Homework 1. Due Sept. 5

Problem 1. *From Classical Mechanics by J. R.Tylor*
Problems: 1.6, 1.8, 1.10, 1.17, 1.19, 1.23

Homework 2. Due September 12.

Problem 1. *Rising Snake*
A snake of length L and linear mass density ρ rises from the table. It's head is moving straight up with the constant velocity v. What force does the snake exert on the table?

Problem 2. *1983-Spring-CM-U-1.*
A ball, mass m, hangs by a massless string from the ceiling of a car in a passenger train. At time t the train has velocity $\vec{v}$ and acceleration $\vec{a}$ in the same direction. What is the angle that the string makes with the vertical? Make a sketch which clearly indicates the relative direction of deflection.

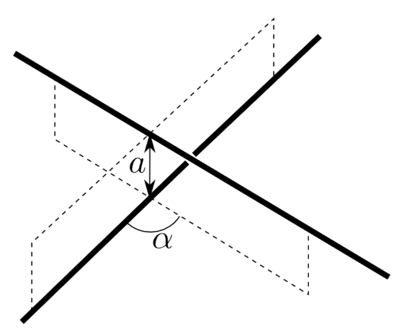
Problem 3. *1984-Fall-CM-U-1.*
Sand drops vertically from a stationary hopper at a rate of 100 gm/sec onto a horizontal conveyor belt moving at a constant velocity, $\vec{v}$, of 10 cm/sec.

1. What force (magnitude and direction relative to the velocity) is required to keep the belt moving at a constant speed of 10 cm/sec?
2. How much work is done by this force in 1.0 second?
3. What is the change in kinetic energy of the conveyor belt in 1.0 second due to the additional sand on it?
4. Should the answers to parts 2. and 3. be the same? Explain.

Problem 4. *1994-Fall-CM-U-1*
Two uniform very long (infinite) rods with identical linear mass density ρ do not intersect. Their directions form an angle α and their shortest separation is a.

1. Find the force of attraction between them due to Newton's law of gravity.
2. Give a dimensional argument to explain why the force is independent of a.
3. If the rods were of a large but finite length L, what dimensional form would the lowest order correction to the force you found in the first part have?

Note: for $A^2 < 1$, $\int_{-\pi/2}^{\pi/2} \frac{d\theta}{1-A^2\sin^2\theta} = \frac{\pi}{\sqrt{1-A^2}}$

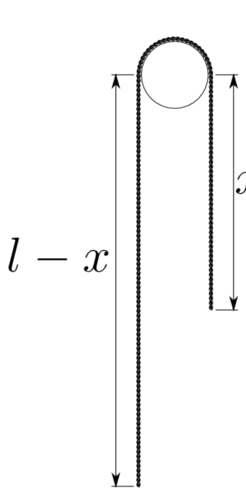


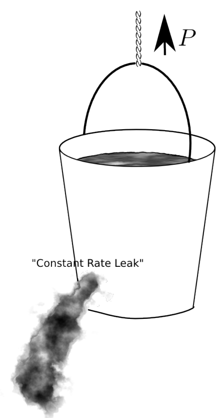
Homework 3. Due September 19.

Problem 1. *Height for quadratic friction.*
A body of mass m was thrown straight up with initial velocity v_0. The force of the air resistance is $-\gamma v|v|$

- How long it will take for the body to reach the top of its trajectory?
- What the answer will be if $\gamma \to 0$.
- How long it will take for the body to reach the top of its trajectory if initial velocity is infinite?
- For finite initial velocity, how far up the body will go?
- What the answer will be if $\gamma \to 0$.

[Hint: $\int \frac{dx}{x^2+1} = \tan^{-1}(x)$, $\int \tan(x)dx = -\log(\cos(x))$, $\cos(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$.]





Problem 2. *Hole of Earth.*
Assume that the earth is a sphere, radius R and uniform mass density, ρ. Suppose a shaft were drilled all the way through the center of the earth from the north pole to the south. Suppose now a bullet of mass m is fired from the center of the earth, with velocity v_0 up the shaft. Assuming the bullet does not go beyond the earths surface and neglecting the air resistance

- calculate in what time the bullet will return back?
- how does this time depend on v_0?
- how does this time depend on m?

