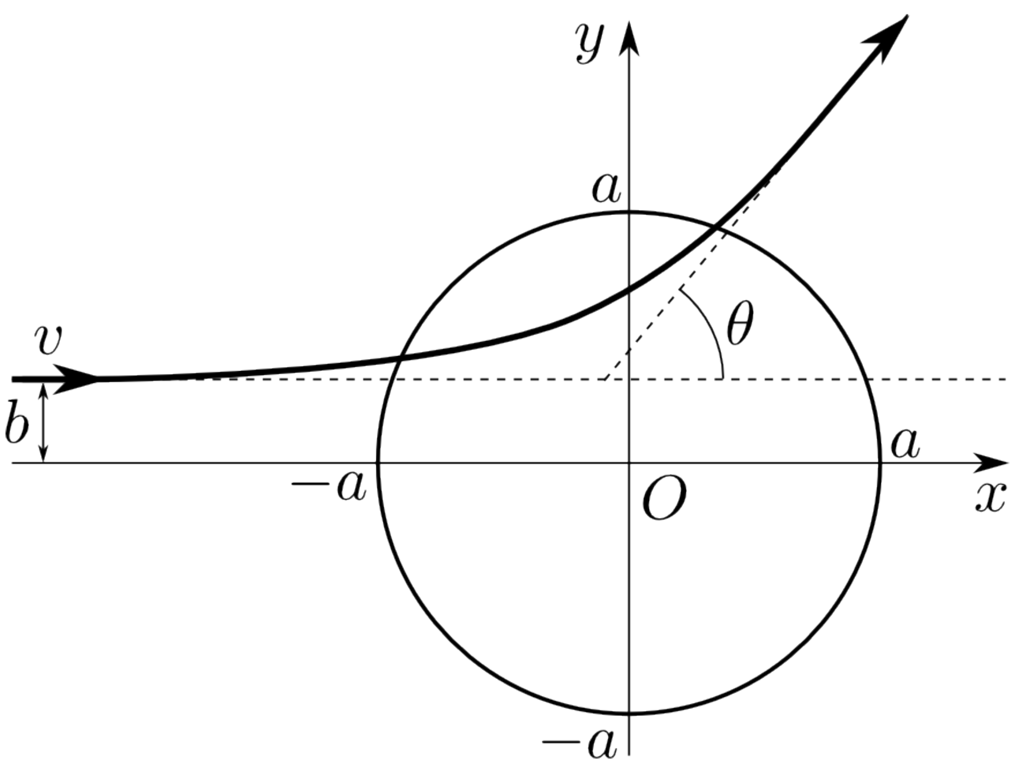
EXAM 1. Tuesday, March 19-21, 2019, take home

Problem 1. *2000-Fall-CM-G-5.jpg*
Particles are scattered classically by a potential:

$$V(r) = \begin{cases} U(1 - r^2/a^2), & \text{for } r \le a \\ 0, & \text{for } r > a \end{cases}, \quad U \text{ is a constant.}$$

Assume that $U > 0$. A particle of mass m is coming in from the left with initial velocity v_0 and impact parameter $b < a$. Hint: work in coordinates (x, y) <u>not</u> (r, ϕ).

1. What are the equations of motion for determining the trajectory $x(t)$ and $y(t)$ when $r < a$?
2. Assume that at $t = 0$ the particle is at the boundary of the potential $r = a$. Solve your equations from the previous part to find the trajectory $x(t)$ and $y(t)$ for the time period when $r < a$. Express your answer in terms of sinh and cosh functions.
3. For initial energy $\frac{1}{2}mv_0^2 = U$, find the scattering angle θ as function of b.



