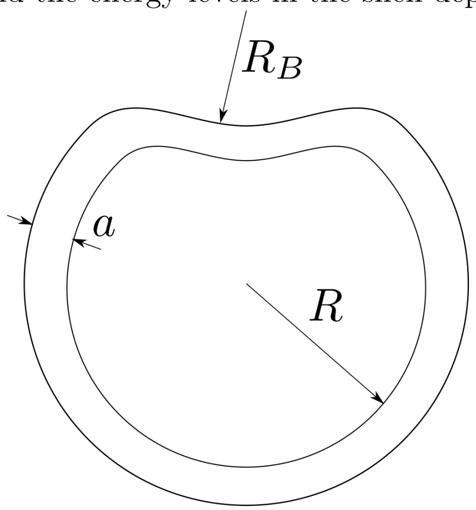
If $\vec{n}$ is a unit radius-vector of a particle find $\langle n_i \rangle$ (where $i = x, y, z$) averaged of a state with a given vector $\vec{m}$ (for example in electric field)

Problem 115. ID:TM-QM-L-327

Find energy levels in a potential of size a in cylinder of radius $R \gg a$.

Problem 116. ID:TM-QM-L-330

Find the energy levels in the shell depicted in the figure, if $a \ll R_B \ll R$.



Problem 117. ID:TM-QM-L-333

Incomprehensible.

Problem 118. ID:TM-QM-L-336

Find a polarizability of a particle in an $3D$ oscillator potential.

Problem 119. ID:TM-QM-L-339

Hydrogen atom in a strong magnetic field.

Problem 120. ID:TM-QM-L-342

N bosons are in the potential $U(x) = \begin{cases} 0, & |x| < a \\ \infty, & |x| > a \end{cases}$. Find the density of particles $\rho(x)$.

Problem 121. ID:TM-QM-L-345

Quasiclassical $3D$ potential well. The number of states $n \gg 1$. Find the number of electrons that can be fitted in this well.

Problem 122. ID:TM-QM-L-348

Scattering of electron off a hydrogen atom.

Problem 123. ID:TM-QM-L-351
Show, that quasiclassical solution gives $E = -\frac{1}{2n^2}$ for the Coulomb potential.

Problem 124. ID:TM-QM-L-354
Show, that quasiclassical solution gives $E = \hbar\omega(n + 1/2)$ for the oscillator potential.

Problem 125. ID:TM-QM-L-357
Spherical $3D$ potential well $U(r) = \begin{cases} 0, & r < a \\ \infty, & r > a \end{cases}$, is deformed to an ellipsoid, which is still close to the sphere. Find the corrections to the energy levels.

Problem 126. ID:TM-QM-L-360
Find the splitting of levels for two $3D$ potential wells which are far from each other.

Problem 127. ID:TM-QM-L-363
The $3D$ potential is given by $U(r) = U_0\left((r_0/r)^{10} - (r_0/r)^6\right)$. At what U_0 the first bound state appears?

Problem 128. ID:TM-QM-L-366
Incomprehensible.

Problem 129. ID:TM-QM-L-369
Find energy levels in strongly smashed ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where $b \gg, a, c$.

Problem 130. ID:TM-QM-L-372
Repeated problem.

Problem 131. ID:TM-QM-L-375
$N \gg 1$ of non-interacting fermions are in the potential $U(r) = -\frac{\alpha}{r}$. Find the Fermi level and the total energy.

Problem 132. ID:TM-QM-L-378
Find the energy levels in potential $U(r) = A/r^2 - B/4$, where $A \gg^{\hbar/m}$.

Problem 133. ID:TM-QM-L-378
Find the energy levels in potential $U(r) = A\left(e^{-2x/a} - 2e^{-x/a}\right)$, where $A \gg^{\hbar^2/ma^2}$..

Problem 134. ID:TM-QM-L-381
A hydrogen atom is placed in a cylinder of radius $R \ll r_B$. Find energy levels.

Problem 135. ID:TM-QM-L-384
Find the dependence of $\sigma(E)$ for $E \to \infty$.

Problem 136. ID:TM-QM-L-387

Find energy levels for an oscillator in magnetic field.

Problem 137. ID:TM-QM-L-390

Two protons at short distance $R \ll r_B$, find the energy of the electron's ground state as a function of R.

Problem 138. ID:TM-QM-L-393

Protons are interacting according to $\frac{\alpha}{r}$. There is an electron in the system. Find the oscillatory and rotation energy levels. Us the fact that a proton is much heavier than electron.

Problem 139. ID:TM-QM-L-396

A particle in a state with $l = 1$, $m = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$ at the angle θ to the $\hat{z}$. A magnetic field is turned on alone $\hat{z}$. Find the probability of the angular momentum projection on $\hat{z}$.

Problem 140. ID:TM-QM-L-399

$N \gg 1$ of non-interacting fermions are in the potential $U = \alpha r^m$. Find

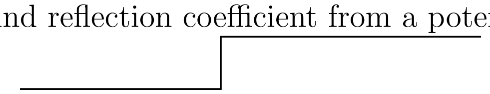
1. the Fermi energy;
2. the total energy.

Problem 141. ID:TM-QM-L-402

Find exact energy levels in the potential $U(x) = -\frac{U_0}{\cosh^2(x/a)}$.

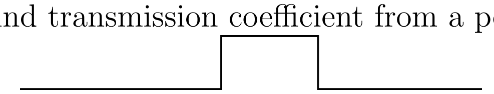
Problem 142. ID:TM-QM-L-405

Find reflection coefficient from a potential on the figure.



Problem 143. ID:TM-QM-L-408

Find transmission coefficient from a potential on the figure.



Problem 144. ID:TM-QM-L-411

Find the average potential of the field created by a nucleus and an electron in the ground state.

Problem 145. ID:TM-QM-L-414

Incomprehensible.

Problem 146. ID:TM-QM-L-417

Find energy levels in the potential $U = \frac{A}{r^2} + Br^2$.

Problem 147. ID:TM-QM-L-420

Find the number of levels in the quasiclassical potential $U(\vec{r})$.

Problem 148. ID:TM-QM-L-423

There are n particles with a constant total angular momentum $L = 1$ and total spin $S = 1$. What is $|\vec{J}|$ if the particles interact according to $\hat{V} = \sum_{i,j} \hat{S}_i \hat{S}_j \left(\hat{L}_i \hat{L}_j + \hat{L}_j \hat{L}_i \right)$.

Problem 149. ID:TM-QM-L-426

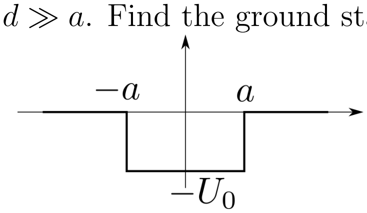
Two electrons are interacting by the Coulomb law and are placed in the oscillator potential with $\omega \ll \frac{me^4}{\hbar^3}$. Find the ground state energy and the energies of rotation states.