Problem 3. *Rope*

A simple Atwoods machine consists of a heavy rope of length l and linear density ρ hung over a pulley (see Fig.). Neglecting the part of the rope in contact with the pulley, find $x(t)$ if the initial conditions are $x(t=0) = l/2$ and $v(t=0) = v_0$.

Problem 4. *Bucket*

A weightless bucket which initially contains mass m_0 of water is being drawn up a well by a rope which exerts a steady force $P = m_0 g$ on the bucket. Initially the bucket is at rest. The water leaks out of the bucket at a constant rate, so that after a time T, before the bucket reaches the top, the bucket is empty. Find the velocity of the bucket just at the instant it becomes half empty. Express your answer in terms of T, and g, the acceleration due to gravity.

Problem 5. Extra for Honors

Plot coordinate vs. time for an overdamped oscillator with initial conditions $x(t=0) = 0$, $v(t=0) = v_0$.

- What is the slope of the graph at $t = 0$?
- What is the dependence of coordinate on time at $t \to \infty$?
- Find the general solution for the damped oscillator in the case $\gamma = \omega_0$. Hint: The solution must still have two independent constants.

Homework 4. Due September 26.

Problem 1. *From Classical Mechanics by J. R.Tylor*
Problem 5.44

Problem 2. *1985-Spring-CM-U-3.*
A damped one-dimensional linear oscillator is subjected to a periodic driving force described by a function $F(t)$. The equation of motion of the oscillator is given by

$$m\ddot{x} + b\dot{x} + kx = F(t),$$

where $F(t)$ is given by

$$F(t) = F_0\left(1 + \sin(\omega t)\right).$$

The driving force is characterized by $\omega = \omega_0$ and the damping by $\phi = b/2m = \omega_0$, where $\omega_0^2 = k/m$. At $t = 0$ the mass is at rest at the equilibrium position, so that the initial conditions are given by $x(0) = 0$, and $\dot{x}(0) = 0$. Find the solution $x(t)$ for the position of the oscillator vs. time.

Problem 3. *1994-Fall-CM-U-2*
Suppose that the payload (e.g., a space capsule) has mass m and is mounted on a two-stage rocket (see figure below). The total mass — both rockets fully fueled, plus the payload — is Nm. The mass of the second-stage plus the payload, after first-stage burnout and separation, is nm. In each stage the ratio of burnout mass (casing) to initial mass (casing plus fuel) is r, and the exhaust speed is v_0.

1. Find the velocity v_1 gained from first-stage burn starting from rest (and ignoring gravity). Express your answer in terms of v_0, N, n, and r.

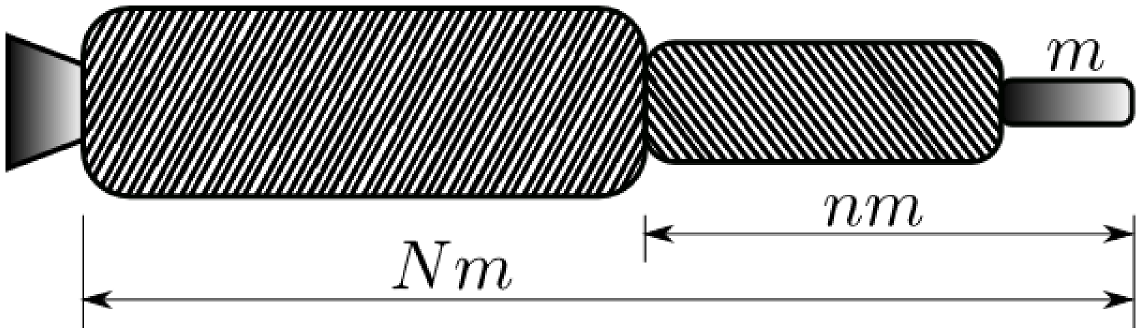
2. Obtain a corresponding expression for the additional velocity, v_2 gained from the second-stage burn.

3. Adding v_i and v_2, you have the payload velocity v in terms of N, n, v_0, and r. Taking N, v_0, and r as constants, find the value for n for which v is a maximum.

