Homework 1. Due Jan. 23

Problem 1. *Work. I*

Consider a cyclic engine operating with n moles of an ideal monoatomic gas in the cycle $a \to b \to c \to d \to a$, where V_a and T_a are the volume and the temperature of the gas in point a.

$a \to b$ pressure doubles at constant volume.

$b \to c$ volume doubles at constant pressure.

$c \to d$ pressure halves constant volume.

$d \to a$ volume halves at constant pressure.

All processes are reversible.

a. What is the net entropy change of the gas in one cycle?

b. What is the net change of the energy of the gas in one cycle?

c. What work is done by the gas during one cycle?

d. How much heat the gas got during one cycle?

e. What is the change of entropy of the gas during each of the four processes $a \to b \to c \to d \to a$?

Problem 2. *Work II*

Consider a cyclic engine operating with one mole of an ideal monoatomic gas in the cycle $a \to b \to c \to d \to a$, where V_a and T_a are the volume and the temperature of the gas in point a.

$a \to b$ is isobaric increase of temperature from T_a to $3T_a$

$b \to c$ isothermal expansion to the volume $4V_a$

$c \to d$ decrease of temperature back to T_a at constant volume

$d \to a$ isothermal compression back to the volume V_a

All processes are reversible.

a. What is the net entropy change of the gas in one cycle?

b. What is the net change of the energy of the gas in one cycle?

c. What work is done by the gas during one cycle?

d. How much heat the gas got during one cycle?

e. What is the change of entropy of the gas during each of the four processes $a \to b \to c \to d \to a$?

Homework 2. Due Jan. 30

Problem 1. *Jacobian*
Prove the Jacobian relations used in LL
a.

$$\frac{\partial(u,y)}{\partial(x,y)} = \left(\frac{\partial u}{\partial x}\right)_y$$

b.

$$\frac{\partial(u,v)}{\partial(x,y)} = -\frac{\partial(u,v)}{\partial(y,x)}$$

c.

$$\frac{\partial(g,f)}{\partial(x,y)} = \frac{\partial(g,f)}{\partial(u,v)}\frac{\partial(u,v)}{\partial(x,y)}$$

Problem 2. *van der Waals*
a. Show that

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

b. For a van der Waals gas, show that the internal energy increases as the volume increases at constant temperature. What is the answer for the ideal gas? The van der Waals equation of state is

$$P = \frac{RT}{V-b} - \frac{a}{V^2}, \qquad a > 0, \quad b > 0, \quad V > b$$

Problem 3. *Ideal gas*
Using the ideal gas expression for entropy

$$S(E,V,N) = N\left\{\log\frac{V}{N} + c\log\frac{2E}{3N} + \log\frac{(2\pi m)^{3/2}e^{5/2}}{h^3}\right\}$$

a. find $E(S,V,N)$ and derive T, P from it;

b. find the free energy $F(T,V,N) = E - TS$ and derive S, and P from it;

c. What is the equation of state of an ideal gas? What is the condition for an adiabatic process for an ideal gas in terms of variables T and V?

d. find specific heats $C_V(T,V)$ and $C_P(T,P)$ for an ideal gas.

e. Find α, β_T, and β_S for an ideal gas.

Problem 4. *Joule-Thomson process*
Consider a Joule-Thomson process. Let's take the difference in pressure to be small. What is the change of volume of filtered gas as it passes through the membrane?

Homework 3. Due Feb. 6.

Read: **LL** 19,15,24

Problem 1. *A Brick*

For a solid body (a brick) C_P and C_V are almost the same, as the thermal expansion is small. In a large range of temperatures the heat capacity can be considered to be independent of temperature.

If temperature of a brick with heat capacity C has changed from T_1 to T_2 what is the change of energy ΔE and entropy ΔS?

Problem 2. *A Brick and an Iceberg.*

The thermodynamic system consists of a hot brick of temperature T_1 and specific heat C and of an iceberg at $T_2 = 0°\mathrm{C}$. What is the maximal work W that can be performed by bringing this system to the state of thermal equilibrium? Consider the effects due to the change of the total volume of the system as negligible.

Problem 3. *Two bricks*

There are two identical bricks with heat capacitance C each. The temperature of the first brick is T_1, the temperature of the second is T_2. What will be the finite temperature of the bricks

a. if we bring the bricks to a contact and leave them touching for a long time?

b. if we extract as much work as possible from the equilibration process?

c. In which case the final temperature is smaller?

d. What is the maximal amount of work we were able to extract in the part **b**?