Problem 150. ID:TM-QM-L-429

Slow ($ka \ll 1$) particles are scattered off the potential $\frac{\gamma}{r^5 + a^5}$. Find the dependence of the scattering amplitude on angle.

Problem 151. ID:TM-QM-L-432

Two particles interact according to the potential shown on the figure, where $U_0 \ll \frac{\hbar^2}{ma^2}$. They are placed in between two parallel planes. The distance between the planes is $d \gg a$. Find the ground state energy.



Problem 152. ID:TM-QM-L-435

Incomprehensible.

Problem 153. ID:TM-QM-L-438

Find energy levels for a charged particle in magnetic field given by $A_x = ay^2$, $A_y = A_z = 0$.

Problem 154. ID:TM-QM-L-441

A hydrogen atom in the state with $l = 1$ is in a large cylinder. The cylinder is rotating about its axis with the frequency ω. Find how the energy level is split. (There are many atoms in the cylinder, so the gas is rotating with the cylinder.)

Problem 155. ID:TM-QM-L-444

In Born approximation, find the scattering amplitude and the scattering cross-section for electrons scattered off a hydrogen atom. The atom is in a strongly excited state.

Problem 156. ID:TM-QM-L-447

Find energy levels for high energy for a particle in the potential $U = U_0 \frac{x^2 y^2}{a^4}$.

Problem 157. ID:TM-QM-L-450

Find energy level for a particle in $2D$ ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the case

1. $b \gg a$;
2. $\frac{|b-a|}{a} \ll 1$.

Problem 158. ID:TM-QM-L-453

Find the transmission coefficient through the barrier $U = \frac{U_0 a}{|x|+a}$ for particles with $E \to 0$.

Problem 159. ID:TM-QM-L-456

Find the transmission coefficient through the barrier $U = \frac{U_0 a}{|x|+a}$ for particles with fixed E in the case $a \to 0$.

Problem 160. ID:TM-QM-L-453

Find the ground state energy in the potential $U = -\frac{U_0}{|x|+a}$ in the case $a \to 0$.

Problem 161. ID:TM-QM-L-459

Find the magnetic moment in a weak magnetic field for a hydrogen atom in the ground state.

Problem 162. ID:TM-QM-L-462

Find scattering amplitude and scattering cross-section in Born approximation for the slow particles scattered off a potential $U = \frac{U_0 a^4}{r^4 + a^4}$, where $r^2 = \sum_{i=1}^{4} x_i^2$.

Problem 163. ID:TM-QM-L-465

In the potential $U(x) = \frac{\hbar^2 \kappa_0}{m} \sum_{n=-\infty}^{\infty} \delta(x - na)$ find the allowed zones and the effective mass for $ka \gg 1$.

Problem 164. ID:TM-QM-L-468

$U(r) = \begin{cases} U_0 \frac{a^2}{r^2}, & r < a \\ 0, & r > a \end{cases}$. Find scattering cross-section for slow particles $ka < 1$.

Problem 165. ID:TM-QM-L-471

$U(x) = -\frac{\hbar^2 \kappa_0}{m} \delta(x - a\cos(\omega t))$, where a is small, and ω is large. Find the probability of escape form the well per unit time.

Problem 166. ID:TM-QM-L-474

Find the energy levels in the infinite $2D$ well shaped as a very long rhombus.

Problem 167. ID:TM-QM-L-477
Is it possible for an electron and neutron to be in a bound state?

Problem 168. ID:TM-QM-L-480
The potential $U(\vec{r}) = -\frac{\alpha}{\sqrt{r^2+b^2\cos^2(\theta)}}$, where $b \ll 1$. Find the perturbation of the Coulomb ground state. The same for the potentials $-\frac{\alpha}{\sqrt{r^2+b^2}}$, $-\frac{\alpha r}{r^2+a^2}$, $-\frac{\alpha}{\sqrt[3]{r^3+a^3}}$.

Problem 169. ID:TM-QM-L-483
Find the level splitting of $l = 1$ level in the field T. The same for $l = 3$ level in the field D_3.

Problem 170. ID:TM-QM-L-486
A particle with $L = N$ and $S = N$ is in the state with $J = 0$. The spin part of the wave function is rotated by the angle θ around the axis $\vec{n}$. In the new state find $\hat{\vec{L}} \times \hat{\vec{S}}$.

Problem 171. ID:TM-QM-L-486
In the cylindrical coordinates the vector potential is given by $A_\rho = A_z = 0$, $A_\phi = \alpha\rho^2$. Find the energy levels for a particle of charge e for the states with large orbital momentum $|m| \gg 1$.

Problem 172. ID:TM-QM-L-489
$U(r) = -\frac{U_0 a^2}{r^2+a^2}$. Find $\frac{\partial n}{\partial E}$ for $E < E_0 < 0$, $|E_0| \ll U_0$.

Problem 173. ID:TM-QM-L-492
A $2D$ electron gas (non-interacting) with density n per unit area is in the magnetic field H perpendicular to the plane. Find the ground state energy as a function of H. Plot $E_{\text{ground}}(1/H)$.

Problem 174. ID:TM-QM-L-495
A particle is in between two coaxial cylinders of radii R, and r, where $\frac{R-r}{R} \ll 1$. There is a magnetic field H inside the inner cylinder. The magnetic field is directed along the cylinder's axis. Find the ground state energy as a function of the magnetic field H.

Problem 175. ID:TM-QM-L-498
A beam of polarized protons is scattered by the unpolarized neutrons at rest. The scattering amplitude is $A + B\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2$. Find the what portion of the protons will flip their spins.

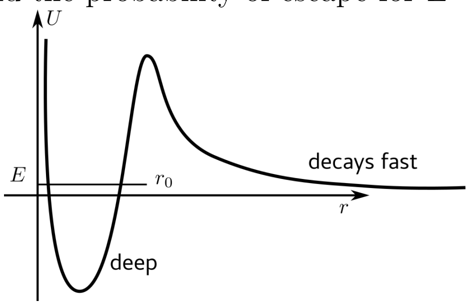
Problem 176. ID:TM-QM-L-501
A plane rotator is in a strong electric field ($ER \gg \frac{\hbar^2}{mR^2}$). Find the low lying energy level, the bandwidth of the low zones, and the dispersion.

Problem 177. ID:TM-QM-L-504

The potential is given by $U = \frac{1}{2}\left(1 + \frac{y^2}{a^2}\right)$, where $a \gg 1$. Find low energy levels.

Problem 178. ID:TM-QM-L-507

Find the probability of escape for $E \to 0$ for a state with $l \neq 0$.



Problem 179. ID:TM-QM-L-510

Find σ as $f(\delta_l)$.

Problem 180. ID:TM-QM-L-513

Full symmetry of a cube is O_h. Find how the levels with $l = 1$ and $l = 2$ are split.

