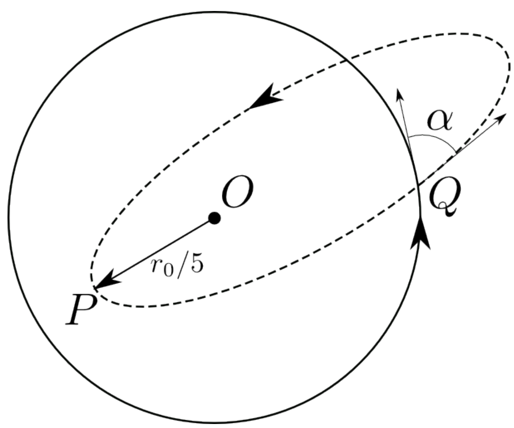
EXAM 1. Wednesday – Friday, Oct. 16 – Oct. 18

Problem 1. *1985-Spring-CM-U-2.*
Two cylinders having radii R_1 and $R_2 < R_1$ and rotational inertias I_1 and I_2 respectively, are supported by fixed axes perpendicular to the plane of the figure. The large cylinder is initially rotating with angular velocity ω_0. The small cylinder is moved until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually, slipping ceases, and the two cylinders rotate at constant rates in opposite directions. Find the final angular velocity ω_2 of the small cylinder in terms of I_1, I_2, R_1, R_2, and ω_0.

Problem 2. *1993-Fall-CM-U-1*
A satellite of mass m is traveling at speed v_0 in a circular orbit of radius r_0 under the gravitational force of a fixed mass M at point O. At a certain point Q in the orbit (see the figure below) the direction of motion of the satellite is suddenly changed by an angle α without any change in the magnitude of the velocity. As a result the satellite goes into an elliptic orbit. Its distance of the closest approach to O (at point P) is $r_0/5$.

1. What is the speed of the satellite at P, expressed as a multiple of v_0?
2. Find the angle α.

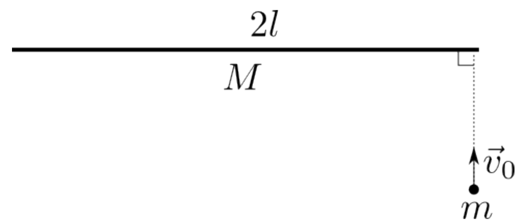


Problem 3. *1993-Fall-CM-U-3.*
A point-like test mass m is placed at the center of a spherical planet of uniform density, mass M and radius R.

1. Calculate the energy required to remove this test mass to an infinite distance from the planet.
2. Calculate the gravitational binding energy of this planet (without the test mass), i.e. the energy required to break the planet up into infinitesimal pieces separated by an infinite distance from each other.

Problem 4. *1995-Spring-CM-U-3*

A uniform line-like bar of mass M, and length $2l$ rests on a frictionless, horizontal table. A point-like particle of mass m slides along the table with velocity v_0 perpendicular to the bar and strikes the bar very near one end, as illustrated below. Assume that the force between the bar and the particle during the collision is in the plane of the table and perpendicular to the bar. If the interaction is elastic (*i.e.*, if energy is conserved) and lasts an infinitesimal amount of time, then determine the rod's center-of-mass velocity V and angular velocity ω, and the particle's velocity v after the collision.



Problem 5. *1996-Fall-CM-U-2*

A particle of mass m slides down a curve $y = kx^2$, $(k > 0)$ under the influence of gravity, as illustrated. There is no friction, and the particle is constrained to stay on the curve. It starts from the top with negligible velocity.

1. Find the velocity v as a function of x.
2. Next, assume that the particle initially slides down the curve under gravity, but this time is not constrained to the curve. Does it leave the curve after it has fallen a certain distance? Prove your answer.