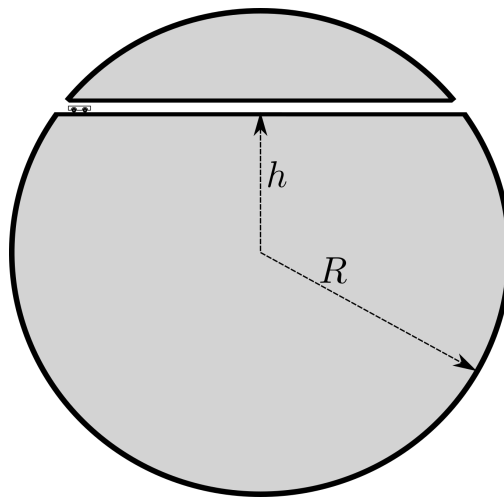


# EXAM 1. Take home. Monday, July 8, 2019, return by Wednesday, July 10, 2:00pm

## Problem 1. *A Cart in the Earth*

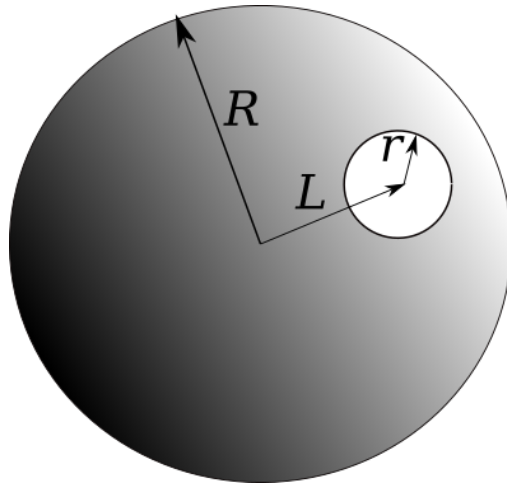
1. A straight narrow shaft is drilled in the Earth as shown in the figure. What time will it take for a cart to travel from one end to another with zero initial velocity? Neglect air resistance, friction, and Earth rotation. Take the Earth to be a uniform sphere. Express your answer through the acceleration of free fall  $g$  and Earth radius  $R$  and length  $h$ .
2. How does this time depend on  $h$ ?



## Problem 2. *Sphere with a cavity*

A uniformly charged sphere of radius  $R$  (not a spherical shell, but the whole sphere) with volume charge density  $\rho$  has an empty spherical cavity of radius  $r$ , as shown. The distance  $L$  is known.

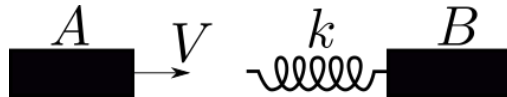
1. Find electric field  $\vec{E}$  at the center of the cavity (both magnitude and direction.)
2. Find the electric field in the arbitrary point inside the cavity (you need to introduce a system of coordinates of your choice).



**Problem 3.** *Bricks and spring*

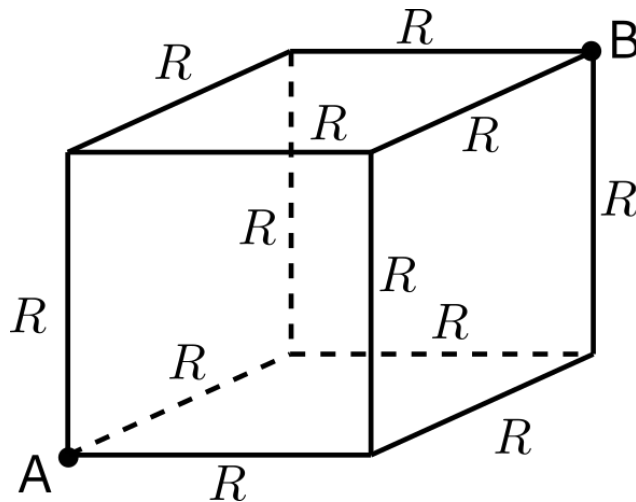
A brick  $A$  of mass  $m$  flying with the velocity  $V$  through space (no gravity, no air) hits head on an identical free brick  $B$ . The brick  $B$  has a spring of spring constant  $k$  attached to it. When the brick  $A$  touches the spring it gets glued to it.

1. What will be the frequency of the oscillations of one brick with respect to another?
2. What will be the amplitude of these oscillations.



**Problem 4.** *Cube of resistors.*

A cube is made of metal wire. The resistance of each link is  $R$ . Find the resistance between the points  $A$  and  $B$ .

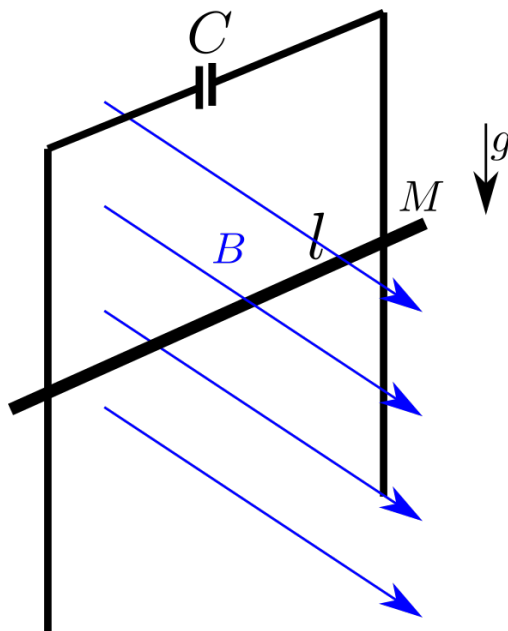


## EXAM 2. Final. Tuesday, August 6, 2019, 3:30pm-5:30pm

### Problem 1. Vertical Rails with capacitor

Long enough vertical rails distance  $l$  from each other made of an ideal conductor are connected by a capacitor  $C$ . A bar of mass  $M$ , also made of an ideal conductor, can slide along the rails without friction and without losing electrical contact with the rails. A uniform magnetic field  $B$  is horizontal and perpendicular to the plane of the rails. At some point the bar was released with zero initial velocity. Initial charge on the capacitor is zero.

Find the acceleration of the bar.



### Problem 2. Wave function and expectations

For a particle in a quantum mechanical state given by a wave function  $\psi(x) = Ce^{-|x|/\lambda}$  find:

1. The normalization constant  $C$ .
2. The probability to find the particle somewhere in the interval between  $-\lambda$  and  $\lambda$ .
3. The average position  $\bar{x}$ .
4. The standard deviation from the average position  $\Delta x = \sqrt{x^2 - \bar{x}^2}$ .

*Hint: you might need the following integrals:  $\int_0^\infty e^{-x} dx = 1$ ,  $\int_0^1 e^{-x} dx = 1 - e^{-1}$ , and  $\int_0^\infty x^2 e^{-x} dx = 2$ .*

**Problem 3.** *Electron-positron annihilation*

An electron (rest mass  $m_e$ ) of total energy  $E$  makes a head-on collision with a positron, which is initially at rest. Positron is electron's antiparticle, it has the same mass as electron, but opposite charge. In collision the two particles annihilate each other and are replaced by two photons ( $\gamma$  rays) of equal energy, each traveling at equal angles  $\theta$  with electron's direction of motion. Find

1. The energy  $\epsilon$  of each photon.
2. The momentum  $p$  of each photon.
3. The angle  $\theta$ .