Problem 181. ID:TM-QM-L-516

A shallow $3D$ potential well is in between two infinite parallel hard walls. The distance between the walls is $d \gg a$, where a is characteristic well's size. Will a bound state appear? What will be its energy?

Problem 182. ID:TM-QM-L-519

Find the polarizability of the hydrogen atom in the ground state.

Problem 183. ID:TM-QM-L-522

Find the probability of ionization of the hydrogen atom as a function of the frequency of the applied weak electric field. The applied field has the form $\vec{E} = \vec{k}\mathcal{E}e^{i\omega t}$, where $\mathcal{E}r_B \ll \hbar\omega$.

Problem 184. ID:TM-QM-L-525

A string is charged with the linear charge density $\lambda > 0$. Find the energy levels of electron with $m \to \infty$, where m is the magnetic number.

Problem 185. ID:TM-QM-L-528

Incomprehensible.

Problem 186. ID:TM-QM-L-531

Two electrons are inside a sphere. Find (qualitatively) how the total spin depends on the radius of the sphere R.

Problem 187. ID:TM-QM-L-534

Find the relationship between the transmission and reflection amplitudes for a scattering off a $1D$ potential from the left and from the right.

Problem 188. ID:TM-QM-L-537

In a $2D$ spherical potential well there is a non-degenerate level at the energy E. Now three of these wells are in the unilateral triangle. Find the energy levels.

Problem 189. ID:TM-QM-L-540

Find the area of the geometrical shadow behind an ideally reflecting sphere of radius a for fast particles ($ka \gg 1$). Find the forward shattering amplitude. Find $f(\theta)$ for all θ.

Problem 190. ID:TM-QM-L-543

A $2D$ charged rotor is placed in uniform perpendicular magnetic field. There is a weak electric field in the plane of the rotor. Find how the energy levels depend on magnetic field.

Problem 191. ID:TM-QM-L-546

Repeated problem

Problem 192. ID:TM-QM-L-549

Find the ground state energy in the $1D$ potential $U(x) = \left\{ \begin{array}{ll} \alpha/|x|, & |x| > a \\ \alpha/a, & |x| < a \end{array} \right.$, where $a \ll a_B$ – Bohr radius.

Problem 193. ID:TM-QM-L-552

Find nonzero magnetic and electric multipoles for a nucleus with $j = 3/2$.

Problem 194. ID:TM-QM-L-555

An atom, consisting of a π^- meson in $1S$ state around a deuterium, decays as $\pi^- + d \to n + n$. Find internal parity of π^-.

Problem 195. ID:TM-QM-L-558

A charged particle is in a bound state of a δ-potential. Find the probability for the particle to escape per unit time if a weak electric field is applied.

Problem 196. ID:TM-QM-L-561

Find the dependence of $\sigma(k)$ for $k \to 0$ in the first non-vanishing order.

Problem 197. ID:TM-QM-L-564

A state with $l = 3$ is in D_3 field. Find the level splitting and corresponding eigen functions.

3 Problems from the entrance exams to Lev Gor'kov's group.

Problem 198. *I* ID:TM-QM-G-3

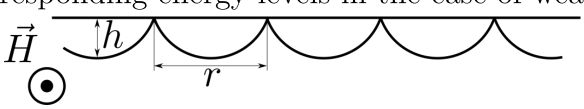
The interaction energy of two spin-1/2 particles is given by $\frac{m\omega^2}{2}(x_1 - x_2)^2$. Find energy level if the total spin of the particles is 1.

Problem 199. *I* ID:TM-QM-G-6

A particle is in the shallow $E < 0$ level in a $3D$ potential well. Suddenly the wel changes and the level's energy becomes $\tilde{E}$. What is the probability that the particle stays on this level?

Problem 200. *I* ID:TM-QM-G-9

Nagnetic field is parallel to a flat surface of a metal. For the electrons with velocity v almost parallel to the surface there are trajectories shown on the figure. Find the corresponding energy levels in the case of weak magnetic field $H \ll \frac{cmv}{e\hbar}$.



Problem 201. *I* ID:TM-QM-G-12

A particle of charge e is in a hard shell. Find the low energy levels in a strong electric field $\mathcal{E} \gg \frac{\hbar^2}{meR^3}$.

Problem 202. *II* ID:TM-QM-G-15

A particle is in a bound state of a potential $U(x) = -\frac{\hbar^2}{m}\delta(x)$. A hard wall is slowly moved from $-\infty$ towards the well. Find at what distance between the wall and the well the particle will escape? Qualitatively draw a dependence of the force acting on the wall as a function of the distance between the wall and the well.

Problem 203. *II* ID:TM-QM-G-18

A roton – a particle with the dispersion $E = \frac{(|p|-p_0)^2}{2m}$ is in the potential $U = -\alpha/r$. Find high energy levels E with $l = 0$.

Problem 204. *II* ID:TM-QM-G-21

In what magnetic field the spins of all electrons in the atom S – atomic number 16 – are parallel? Consider all electrons are in the Coulomb potential of the nucleolus and do not interact with each other.

Problem 205. *II* ID:TM-QM-G-24

Find scattering cross-section for slow particles $ka \ll 1$ scattered by a potential $U = \begin{cases} 0, & r > a \\ U_0 a/r, & r < a \end{cases}$.

Problem 206. *III* ID:TM-QM-G-27
Find quasi-stationary levels and their widths for a particle in between a hard wall and a potential hump $U(x) = \frac{\hbar^2\kappa}{m}\delta(x)$, if a distance a between the wall and the hump is large $a\kappa \gg 1$.

Problem 207. *III* ID:TM-QM-G-30
A particle is in a shallow $1D$ potential well with a level at $E < 0$. The well is suddenly moved by the distance a. Find the probability for th particle to escape.

Problem 208. *III* ID:TM-QM-G-33
A charged particle is on a shallow level in a $3D$ potential well. Find its magnetic moment in a weak magnetic field.

Problem 209. *III* ID:TM-QM-G-36
A particle is close to a hard wall. Behind the wall at a distance R there is a source of attracting Coulomb's field acting on the particle $U = -\alpha/r$, for $\alpha/r \gg \hbar^2/mR^2$. Find the low energy levels.

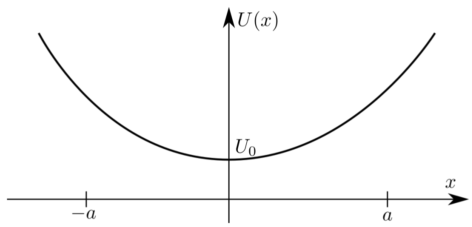
Problem 210. *IV* ID:TM-QM-G-39
Find at which α in a $3D$ potential well $U(r) = \begin{cases} \frac{m\omega^2 r^2}{2} - \alpha, & r < r_0 \\ 0, & r > r_0 \end{cases}$, where $r_0 \gg \sqrt{\frac{\hbar}{m\omega}}$ the first bound state appears.