4. Show that the condition for v to be a maximum corresponds to having equal gains of velocity in the two stages. Find the maximum value of v, and verify that it makes sense for the limiting cases described by $r = 0$ and $r = 1$.

5. You need to build a system to obtain a payload velocity of 10km/sec, using rockets for which $v_0 = 2.5$km/sec and $r = 0.1$. Can you do it with a two-stage rocket?

6. Find an expression for the payload velocity of a single-stage rocket with the same values of N, r, and v_0. Can you do it with a single-stage rocket by taking the same conditions as in the previous point?



Problem 4. *Hovering rocket*

How the rocket should burn its fuel (what is $m(t)$) in order to hover in the Earth gravitational field close to the Earth surface.

Problem 5. **Extra for Honors**

A small rocket burns fuel according to the law $m(t) = m_0 e^{-\alpha t}$. It starts from the ground with no initial velocity and flies straight up.

- How large α must be in order for the rocket to take off.
- Find how high the rocket is at time t if the air resistance is $-\gamma v$.
- What is the maximum velocity the rocket will have? At what time after the start will it happen?

Homework 5. Due October 3.

Problem 1. *Tensor of inertia.*
In the frame where the tensor of inertia is diagonal calculate it for:

1. A uniform disc of mass m and radius R.
2. A uniform solid (not hollow) sphere of mass m and radius R.
3. A thin stick of length L and mass m, if the origin is at one end of the stick.
4. A thin stick of length L and mass m, if the origin is at the center of the stick.

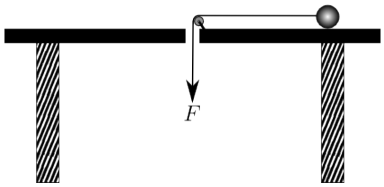
Problem 2. *1989-Spring-CM-U-2.*
A platform is free to rotate in the horizontal plane about a frictionless, vertical axle. About this axle the platform has a moment of inertia I_p. An object is placed on a platform a distance R from the center of the axle. The mass of the object is m and it is very small in size. The coefficient of friction between the object and the platform is μ. If at $t = 0$ a torque of constant magnitude τ_0 about the axle is applied to the platform when will the object start to slip?

Problem 3. *1990-Spring-CM-U-1.*
A particle of mass m and charge e is moving in an electric field $\vec{E}$ and a magnetic field $\vec{H}$.

1. What is the general formula for the force acting on a charged particle by the fields? Write down the vector equation of motion for this charged particle in three dimensions.
2. Let $\vec{E} = 0$, and $\vec{B}$ be uniform in space and constant in time. If the particle has an initial velocity which is perpendicular to the magnetic field, integrate the equation of motion in component form to show that the trajectory is a circle with radius $r = v/\omega$, where $\omega = eB/m$, (or eB/mc if you work in the Gaussian unit system).
3. Now suppose that $\vec{E} = (0, E_y, E_z)$, and $\vec{B} = (0, 0, B)$ are both uniform in space and constant in time, integrate the equation of motion in component form again, assuming that the particle starts at the origin with zero initial velocity.

Problem 4. *A ball on a string.*
A ball is on a frictionless table. A string is connected to the ball as shown in the figure. The ball is started in a circle of radius R with angular velocity ω_0. The force exerted on a string is then increases so that the distance between the hole and the ball decreases and is a given function of time $r(t)$. Assuming the string stays straight and that it only exerts a force parallel to its length.

1. Find the velocity of the ball as a function of time.
2. What is polar ϕ component of the force must be in order for this motion to be possible?



Extra for Honors

Problem 5. *1990-Spring-CM-U-2.*

A uniform rod, of mass m and length $2l$, is freely rotated at one end and is initially supported at the other end in a horizontal position. A particle of mass αm is placed on the rod at the midpoint. The coefficient of static friction between the particle and the rod is μ. If the support is suddenly removed:

1. Calculate by what factor the reaction at the hinge is instantaneously reduced?

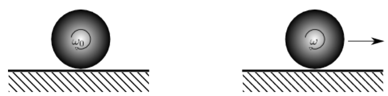
2. The particle begins to slide when the falling rod makes an angle θ with the horizontal. Calculate this angle.

