## EXAM III Physics 218

Name...................................................Section Number......

## USEFUL INFORMATION

$$
\begin{gathered}
\text { If } f(x)=k x^{n} \quad \frac{d f}{d x}=n k x^{n-1} \\
\text { If } f(x)=k x^{n} \quad \int_{A}^{B} f(x) d x=\frac{1}{n+1} k\left(B^{n+1}-A^{n+1}\right) \\
\int_{\vec{r}_{1}}^{\vec{r}_{2}} \vec{F}_{t o t} \cdot d \vec{r}=\frac{1}{2} m v^{2}\left(\vec{r}_{2}\right)-\frac{1}{2} m v^{2}\left(\vec{r}_{1}\right)
\end{gathered}
$$

If $\vec{F}$ is conservative:

$$
\int_{\vec{r}_{1}}^{\vec{r}_{2}} \vec{F} \cdot d \vec{r}=-\left[U\left(\vec{r}_{2}\right)-U\left(\vec{r}_{1}\right)\right]
$$

and

$$
\begin{array}{cc}
F_{x}=-\frac{\partial U}{\partial x} & F_{y}=-\frac{\partial U}{\partial y} \\
\vec{L}=\vec{r} \times \vec{p} & \vec{\tau}=\vec{r} \times \vec{F} \\
& I=\sum m_{i} r_{2}^{2} \\
& 1 . \\
& 2 . \\
& 4 \\
& \\
& \\
& \\
& \\
&
\end{array}
$$

1. (25 points) Derive the expressions for the $\vec{i}_{r}$ and $\vec{i}_{\theta}$ components of the velocity and acceleration.
2. (25 points) A vertical axle is free to rotate. A massless rod of length $S$ is attached to the axle, as shown, and a small wooden block of mass $m_{1}$ is attached to the rod. A bullet, mass $m_{2}$, is shot at the block with velocity of magnitude $v_{1}$ at the angle $\phi$ as shown. The bullet slows down inside the block, finally coming out in the direction shown with $\frac{1}{4}$ of its initial kinetic energy when the block has rotated through the angle $\frac{\pi}{2}$. What will be the angular velocity of the block after the bullet emerges? (Ignore gravity in this problem.)


What would be the angular velocity of the block after the bullet emerges if instead of a massless rod, the rod had moment of inertia $I$ about the axle?
3. (25 points) A small object of mass $m$ is on a frictionless surface. Because of a complicated force that acts on the object it moves so that its distance from the origin is observed to be $h \cos \theta$ where $h$ is known and $\theta$ is the angle shown below. The angle $\theta$ is observed to vary with time according to $\theta(t)=c_{1} t^{3}$ where $c_{1}$ is known. Consider the motion only until the object reaches the origin.

a. Find the object's velocity as a function of time.
b. Find the torque about the origin that is exerted on the object as a function of time.
4. (25 points) A block is moving with a known velocity of magnitude $V$ along the x axis on a frictionless table. It explodes into three pieces that remain in the plane of the table. The pieces have masses $m_{1}, m_{2}$, and $m_{3}$ and the first two pieces have velocities $v_{1}$ and $v_{2}$ as shown, with $\theta$ a known angle. Find the direction of the velocity of the third piece.