Problem 211. *IV* ID:TM-QM-G-42
A meson can be found in two ground states — state 1, and state 2 — with amplitude (matrix element) K_{12} between them. In the state 1 the meson's half life time is T_1, in the state 2 the half life time is T_2, where $T_2 \gg 1/K_{12}$. Find the time dependence of the rate of decay products escape, if at $t = 0$ all mesons are in the state 1.

Problem 212. *IV* ID:TM-QM-G-45
In what magnetic field the spins of all electrons of an atom Ne in the ground state are parallel? Consider electrons as non-interacting fermions in the nucleus' Coulomb field.

Problem 213. *IV* ID:TM-QM-G-48
Using the Born approximation, find differential scattering cross section of rotons (particle with a dispersion relation $E = \frac{(|p|-p_0)^2}{2m}$) on a potential α/r.

Problem 214. *V* ID:TM-QM-G-51
A particle is in a $1D$ potential well given by $V(x) = U(x) - U(x - l)$, where $U(x)$ is shown in the figure. Fins the force of interaction between the well and the particle for $\frac{\hbar^2}{mal} \ll |U_0| \ll \frac{\hbar^2}{ml^2}$.



Problem 215. *V* ID:TM-QM-G-54

A flat rotator with moment of inertia I and dipole moment d is in electric field $\mathcal{E}$. The electric field is in the plane of the rotation. Find the dependence of the dipole moment on the electric field if $\mathcal{E} \gg \frac{\hbar^2}{Id}$.

Problem 216. *V* ID:TM-QM-G-57

Estimate the magnetic moment of the atom C in a weak magnetic field.

Problem 217. *V* ID:TM-QM-G-60

For a scattering of fast ($ka \gg 1$) particles flying along the axis on to the $2D$ potential barrier $U_0 e^{-\rho^2/a^2}$, where $\rho^2 = x^2+y^2$, find $f(\theta) - f(0)|_{\theta\to 0}$. Consider $E \gg U_0 \gg \frac{\hbar^2 k^2}{2m}$.

Problem 218. *VI* ID:TM-QM-G-63

Find the coefficient of reflection from $U(x) = \frac{U_0 a}{a+|x|}$.

Problem 219. *VI* ID:TM-QM-G-66

A flat rotator with moment of inertia I and dipole moment d is in electric field $\mathcal{E}$. The electric field is in the plane of the rotation. Find the dependence of the dipole moment on the electric field if $\mathcal{E} \ll \frac{\hbar^2}{Id}$.

Problem 220. *VI* ID:TM-QM-G-69

Find energy levels for a particle of mass m in a spherical hard shell for large orbital moments.

Problem 221. *VI* ID:TM-QM-G-72

Find the dependence of the scattering amplitude on the angle θ for scattering of slow, $ka \ll 1$, particles off a potential $U(r) = \begin{cases} U_0, & r < a \\ 0, & r > a \end{cases}$, for $U_0 \gg \frac{\hbar^2}{ma^2}$.

Problem 222. *VII* ID:TM-QM-G-75

A particle with spin $S = 1$ is in a state with $L = 1$. Find what is its total angular momentum if there is a weak spin-orbit interaction $\alpha(\vec{S}\cdot\vec{n})^2$; $\alpha < 0$; $\vec{n} = \vec{r}/|\vec{r}|$.

Problem 223. *VII* ID:TM-QM-G-78

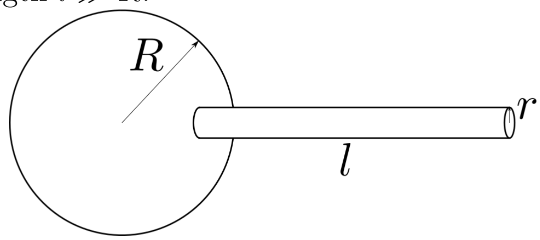
Two electrons are in a $3D$ potential αr. Find the ground state energy and energies of rotating states in the limit $0 < \alpha \ll 1$ (in atomic units).

Problem 224. *VII* ID:TM-QM-G-81

Find the probability $W(\omega)$ of escape for a particle in a bound state of a potential $-\frac{\hbar^2}{2m}\kappa\delta(x)$ under the action of a time depended field $\alpha x e^{-i\omega t}$.

Problem 225. *VII* ID:TM-QM-G-84

Find the width of the ground state level for a particle of mass m escaping from a hard spherical shell of radius R through a narrow hard channel of radius $r \ll R$ and length $l \gg R$.



Problem 226. *VIII* ID:TM-QM-G-87

A weak time dependent electric field $\mathcal{E}(t) = \mathcal{E}_0 \frac{\tau^2}{\tau^2+t^2}$ is acting on a charged $1D$ oscillator initially in the ground state. Find the probability to excite the oscillator.

Problem 227. *VIII* ID:TM-QM-G-90

Two particles are in a bound state with orbital angular momentum $l = 1$. Find out which projection of the orbital angular momentum of the axis z is more energetically favorable if the particles in addition to the bounding potential are weakly interacting as magnetic dipoles with magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$, directed along z.

Problem 228. *VIII* ID:TM-QM-G-93

In a potential $U(x) = \begin{cases} \alpha x, & x > 0 \\ \infty, & x < 0 \end{cases}$ find the average kinetic energy of a state n for $n \gg 1$.

Problem 229. *VIII* ID:TM-QM-G-96

Find the scattering cross-section for fast particles ($kr_0 \gg 1$) scattered off the potential $U(\vec{r}) = \lambda\delta(|\vec{r}| - r_0)$. Consider both cases:

1. $\lambda \gg \frac{\hbar^2}{m}k$;
2. $\lambda \ll \frac{\hbar^2}{m}k$.

Problem 230. *IX* ID:TM-QM-G-99

Two particles with spins $1/2$ are in a bound state with $S = 1$ and $L = 1$. What is the total angular momentum J if in addition to the bounding potential the particles weakly interact by a spin-orbit interaction $\alpha \hat{S}_{1i}\hat{S}_{ij}\left(\hat{L}_{1i}\hat{L}_{2j} + \hat{L}_{1j}\hat{L}_{2i}\right)$; $\alpha < 0$?

Problem 231. *IX* ID:TM-QM-G-102

Two electrons are in the $3D$ potential $\frac{m\omega^2 r^2}{2}$ and repel each other via a Coulomb interaction. Find the energy of the ground state and the energy of rotating states for $\omega \ll \frac{me^4}{\hbar^3}$.

Problem 232. *IX* ID:TM-QM-G-105

In the Born approximation find how the scattering amplitude for slow ($ka \ll 1$) particles on the potential $\frac{\gamma}{r^5+a^5}$ depends on the scattering angle.

Problem 233. *IX* ID:TM-QM-G-108

There is a weak attracting potential $U(x) = \begin{cases} -U_0, & r < a \\ 0, & r > a \end{cases}$, where $U_0 \ll \frac{\hbar^2}{ma^2}$. Find the energy of a bound state if the particles are in between two infinite hard walls with the distance $d \ll a$ between them.