   [**Hint:** To do this part, you need both components of the force equation on the mass αm; plus the torque equation and the equation for the conservation of the energy of the whole system.]

Homework 6. Due October 10.

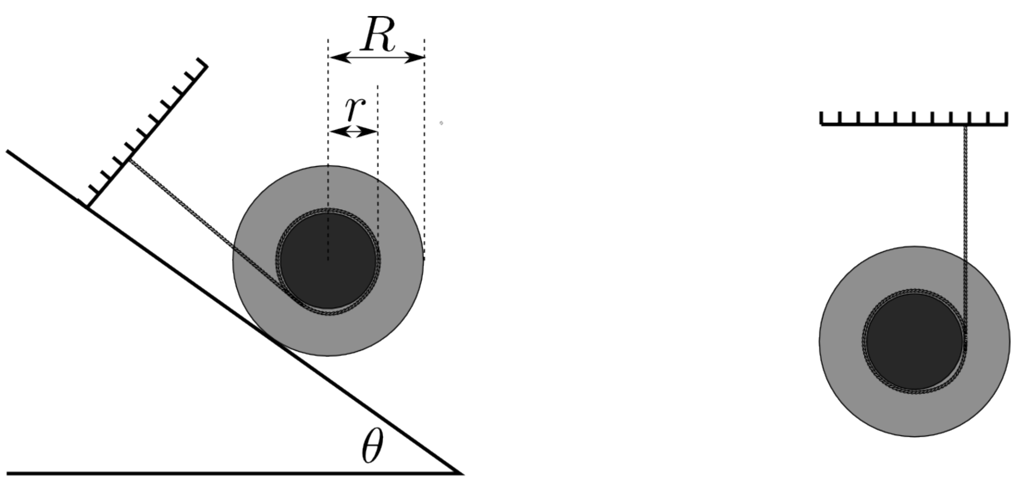
Problem 1. *1991-Spring-CM-U-3.*
A uniform sphere with a mass M and radius R is set into rotation with a horizontal angular velocity ω_0. At $t = 0$, the sphere is placed without bouncing onto a horizontal surface as shown. There is friction between the sphere and the surface. Initially, the sphere slips, but after a time T, it rolls without slipping.

1. What is the angular speed of rotation when the sphere finally rolls without slipping at time T?
2. How much energy is lost by the sphere between $t = 0$ and $t = T$?
3. Show that amount of energy lost is equal to the work done against friction causing the sphere to roll without slipping?



Problem 2. *1993-Fall-CM-U-2.*
A "Yo-Yo" (inner and outer radii r and R) has mass M and moment of inertia about its symmetry axis Mk^2.

1. The Yo-Yo is attached to a wall by a massless string and located on the slope as shown in Fig. a). The coefficient of static friction between the slope and Yo-Yo is μ. Find the largest angle θ before the Yo-Yo starts moving down.
2. Now, consider Yo-Yo which is falling down as shown in Fig. b). Find the ratio of its translational and rotational kinetic energies.

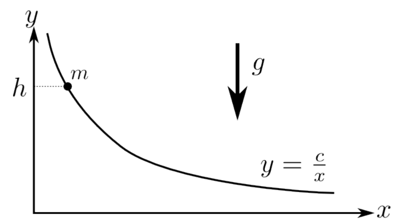


a) b)

Problem 3. *1994-Spring-CM-U-3*

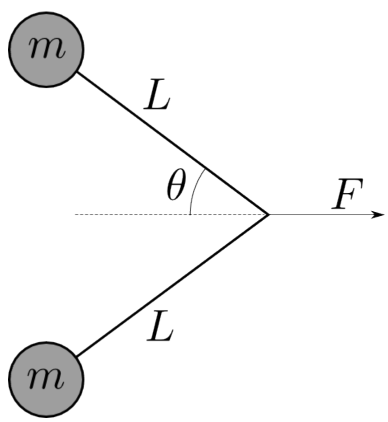
A wire has the shape of a hyperbola, $y = c/x$, $c > 0$. A small bead of mass m can slide without friction on the wire. The bead starts at rest from a height h as shown in the figure

1. Find the velocity vector $\vec{v}$ of the bead as a function of x.
2. Find the force $\vec{F}$ that the bead exerts on the wire as a function of x.