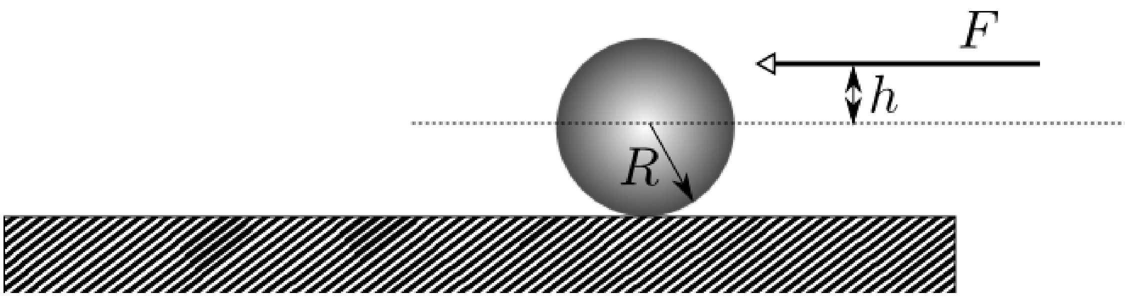
Problem 2. *1984-Spring-CM-U-2.*
A particle of mass m moves subject to a central force whose potential is $V(r) = Kr^3$, $K > 0$.

1. For what kinetic energy and angular momentum will the orbit be a circle of radius a about the origin?

2. What is the period T of this circular motion?

3. If the motion is slightly disturbed from this circular orbit, what will be the period τ of small radial oscillations about $r = a$? Express τ through T.

Problem 3. *1996-Spring-CM-U-1*
A billiard ball initially at rest, is given a sharp impulse by a cue stick. The cue stick is held horizontally a distance h above the centerline as in the figure below. The ball leaves the cue with a horizontal speed v_0 and, due to its spin, eventually reaches a final speed of $9v_0/7$. Find h in terms of R, where R is the radius of the ball. You may assume that the impulsive force F is much larger than the frictional force during the short time that the impulse is acting.



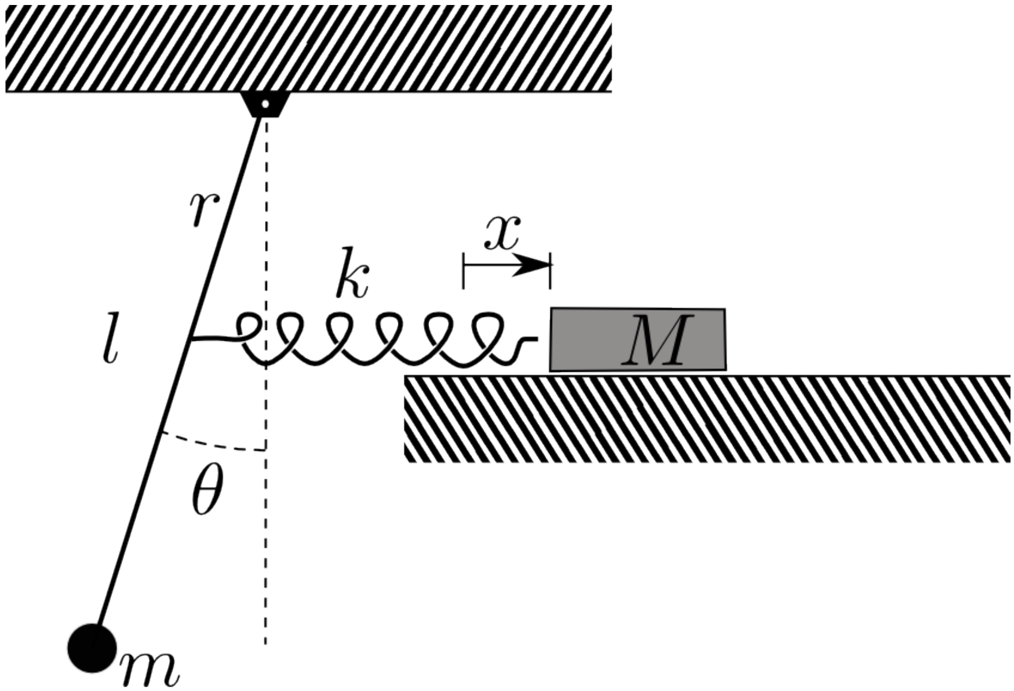
Problem 4. *2001-Spring-CM-U-2*
Consider the system, pictured below, which consists of a ball of mass m connected to a massless rod of length l. This is then joined at point r to a spring of spring constant k connected to a block of mass M which rests on a frictionless table. When $\theta = 0$ and $x = 0$ the spring is unstretched.