Problem 234. *X* ID:TM-QM-G-111

Find the probability for the hydrogen atom to transition from the ground state to the first exited state after sudden turn on of a weak electric field.

Problem 235. *X* ID:TM-QM-G-114

Two particles with spins 1/2 and magnetic moments $\hat{\vec{\mu}}_i = g\hat{\vec{S}}_i$, where $i = 1, 2$, are at some distance form each other. The particles are interacting as magnetic dipole. Find the most energetically favorable spin configuration.

Problem 236. *X* ID:TM-QM-G-117

Find the density of states $\frac{\partial n}{\partial E}$ for $E \to 0$ in a potential well $U(x) = U_0 \begin{cases} 0, & |x| > a \\ -(1+\cos(\pi x/a)), & |x| < a \end{cases}$, for $U_0 \gg \frac{\hbar^2}{ma^2}$.

Problem 237. *X* ID:TM-QM-G-120

Find the scattering cross-section for slow ($kr_0 \ll 1$) particles on the potential $U(\vec{r}) = \lambda\delta(|\vec{r}| - r_0)$. Find out at what conditions the real or virtual state exists.

Problem 238. *XI* ID:TM-QM-G-123

A particle is in a bound state of a potential $U(x) = -\frac{\hbar^2}{m}\delta(x)$. A hard wall is slowly moved from $-\infty$ towards the well. Find at what distance between the wall and the well the particle will escape? Qualitatively draw a dependence of the force acting on the wall as a function of the distance between the wall and the well.

Problem 239. *XI* ID:TM-QM-G-126

A roton – a particle with the dispersion $E = \frac{(|p|-p_0)^2}{2m}$ is in the potential $U = -\alpha/r$. Find high energy levels E with $l = 0$.

Problem 240. *XI* ID:TM-QM-G-129

In what magnetic field the spins of all electrons in the atom S – atomic number 16 – are parallel? Consider all electrons are in the Coulomb potential of the nucleolus and do not interact with each other.

Problem 241. *XI* ID:TM-QM-G-132

Find scattering cross-section for slow particles $ka \ll 1$ scattered by a potential $U = \begin{cases} 0, & r > a \\ U_0 a^2/r^2, & r < a \end{cases}$.

Problem 242. *XII* ID:TM-QM-G-135

$N \gg 1$ fermions with dispersion relation $E = cp$ are in the oscillator's potential $U = \alpha r^2$. Find the Fermi energy.

Problem 243. *XII* ID:TM-QM-G-138

A $3D$ particle with anisotropic mass: $E(p) = \frac{p_z^2}{2m_1} + \frac{p_\perp^2}{2m_2}$, where $m_1 \gg m_2$ is in a Coulomb potential. Find the low energy levels.

Problem 244. *XII* ID:TM-QM-G-141

Find the splitting of the first excited state of the hydrogen atom under the perturbation $U = \alpha \frac{xy}{r^2}$.

Problem 245. *XII* ID:TM-QM-G-144

Find the ionization probability for a hydrogen atom when it suddenly converts into deuterium.

Problem 246. *XIII* ID:TM-QM-G-147

Find the correction to the ground state energy for an electron in the $3D$ oscillator's potential under a weak perturbation $\alpha \hat{\vec{s}} \cdot \hat{\vec{p}}$, where $\hat{\vec{s}}$ is the spin operator and $\hat{\vec{p}}$ is the momentum operator.

Problem 247. *XIII* ID:TM-QM-G-150

Find the degeneracy of the energy levels for a particle in the potential $U = \alpha(x^4 + y^4 + z^4) + \beta x^2 y^2 z^2$.

Problem 248. *XIII* ID:TM-QM-G-153

Find the scattering amplitude for a slow particle in $4D$ potential decaying as $1/r^4$, where $r^2 = x^2 + y^2 + z^2 + u^2$.

Problem 249. *XIII* ID:TM-QM-G-156

Find the low energy levels for an electron with large angular momentum $m \gg 1$ in a attractive field of an infinite charged line with linear charge density σ.

Problem 250. *XIV* ID:TM-QM-G-159

Find the splitting of the first excited energy level in a $3D$ oscillator potential under a weak perturbation $U = a\cos^2\theta$, where θ is the usual polar angle in the spherical coordinates.

Problem 251. *XIV* ID:TM-QM-G-162

Find the degeneracy of the energy levels for a particle in a potential $U = a(x^4 + y^4 + z^4) + b(x^2 + y^2 + z^2)$.

Problem 252. *XIV* ID:TM-QM-G-165

Find the low energy levels of a proton in a magnetic field $H = H_0 e^{-\alpha z^2}$ directed along z. Assume that $\frac{\alpha \hbar c}{e H_0} \ll 1$.

Problem 253. *XIV* ID:TM-QM-G-168

Find the scattering amplitude for a slow particle in $4D$ potential decaying as $1/r^3$, where $r^2 = x^2 + y^2 + z^2 + u^2$.

Problem 254. *XV* ID:TM-QM-G-171

Find energy levels for a particle in $1D$ potential $\frac{\hbar^2}{ma^2}\left(\frac{x}{a}\right)^{2n}$ for $n \gg 1$.

Problem 255. *XV* ID:TM-QM-G-174

Find the splitting of $2s$ and $2p$ levels in the hydrogen atom if the position of the proton is oscillating $\vec{r}(t) = a\left(\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t) + \hat{z}\cos(\omega t/2)\right)$, where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are coordinate unit vectors, $a \ll \frac{\hbar^2}{me^2}$, $\hbar\omega \gg \frac{me^4}{\hbar}$. Consider the proton as a point charge.

Problem 256. *XV* ID:TM-QM-G-177

Two particles of mass m and opposite charges e and $-e$ are separated by a flat infinite slab of thickness d. The particles cannot penetrate through this slab. Find the low energy levels if $d \gg \frac{\hbar^2}{me^2}$.

Problem 257. *XV* ID:TM-QM-G-180

Find the dependence of the transport cross-section $\sigma_{tr} = \int (1 - \cos\theta) d\sigma$ on energy for a particle with spin $S = 1/2$ scattered off a potential $U = U_0 e^{-r^2/a^2}\left(\vec{S}\cdot\hat{\vec{l}}\right)$, where $\hat{\vec{l}} = -i\vec{r} \times \frac{\partial}{\partial \vec{r}}$, $ka \gg 1$, $U_0 \ll \left(\frac{\hbar^2}{ma^2}\right) ka$.

Problem 258. *XVI* ID:TM-QM-G-183

Find the correction to the ground state energy of a hydrogen atom due to relativistic dependence of the electron's mass on velocity.

Problem 259. *XVI* ID:TM-QM-G-186

A particle is in a bound state in the potential $-\frac{\hbar^2}{m}\kappa\delta(x - a)$. Find the probability per unit time for the particle to transition to a state of the continuous part of the spectrum if the point a is oscillating according to $a = a_0 + b\cos(\omega t)$; $b \gg \kappa^{-1}$; $\hbar\omega \gg \frac{\hbar^2\kappa^2}{2m}$.