Problem 4. *1996-Fall-CM-U-1*

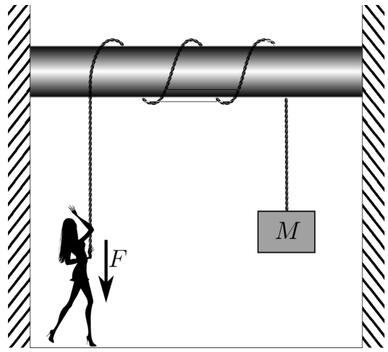
Two pucks, each of mass m, are connected by a massless string of length $2L$ as illustrated below. The pucks lie on a horizontal, frictionless sheet of ice. The string is initially straight (*i.e.*, $\theta = 90^o$). A constant, horizontal force F is applied to the middle of the string in a direction perpendicular to the line joining the pucks. When the pucks collide, they stick together. How much mechanical energy is lost in the collision?



Extra for Honors

Problem 5. *1994-Spring-CM-U-4*

A person wants to hold up a large object of mass M by exerting a force F on a massless rope. The rope is wrapped around a fixed pole of radius R. The coefficient of friction between the rope and the pole is μ. If the rope makes $n+1/2$ turns around the pole, what is the maximum weight the person can support?

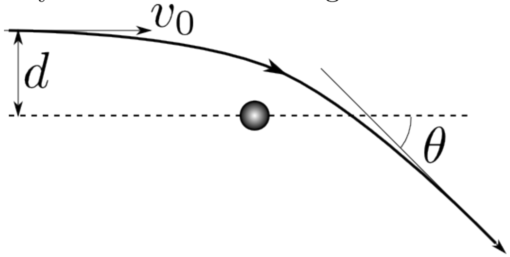

F
M

Homework 7. Due October 17.

Problem 1. *Kepler orbits.*
The shape of the Kepler's orbits (bounded and unbounded) is defined by its eccentricity ϵ. Find the shape (its equation in the Cartesian coordinates) for

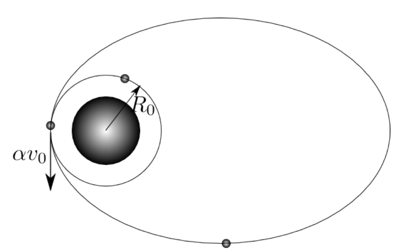
1. $\epsilon = 0$.
2. $0 < \epsilon < 1$.
3. $\epsilon = 1$. What is the energy E of the body?
4. $\epsilon > 1$.

Problem 2. *Scattering.*
A comet is approaching the sun. Its velocity at infinity was v_0. The impact parameter at infinity was d. Find the angle of deflection θ.



Problem 3. *1990-Fall-CM-U-2.jpg*
An artificial Earth satellite is initially orbiting the earth in a circular orbit of radius R_0. At a certain instant, its rocket fires for a negligibly short period of time. This firing accelerates the satellite in the direction of motion and increases its velocity by a factor of α. The change of satellite mass due to the burning of fuel can be considered negligible.

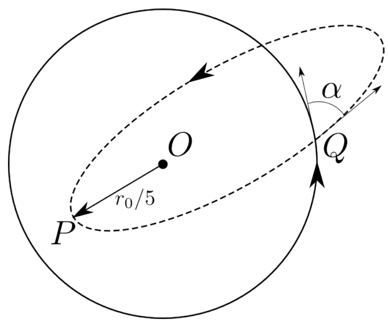
1. Let E_0 and E denote the total energy of the satellite before and after the firing of the rocket. Find E solely in terms of E_0 and α.
2. For $\alpha > \alpha_{\text{es}}$ the satellite will escape from earth. What is α_{es}?
3. For $\alpha < \alpha_{\text{es}}$ the orbit will be elliptical. For this case, find the maximum distance between the satellite and the center of the earth, R_{max}, in terms of R_0 and α.



Problem 4. *1993-Fall-CM-U-1*

A satellite of mass m is traveling at speed v_0 in a circular orbit of radius r_0 under the gravitational force of a fixed mass M at point O. At a certain point Q in the orbit (see the figure below) the direction of motion of the satellite is suddenly changed by an angle α without any change in the magnitude of the velocity. As a result the satellite goes into an elliptic orbit. Its distance of the closest approach to O (at point P) is $r_0/5$.