1. Write the Lagrangian for the system in terms of the coordinates θ and x assuming small displacements of the pendulum

2. Write the equations of motion for the system.

3. Making the simplifying assumptions that $M = m$, $l = 2r$, and setting $k/m = g/l = \omega_0^2$, find the normal-mode frequencies of this system for small oscillations in terms of ω_0

4. Assuming the same conditions, calculate the ratio of amplitudes (for each of the two masses) of the two normal modes of oscillation.

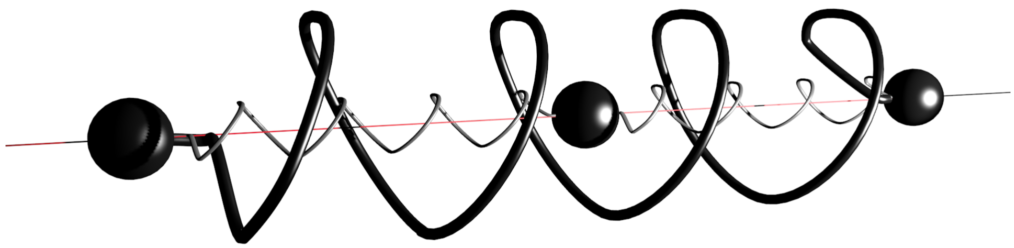

r
k
x
l
M
θ
m

EXAM 2. Final. Tuesday, May 7, 2019, 8-10am

Problem 1. *2001-Fall-CM-U-3*
Three spheres of equal mass m are constrained to move in one dimension along the line connecting their centers. The three spheres are connected by three springs, as shown in the figure. The three springs have equal spring constants k In equilibrium, all three of the springs are at their respective natural lengths.

1. Choose a reasonable set of coordinates and find the equations of motion.

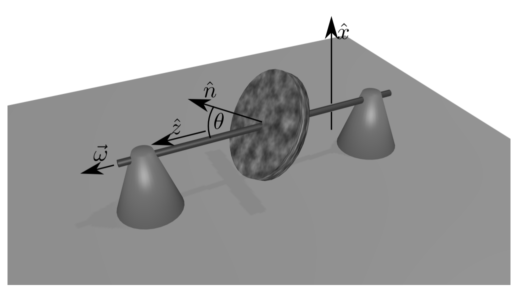
2. Find the normal-mode frequencies.



Problem 2. *1985-Fall-CM-G-6*
A disk is rigidly attached to an axle passing through its center so that the disc's symmetry axis $\hat{n}$ makes an angle θ with the axle. The moments of inertia of the disc relative to its center are C about the symmetry axis $\hat{n}$ and A about any direction $\hat{n}'$ perpendicular to $\hat{n}$. The axle spins with constant angular velocity $\vec{\omega} = \omega\hat{z}$ ($\hat{z}$ is a unit vector along the axle.) At time $t = 0$, the disk is oriented such that the symmetry axis lies in the $X - Z$ plane as shown.

1. What is the angular momentum, $\vec{L}(t)$, expressed in the space-fixed frame.

2. Find the torque, $\vec{\tau}(t)$, which must be exerted on the axle by the bearings which support it. Specify the components of $\vec{\tau}(t)$ along the space-fixed axes.



Problem 3. *1993-Spring-CM-G-4.jpg*

Because of the gravitational attraction of the earth, the cross section for collisions with incident asteroids or comets is larger than πR_e^2 where R_e is the physical radius of the earth.

1. Write the Lagrangian and derive the equations of motion for an incident object of mass m. (For simplicity neglect the gravitational fields of the sun and the other planets and assume that the mass of the earth, M is much larger than m.)

2. Calculate the effective collisional radius of the earth, R, for an impact by an incident body with mass, m, and initial velocity v, as shown, starting at a point far from the earth where the earth's gravitational field is negligibly small. Sketch the paths of the incident body if it starts from a point 1) with $b < R_e$ 2) with $b \gg R_e$, and 3) at the critical distance R. (Here b is the impact parameter.)

3. What is the value of R if the initial velocity relative to the earth is $v = 0$? What is the probability of impact in this case?