Problem 260. *XVI* ID:TM-QM-G-189

Two particles of equal masses m and charges e freely move on a circle of radius R. Find all low energy levels for $R \gg \frac{\hbar^2}{me^2}$.

Problem 261. *XVI* ID:TM-QM-G-192

Find the dependence of the differential cross-section on energy for a slow ($ka \ll 1$) particle with spin $S_1 = 1/2$ scattered off a particle with spin $S_2 = 1/2$, if they interact according to $U(\vec{r}) = \frac{U_0 a^5}{r^5+a^5}\left(\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2 - 3\frac{(\hat{\vec{S}}_1\cdot\vec{r})(\hat{\vec{S}}_2\cdot\vec{r})}{r^2}\right)$, where $U_0 \ll \frac{\hbar^2}{ma^2}$.

Problem 262. *XVII* ID:TM-QM-G-195
Find quasi-stationary levels and their widths for a particle in between a hard wall and a potential hump $U(x) = \frac{\hbar^2\kappa}{m}\delta(x)$, if a distance a between the wall and the hump is large $a\kappa \gg 1$.

Problem 263. *XVII* ID:TM-QM-G-198
A particle is in a shallow $1D$ potential well with a level at $E < 0$. The well is suddenly moved by the distance a. Find the probability for th particle to escape.

Problem 264. *XVII* ID:TM-QM-G-201
A charged particle is on a shallow level in a $3D$ potential well. Find its magnetic moment in a weak magnetic field.

Problem 265. *XVII* ID:TM-QM-G-204
A particle is close to a hard wall. Behind the wall at a distance R there is a source of attracting Coulomb's field acting on the particle $U = -\alpha/r$, for $\alpha/r \gg \hbar^2/mR^2$. Find the low energy levels.

Problem 266. *XVIII* ID:TM-QM-G-207
Electrons injected into liquid helium, form bubbles with many $N \gg 1$ electrons per bubble. Find the bubble's radius if the surface tension coefficient of the helium is σ. Assume that electrons cannot penetrate into helium. Neglect the Coulomb interaction between the electrons.

Problem 267. *XVIII* ID:TM-QM-G-210
Find the quasi-classical energy levels in $3D$ potential $U(r) = \alpha r^4$ for the particles of anisotropic mass $E(\vec{p}) = \frac{p_z^2}{2m_z} + \frac{p_\perp^2}{2m_\perp}$, where $m_z \ll m_\perp$.

Problem 268. *XVIII* ID:TM-QM-G-213
Find the splitting of the first excited level of the hydrogen atom under a weak perturbation $U = \alpha\hat{\vec{S}}\cdot\hat{\vec{L}}$.

Problem 269. *XVIII* ID:TM-QM-G-216
The nucleus of a hydrogen atom in the ground state captures two neutrons and suddenly becomes tritium. Find the probability for the electron to transition to the excited state.

Problem 270. *XIX* ID:TM-QM-G-219
A particle of mass m and charge e is in between two hard coaxial cylinders of radii R_1 and R_2, $(R_2 > R_1)$ Inside the cylinder R_1 there is a uniform magnetic field H parallel to the cylinders axis. Find how the energy of the ground state E depends on the magnetic field H.

Problem 271. *XIX* ID:TM-QM-G-222

A hydrogen atom is in uniform time-independent electric field $\mathcal{E}$. Find the ionization probability per unit time without using parabolic coordinate system.

Problem 272. *XIX* ID:TM-QM-G-225

An electron is in a magnetic field with vector potential $A_x = y^2$, $A_y = A_z = 0$. Find the dependence of its energy on x component of the momentum p_x for large positive (negative) p_x.

Problem 273. *XIX* ID:TM-QM-G-228

A vessel with atomic hydrogen is rotating with the angular velocity ω. Find the splitting of the level with $l = 1$. How the result will change if there is spin orbit interaction?

Problem 274. *XX* ID:TM-QM-G-231

How the $l = 2$ (or $l = 3$) level of a hydrogen atom will split in a weak crystal field with D_3 or C_4 symmetry?

Problem 275. *XX* ID:TM-QM-G-234

A particle is elastically scattered off a hydrogen atom which is in the excited state with global quantum number n. In the Born approximation find the differential scattering cross-section for scattering on small angles in the case of $n \gg 1$.

Problem 276. *XX* ID:TM-QM-G-237

Ignoring spin-orbit interaction find the splitting of energy levels of positronium in the weak magnetic field if ultra-fine interaction is given by $\alpha\hat{\vec{S}}_p \cdot \hat{\vec{S}}_e$.

Problem 277. *XX* ID:TM-QM-G-240

A hydrogen atom is in the excited state with large n and $L = 0$. Find how it's dia-magnetic susceptibility depends on n.

4 Problems from the entrance exams to Karen Ter-Martirosyan's group.

Problem 278. *#1* ID:TM-QM-T-3
Find the normalized wave function of a particle in a bound state with $l = 0$ in a square potential well of radius r_0 and depth V_0. Equation for the eigen values should be solved numerically. How one should increase V_0 while decreasing r_0 to zero so as to keep only one bound state. Find the wave function of that state explicitly.

Problem 279. *#2* ID:TM-QM-T-6
Find the normalized wave function of a particle in a bound state of arbitrary angular momentum l in a square potential well. Find the equation for the energy eigen values.

Problem 280. *#3* ID:TM-QM-T-9
Express scattering amplitude through $k\cot(\delta_l(E))$, where $k = |\vec{k}|$ and $\hbar\vec{k}$ is the momentum and $\delta_l(E)$ is the scattering phase shift.

Problem 281. *#4* ID:TM-QM-T-12
Find the scattering amplitude (or $k\cot(\delta_l(E))$) for a particle with given orbital momentum l scattered off a square potential well of radius r_0 and depth V_0.

Problem 282. *#5* ID:TM-QM-T-15
Using Born approximation, find the scattering amplitude and cross-section for a particle scattered off a square potential well of radius r_0 and depth V_0. What does Born's formula give for the scattering amplitude with given orbital momentum l?

Problem 283. *#6* ID:TM-QM-T-18
For a spherical potential well of radius r_0 (the potential is zero for $r > r_0$) but arbitrary shape find the scattering amplitude through the logarithmic derivative $d_l(E) = -\frac{R_l'(r)}{R_l(r)}\Big|_{r=r_0}$ of the radial part $R_l(r)$ of the wave function of a particle on the well's boundary $r = r_0$ for a state with a given l.