1. What is the speed of the satellite at P, expressed as a multiple of v_0?
2. Find the angle α.



Homework 8. Due October 24.

Problem 1. *1994-Fall-CM-U-4*
The most efficient way to transfer a spacecraft from an initial circular orbit at R_1 to a larger circular orbit at R_2 is to insert it into an intermediate elliptical orbit with radius R_1 at perigee and R_2 at apogee. The following equation relates the semi-major axis a, the total energy of the system E and the potential energy $U(r) = -GMm/r \equiv -k/r$ for an elliptical orbit of the spacecraft of mass m about the earth of mass M:

$$R_1 + R_2 = 2a = \frac{k}{|E|}.$$

1. Derive the relation between the velocity v and the radius R for a circular orbit.

2. Determine the velocity increase required to inject the spacecraft into the elliptical orbit as specified by R_1 and R_2. Let v_1 be the velocity in the initial circular orbit and v_p be the velocity at perigee after the first boost so $\Delta v = v_p - v_1$.

3. Determine the velocity increment required to insert the spacecraft into the second circular orbit when it reaches apogee at $r = R_2$. In this case let v_2 be the velocity in the final orbit and v_a be the velocity at apogee so $\Delta v = v_2 - v_a$.

Problem 2. *2000-Spring-CM-U-1*
A populated spherical planet, diameter a, is protected from incoming missiles by a repulsive force field described by the potential energy function:

$$V(r) = ka(a + r)e^{-r/a}, \qquad r > a/2.$$

Here $k > 0$ and r is the distance of the missile from the center of the planet. Neglect all other forces on the missile.

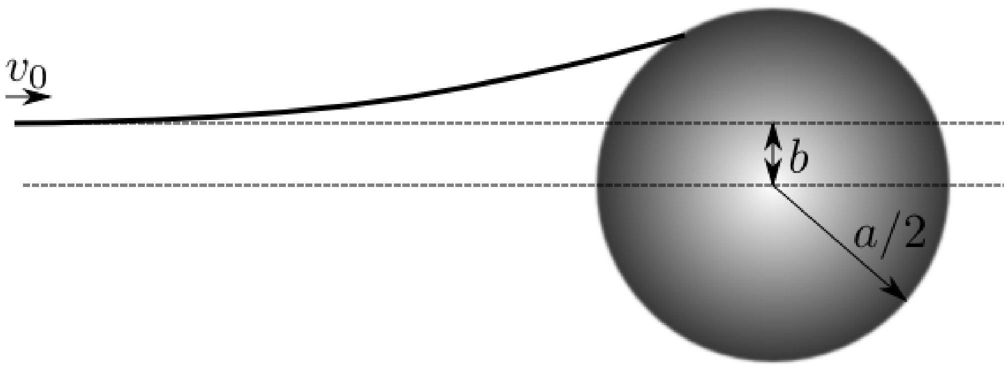
The initial speed of a missile of mass m relative to the planet is v_0 when it is a long way away, and the missile is aimed in such a way that the closest it would approach the center of the planet, if it were not deflected at all by the force field or contact with the surface, would be at an impact parameter b (see the diagram). The missile will not harm the planet if it does not come into contact with its surface. Therefore, we wish to explore, as a function of v_0 the range of values of b:

$$0 \le b \le B$$

such that the missile will hit the planet.

1. If v_0 is less than a certain critical velocity, v_c, the missile will not be able to reach the planet at all, even if $b = 0$. Determine v_c.

