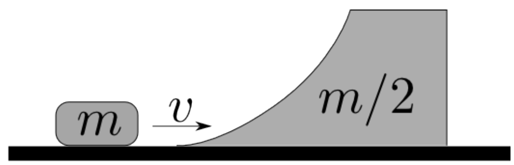
EXAM 1. Wednesday Oct.7, 9:20am – Friday, Oct. 9, 9:20am

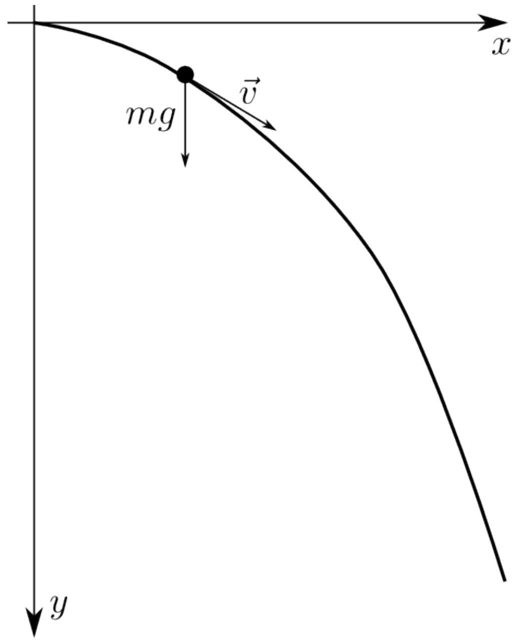
Problem 1. *1987-Fall-CM-U-1.*
A block of mass m slides on a frictionless table with velocity v. At $x = 0$, it encounters a frictionless ramp of mass $m/2$ which is sitting at rest on the frictionless table. The block slides up the ramp, reaches maximum height, and slides back down.

1. What is the velocity of the block when it reaches its maximum height?
2. How high above the frictionless table does the block rise?
3. What are the final velocities of the block and the ramp?



Problem 2. *1996-Fall-CM-U-2*
A particle of mass m slides down a curve $y = kx^2$, $(k > 0)$ under the influence of gravity, as illustrated. There is no friction, and the particle is constrained to stay on the curve. It starts from the top with negligible velocity.

1. Find the velocity (vector!) $\vec{v}$ as a function of x. (use the system of coordinates shown in the figure.)
2. Next, assume that the particle initially slides down the curve under gravity, but this time it is only supported (but not constrained) by the curve. Does it leave the curve after it has slid a certain distance? Prove your answer.



Problem 3. *2000-Spring-CM-U-1*

A populated spherical planet, diameter a, is protected from incoming missiles by a repulsive force field described by the potential energy function:

$$V(r) = ka(a+r)e^{-r/a}, \qquad r > a/2.$$

Here $k > 0$ and r is the distance of the missile from the center of the planet. Neglect all other forces on the missile.

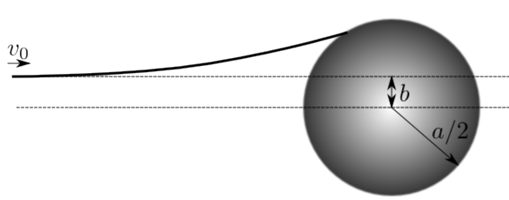
The initial speed of a missile of mass m relative to the planet is v_0 when it is a long way away, and the missile is aimed in such a way that the closest it would approach the center of the planet, if it were not deflected at all by the force field or contact with the surface, would be at an impact parameter b (see the diagram). The missile will not harm the planet if it does not come into contact with its surface. Therefore, we wish to explore, as a function of v_0 the range of values of b:

$$0 \le b \le B$$

such that the missile will hit the planet.

1. If v_0 is less than a certain critical velocity, v_c, the missile will not be able to reach the planet at all, even if $b = 0$. Determine v_c.

2. For missiles with velocity greater than v_c find B as a function of v_0. Write this function of v_0 in terms of v_c and a.



Problem 4. *2001-Spring-CM-U-1*

We wish to extend a flexible wire of length L from a point on the equator of the earth, reaching into space in a straight line. Its uniform linear density is ρ. Assume the earth is spherical, radius R, and rotating at Ω radians per second. The acceleration due to gravity at the surface of the earth is g, which of course decreases according to Newton's Law of gravity as the distance from the center of the earth is varied.

Once set in equilibrium with respect to the rotating earth we assume the wire is strong enough not to break, and is held only at its contact point with the earth's surface, where the tension is T_s. There are no other forces on the wire except gravity.

1. Find the tension T of the wire at an arbitrary distance along the wire, assuming that the wire is long enough so that it will not fall dawn.
2. Find T_s, the tension at the earth's surface.
3. If the wire is too short it will fall down. Find the critical length, L_c, of the wire in this case. You may assume for the purpose of solving the equations involved, that the length, L_c, of the wire is much bigger than the radius of the earth, R.