Problem 284. *#7* ID:TM-QM-T-21
For a spherical potential well of radius r_0 (the potential is zero for $r > r_0$) but arbitrary shape find the equation for the energies of the bound states through the logarithmic derivative $d_l(E) = -\frac{R_l'(r)}{R_l(r)}\Big|_{r=r_0}$ of the radial part $R_l(r)$ of the wave function of a particle on the well's boundary $r = r_0$ for a state with a given l. Show that this equation can be written as $\frac{1}{a_l(E)}$, where $a_l(E)$ is the scattering amplitude for the state with angular momentum l.

Problem 285. *#8* ID:TM-QM-T-24
Find the scattering length (i.e. scattering amplitude at $E = 0$) for a particle scattered off a square potential well of small radius. Find the scattering cross-section for $E = 0$. Can you express both these quantities through the energy E of the bound state?

Problem 286. *#9* ID:TM-QM-T-27

For the scattering of slow particles off the rectangular potential well of small radius find $k\cot(\delta_{l=0}(E))$ as a series of $k^2 = 2mE/\hbar^2$, i.e. $k\cot(\delta_{l=0}(E)) = -\frac{1}{a_0} + k^2\frac{R_0}{2} + \dots$. Find the coefficients a_0 and R_0 explicitly. Find how the scattering cross-section and the bound state energy are expressed through a_0 and R_0.

Problem 287. *#10* ID:TM-QM-T-30

Express the scattering cross-section for neutrons scattered off protons at low energies through the bound state energy of deuterium E. Assume that the interaction potential is a square potential of small radius. Do not take the spins into account.

Problem 288. *#11* ID:TM-QM-T-33

Express the scattering cross-section for neutrons scattered off protons at low energies through the bound state energy of deuterium E. Assume that the interaction potential is a square potential of small radius. Take the spins into account by the following procedure. Assume that $k\cot(\delta_0) = -\frac{1}{a_0} + \frac{1}{2}R_0k^2$, where the constants a_0 and R_0 are given in triplet and singlet states. (Also find the constraints on these constants that arise if the bounding energy of deuterium $\mathcal{E}$ is given)

Problem 289. *#12* ID:TM-QM-T-36

Find how the constants a_0 and R_0 ($k\cot(\delta_0) = -\frac{1}{a_0} + \frac{1}{2}R_0k^2$) is connected with the logarithmic derivative $\alpha_0(E)$ of the radial part of the wave function with small energy E in a potential of arbitrary shape, but such that it is zero for $r > r_0$.

Problem 290. *#13* ID:TM-QM-T-39

Find the wave functions of the bound states with $l = 0$ and the scattering phase shift $\delta_0(E)$ for a particle in the potential $V = -\frac{V_0}{1+e^{\frac{r-r_0}{a}}}$, where V_0, r_0, and a are given constants. (This potential is called Woods–Saxon potential.)

Problem 291. *#14* ID:TM-QM-T-42

Fin the probability that a free particle in the state with the orbital moment l is at distance in the interval between r and $r + dr$ from the origin and its momentum is along $\vec{n}_0(\theta, \phi)$, i.e. the direction of the momentum is in the interval $d\Omega$ of the solid angle around the direction given by $\vec{n}_0$.

Problem 292. *#15* ID:TM-QM-T-45

Find the Fourier coefficients $b(\vec{k}', \vec{k}_0)$ for the spherical wave $\frac{e^{ik_0r}}{r} = \int \frac{d^3\vec{k}'}{(2\pi)^3} b(\vec{k}', \vec{k}_0) e^{i\vec{k}'\cdot\vec{r}}$. Using the result prove that $\int e^{i\vec{k}'\cdot\vec{r}} b(\vec{k}', \vec{k}_0) \frac{d^3\vec{k}'}{(2\pi)^3}\Big|_{r\to 0} \approx \frac{b(|\vec{k}_0|\vec{n}, \vec{k}_0)}{r} e^{i\vec{k}_0\cdot\vec{r}}$, where $\vec{n} = \vec{r}/r$.

Problem 293. *#16* ID:TM-QM-T-48

Using the result of the problem # 15 and writing the wave function as $\psi(\vec{r}) = \int \phi(\vec{k}) e^{i\vec{k}\cdot\vec{r}} \frac{d^3\vec{k}}{(2\pi)^3}$, and $\phi(\vec{k}) = \phi_0(\vec{k}) + b(\vec{k}, \vec{k}_0)$, where $\phi_0(\vec{k})$ is the solution for the free motion, find what the coefficients $b(\vec{k}, \vec{k}_0)$ must be in order for the wave function $\psi(\vec{r})$ to be the solution of the Schrödinger equation.

Problem 294. *#17* ID:TM-QM-T-51
Choosing $\psi_0(\vec{r})$ such that it satisfies the boundary condition at $r \to \infty$, and using the result of the problem #16, write the Schrödinger equation in the integral form for

1. a bound state;
2. a state which describes the scattering.

Problem 295. *#18* ID:TM-QM-T-54
Using the Schrödinger equation in the integral form from the problem #17 obtain the integral equation for the scattering amplitude $a(\vec{k}', \vec{k})$, where $\frac{d\sigma}{d\Omega} = |a(\vec{k}, \vec{k}')|^2$, for $|\vec{k}| = |\vec{k}'|$.

Problem 296. *#19* ID:TM-QM-T-57
Using the iteration procedure for the integral equation for the scattering amplitude form from the problem #18 find the scattering amplitude as a series in the powers of the potential – the series of Born's approximations.

Problem 297. *#20* ID:TM-QM-T-60
In the first Born approximation find the amplitude and the cross-section for scattering on the potentials:

1. Yukawa, $V(r) = \frac{g^2}{r} e^{-\rho r}$;
2. Gauss, $V = V_0 e^{-\rho^2 r^2}$.

For which of these potentials the angle distribution is narrower at large energies?

Problem 298. *#21* ID:TM-QM-T-63
In the second Born approximation find the scattering amplitude for scattering on the Yukawa potential $V(r) = \frac{g^2}{r} e^{-\rho r}$.

Problem 299. *#22* ID:TM-QM-T-66
Find the form of the scattering wave function in momentum representation. (You need to Fourier transform the integral equation of problem #16.)

Problem 300. *#23* ID:TM-QM-T-69
Write the Schrödinger equation which describes the scattering in the momentum representation. (The potential $\hat{V}$ is an operator and is given by its matrix elements $V(\vec{r}', \vec{r})$ – the operator kernel) For the usual local potential these matrix elements are given by $V(\vec{r}', \vec{r}) = V(\vec{r})\delta(\vec{r}' - \vec{r})$.

Problem 301. *#24* ID:TM-QM-T-72

Solve the equation obtained in the problem #23 for a particular case of $V(\vec{r}', \vec{r}) = f(\vec{r}')f(\vec{r})$, where $f(\vec{r})$ is some function decaying at $r \to \infty$. (This is so-called separable potential.)

Problem 302. *#25* ID:TM-QM-T-75

Find the scattering amplitude and cross-section for scattering off a sphere, i.e. $V(r) = V_0\delta(r-r_0)$. Solve this problem by two different methods: in $\vec{r}$ and in $\vec{p}$ representations. Consider the sates with different l.