2. For missiles with velocity greater than v_c find B as a function of v_0.

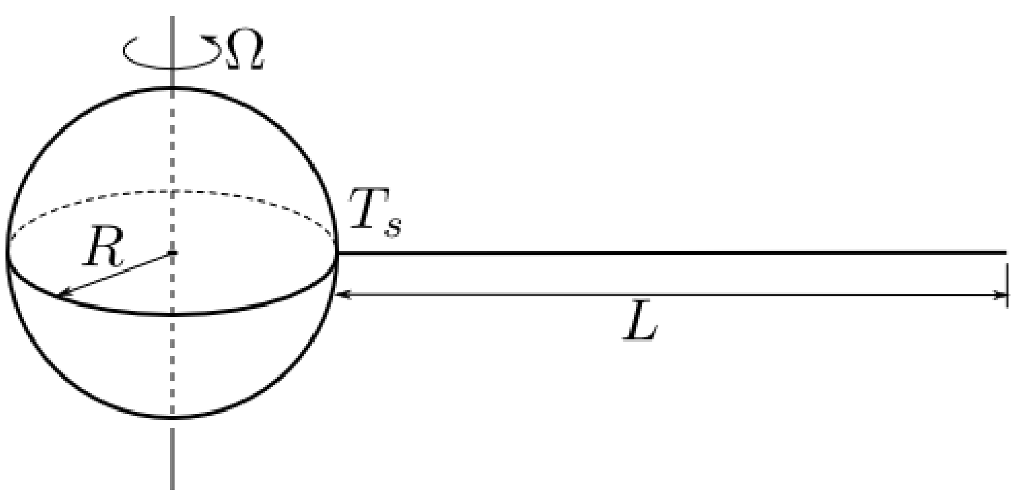


Problem 3. *2001-Spring-CM-U-1*

We wish to extend a flexible wire of length L from a point on the equator of the earth, reaching into space in a straight line. Its uniform linear density is ρ. Assume the earth is spherical, radius R, and rotating at Ω radians per second. The acceleration due to gravity at the surface of the earth is g, which of course decreases according to Newton's Law of gravity as the distance from the center of the earth is varied.

Once set in equilibrium with respect to the rotating earth we assume the wire is strong enough not to break, and is held only at its contact point with the earth's surface, where the tension is T_s. There are no other forces on the wire except gravity.

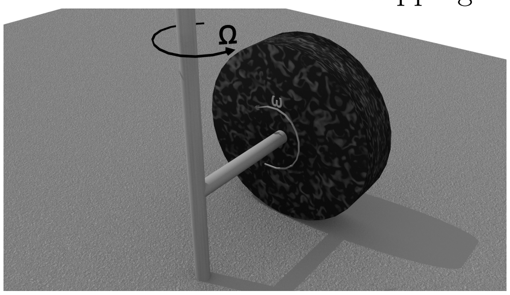
1. Find the tension T of the wire at an arbitrary distance along the wire, assuming that the wire is long enough so that it will not fall dawn.
2. Find T_s, the tension at the earth's surface.
3. If the wire is too short it will fall down. Find the critical length, L_c, of the wire in this case. You may assume for the purpose of solving the equations involved, that the length, L_c, of the wire is much bigger than the radius of the earth, R.



Problem 4. *Problem 6.2 from Taylor*

Problem 5. **Extra for Honors**

In a disk mill a massive cylinder of radius R and mass M can rotate around its geometrical axis. After the initial push it freely rotates with angular velocity Ω around vertical axis and is rolling on the horizontal plate, as shown on the figure. What force the disk applies to the plate? The disk rolls without slipping.



Homework 9. Due October 31.

Problem 1. *From Classical Mechanics by J. R.Tylor*
Problems: 6.5, 6.7, 6.22, 6.24

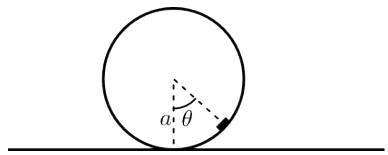
Problem 2. **Extra for Honors** *(and fun for the rest of us.)*
A string of tension T and linear mass density ρ connects two horizontal points distance L apart from each other. y is the vertical coordinate pointing up, and x the is horizontal coordinate.

- Write down the functional of potential energy of the string vs. the shape of the string $y(x)$. Specify the boundary conditions for the function $y(x)$.
- Write down the equation which gives the shape of minimal energy for the string.
- Find the solution of the equation which satisfies the boundary conditions. (Do not try to solve the transcendental equation for one of the constant. Just write it down.)
- In the case $T \gg \rho g L$, the shape is approximately given by $y \approx -\frac{\alpha}{2}x(L-x)$. Find α.

Homework 10. Due November 7.