Problem 303. *#26* ID:TM-QM-T-78

How does the scattering amplitude for the scattering potential from the problem #25 $V(r) = V_0\delta(r-r_0)$ for the states of different l depend on the particle's energy E, for $E \to 0$.

Problem 304. *#27* ID:TM-QM-T-81

Find the energy levels and wave functions of bound states of a particle in the field produced by two centers at $\vec{R}_1 = \frac{\vec{R}}{2}$ and $\vec{R}_2 = -\frac{\vec{R}}{2}$ (deep, small potentials with radii $r_0 \to 0$). The interaction with each center is defined by the quantity $\alpha_i = -\left.\frac{(\rho_i\psi(\vec{r}))'}{\rho_i\psi(\vec{r})}\right|_{\vec{r}\to\vec{R}_i}$, where $\rho_i = |\vec{r} - \vec{R}_i|$ — the logarithmic derivative of the wave function on the well's boundary. How does the energy of the bound states depend on the distance R between the wells, for $R \to 0$? Consider both identical and different wells.

Problem 305. *#28* ID:TM-QM-T-84

Repeat of the problem #27.

Problem 306. *#29* ID:TM-QM-T-87

Find the scattering amplitude and the scattering cross-section for scattering of the potential of the problem #27: the potential is produced by two centers at $\vec{R}_1 = \frac{\vec{R}}{2}$ and $\vec{R}_2 = -\frac{\vec{R}}{2}$ (deep, small potentials with radii $r_0 \to 0$). The interaction with each center is defined by the quantity $\alpha_i = -\left.\frac{(\rho_i\psi(\vec{r}))'}{\rho_i\psi(\vec{r})}\right|_{\vec{r}\to\vec{R}_i}$, where $\rho_i = |\vec{r} - \vec{R}_i|$ — the logarithmic derivative of the wave function on the well's boundary. Average the cross-section over the direction of the vector $\vec{R}$ – the vector between the two centers, and over the directions of $\vec{k}$.

Problem 307. *#30* ID:TM-QM-T-90

Find the shift and the splitting of the energy levels of the system proton-antiproton due to nuclear interaction. Assume, that the nuclear interaction is defined by the logarithmic derivative of the wave function on the well's boundary ($\left.\frac{(r\psi)'}{r\psi}\right|_{\vec{r}\to\vec{\rho}} = C$, for $\rho_i \ll \frac{\hbar^2}{me^2}$). Use

1. the perturbation theory;
2. exact solution of the Schrödinger equation.

Problem 308. *#31* ID:TM-QM-T-93

Find the photo-effect cross-section on the hydrogen atom. Assume, that the electromagnetic wave is a small perturbation.

Problem 309. *#32* ID:TM-QM-T-96

Find the elastic cross-section, the cross-section of exciting, and the cross-section of ionizing a hydrogen atom by the stream of fast electrons. Consider the interaction of the stream and the hydrogen electron as a perturbation.

Problem 310. *#33* ID:TM-QM-T-99

Find the probability for a neutron to be captured by protons in a film of liquid hydrogen of given thickness. The capture is happening due to the process $n + p \to d + \gamma$. (More difficult problem is to find the probability of the conversion of the γ quantum on the shell's electron.)

Problem 311. *#34* ID:TM-QM-T-102

Neutron is scattered by a nucleus's field of the form $V = V_0(r) + \frac{e}{r^2}\frac{\partial V(r)}{\partial r} i(\hat{\vec{\sigma}} \cdot \hat{\vec{L}})$, where $\hat{\vec{L}} = \vec{r} \times \hat{\vec{p}}$ – orbital momentum and $V_0(r) = V_1(r) + iV_2(r)$ – is some optical potential. Find the polarization of the scattered neutrons in the Born approximation.

Problem 312. *#35* ID:TM-QM-T-105

Find the scattering amplitude for neutron scattered by a Coulomb field. The scattering happens due to interaction of the Coulomb field with the field of the magnetic moment of the neutron.

Problem 313. *#36* ID:TM-QM-T-108

Find the scattering amplitude for neutron scattered by a Coulomb field. The scattering happens due to interaction of the Coulomb field with the field of the magnetic moment of the neutron. Take into account also that the neutron is scattered by the nucleus at the center of the field with a given amplitude $a_0 = \text{const}$.

Problem 314. *#37* ID:TM-QM-T-111

A neutron flies by a nucleus. Assuming that the interaction between the neutron and the nucleus is weak find in the first Born approximation the inelastic scattering cross-section of a neutron on a deuterium for which the deuterium breaks apart. (The neutron-proton interaction after breaking must be taken into account exactly)

Problem 315. *#38* ID:TM-QM-T-114

Find the photo-effect cross-section for the light on q deuterium. The light interacts with the deuterium due to its electric and magnetic dipole moments.

Problem 316. *#39* ID:TM-QM-T-117

Find the depolarization of a μ-meson. The μ-meson initially is completely polarized along the magnetic field $\vec{H}$ in muonium, which is created in a target bombardment.

Problem 317. *#40* ID:TM-QM-T-120
Find the energy and the wave function of the bound states of a particle in the $3D$ potential $V(r) = -C\delta(r - R_0)$, where $C > 0$. Obtain the scattering amplitude for this potential.

Problem 318. *#41* ID:TM-QM-T-123
Construct spin and isospin wave function for three nucleons (in particular for the case of tritium). Represent the full wave function as a product of radial, spin, and isospin parts. What are the symmetry properties of the radial part?

Problem 319. *#42* ID:TM-QM-T-126
A deuterium with large energy (non-relativistic) while flying by a nucleus suddenly loses neutron. Find the angle and energy distribution of the resulting protons in the laboratory frame of reference. Assume that the kinetic energy E_d of the deuterium, the direction of its motion, and its binding energy ϵ_d are known.

Problem 320. *#43* ID:TM-QM-T-129
A nucleus in an excited state is in the ground state of an oscillation potential in a molecule. Find the probability that the nucleus remains in the same ground state after emitting a photon of given energy.

Problem 321. *#44* ID:TM-QM-T-132
A free particle is in a state with given orbital moment l and m. Find the probability to find this particle at the distance r from the coordinate origin with a specific direction of the momentum $\vec{p}$.

Problem 322. *#45* ID:TM-QM-T-135
Find the probability of a decay of ion $H^3 \to He^3 + e^- + \tilde{\nu}$ with capturing the electron to one of the Coulomb level of the nucleus He^3. To which level the electron is most likely to be captured? How the capture probability depends on the quantum numbers of the capturing level? Assume that the matrix element of β-decay $\frac{G}{\sqrt{2}} = \langle He^3, e^-, \tilde{\nu}|\hat{V}|H^3\rangle$ is a known constant, and the total energy of decay $\epsilon_0 = M_{H^3} - M_{He^3} \approx 17$KeV is known.