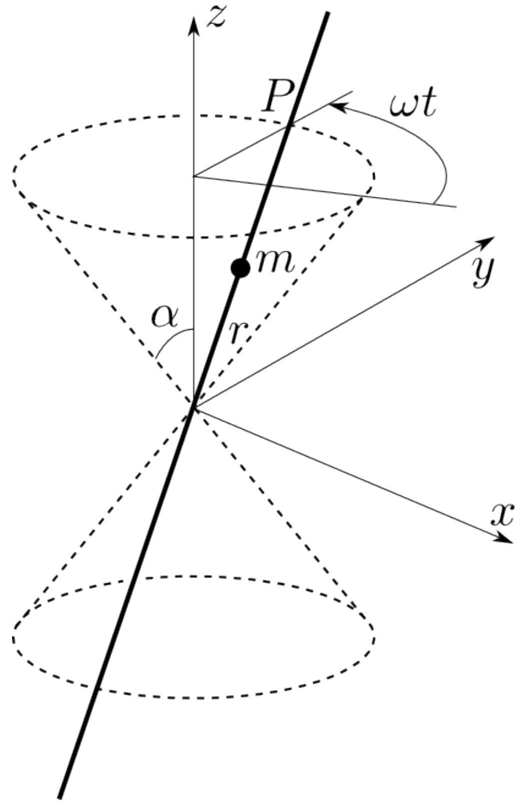
Problem 1. *1991-Fall-CM-U-3.*
A small block of mass m is attached near the outer rim of a solid disk of radius a which also has mass m. The disk rolls without slipping on a horizontal straight line.

1. Find the equation of motion for the angle $\theta(t)$ (measured with respect to the vertical as shown) for all θ.
2. Find the system's small amplitude oscillation frequency about its stable equilibrium position.



Problem 2. *1999-Spring-CM-U-2*
An infinitely long straight frictionless wire, which passes through the origin of the coordinate system, is held at a constant angle α with respect to the (vertical) z axis. The wire rotates about the z axis at constant angular velocity ω, so that it describes the surface of a pair of right circular cones, centered on the vertical axis, with their common vertices at the origin. Hence, an arbitrary point P, fixed on the wire, describes a horizontal circle as the wire rotates. A bead of mass m is free to slide along this wire under the influence of gravity and without friction. Let r be the distance of the bead from the apex of the cone, positive if above and negative if below the vertex.

1. Write the Lagrangian for this system in terms of r, α, ω, m, and g.
2. From the Lagrangian, obtain the differential equation of motion for the bead.
3. Solve the equation of motion subject to the initial conditions that $r = r_0$ and $dr/dt = 0$ at $t = 0$. Find the condition on r_0 which determines whether the bead will rise or fall on the wire.
4. Use your solution to the equation of motion to find $r(t)$ in the limit that ω goes to zero and show that it is consistent with simple kinematics.



Problem 3. *From Classical Mechanics by J. R.Tylor*
Problems: 7.29

Problem 4. *From Classical Mechanics by J. R.Tylor*
Problems: 7.31

Homework 11. Due November 14.

Problem 1. *From Classical Mechanics by J. R.Tylor*
Problems: 7.33

Problem 2. *From Classical Mechanics by J. R.Tylor*
Problems: 7.35

Problem 3. *From Classical Mechanics by J. R.Tylor*
Problems: 7.48

Problem 4. *From Classical Mechanics by J. R.Tylor*
Problems: 7.49

Homework 12. Due November 21.

Problem 1. *From Classical Mechanics by J. R.Tylor*
Problems: 7.34

Problem 2. *From Classical Mechanics by J. R.Tylor*
Problems: 7.37

Problem 3. *From Classical Mechanics by J. R.Tylor*
Problems: 7.38

Problem 4. *From Classical Mechanics by J. R.Tylor*
Problems: 7.41

Homework 13. Due November 28 (may be December 1).

Problem 1. *From Classical Mechanics by J. R.Tylor*
Problems: 13.2

Problem 2. *From Classical Mechanics by J. R.Tylor*
Problems: 13.3

Problem 3. *From Classical Mechanics by J. R.Tylor*
Problems: 13.5

Problem 4. *From Classical Mechanics by J. R.Tylor*
Problems: 13.6