Problem 4. *Three bricks*

There are three identical bricks with temperatures T_1, T_2, and T_3. You are allowed to use any engines, but you cannot use external work or heat. What is the larges temperature you can give to one of the bricks?

Homework 4. Due date Feb. 13.

Problem 1. *Magnetization*
In the presence of magnetic field H one defines magnetization of the body as $M = -(\partial E/\partial H)_{S,V}$. Then one thinks about the energy as of function of three independent thermodynamic variables $E = E(S, V, H)$ with $dE = TdS - PdV - MdH$.

Assume that we know the complete equation of state $P(T, V, H)$, the temperature dependence of (constant volume, constant magnetic field) specific heat $C_V(T, V_0, H_0)$ at some given V_0 and H_0 and temperature and magnetic field dependence of magnetization $M(T, V_0, H)$ at the same given V_0 .

Find the full dependence of $C_V(T, V, H)$ and $M(T, V, H)$ from this data.

Problem 2. *Membrane*
A circular membrane has a temperature dependent surface tension $\sigma(T)$. How does the heat capacity of the membrane depend on the small displacement h of the center of the membrane perpendicular to the membrane?

Problem 3. *Equilibrium*
Let's consider a macroscopic system which has an internal parameter ϕ. This parameter is not a conserved quantity (for example, magnetization). Assume that we know the free energy $F(\phi, T, V)$ of the system as a function of all variables, including the variable ϕ.

a. Show that

$$\left(\frac{\partial S}{\partial \phi}\right)_{E,V} = -\frac{1}{T}\left(\frac{\partial F}{\partial \phi}\right)_{T,V}$$

b. Find the average value of $\langle h \rangle$ for the membrane in the equilibrium in the problem **Membrane**.

c. How will the formula of the part **a.** change if the pressure is constant, instead of volume?

Problem 4. *Second order*
Let's consider a macroscopic system, which has the free energy of the form

$$F(\phi, T) = F_0(T) + a(T - T_c)\phi^2 + b\phi^4,$$

where $a > 0$, $b > 0$, $T_c > 0$ are temperature independent constants and ϕ is a parameter. The value of this parameter is defined by the equilibrium condition.

a. Calculate the value of ϕ in equilibrium, for both $T > T_c$ and $T < T_c$.

b. Calculate the value of F in equilibrium, for both $T > T_c$ and $T < T_c$.

c. Calculate the value of C_V for both $T > T_c$ and $T < T_c$.

Hint: ϕ is a physical parameter, and as such cannot be a complex number.

Homework 5. Due Feb. 20

Problem 1. *Gas and vacuum*
A cylinder of volume $V_0 + V_1$ is divided by a partition into two parts of volume V_0 and V_1, where $V_1 \ll V_0$. The volume V_0 contains 1 mole of a gas at temperature T_0 and pressure P_0. The volume V_1 has vacuum. You know the gas's coefficient of thermal expansion $\alpha(P_0, T_0)$, its $\beta_T(P_0, T_0)$, and its heat capacity $C_V(T, V)$. At some moment the partition disappears.

1. What will be the temperature of the gas after it equilibrates?
2. A piston now adiabatically returns the gas back to the volume V_0, what will be it's temperature?
3. What are these results for the ideal gas?

Problem 2. *Oscillator*
N particles of mass m of ideal gas at temperature T are in a $3D$ harmonic potential $u(r) = \frac{m\omega^2 r^2}{2}$. Find the particle density at distance r from the center.

Problem 3. *1983-Spring-SM-U-3*
The heat capacity of a normal metal C_n at low temperatures is given by $C_n = \gamma T$, where γ is a constant. If the metal is superconducting below T_c, then the heat capacity C_S in the temperature range $0 < T < T_c$ is given by the relation $C_S = \alpha T^3$, where α is a constant. The entropy S_n, S_S of the normal metal and superconducting metal are equal at T_c, also $S_n = S_S$ as $T \to 0$.

1. Find the relation between C_S and C_n at T_c.
2. Is the transition first or second order?

Problem 4. *1985-Spring-SM-G-7.*
A thermally isolated container is divided by a partition into two compartments, the right hand compartment having a volume b times larger than the left one. The left compartment contains ν moles of an ideal gas at temperature T, and pressure P. The right compartment also contains ν moles of an ideal gas at temperature T. The partition is now removed. Calculate:

a. The final pressure

b. The total change in the entropy if the gases are different.

c. The total change in the entropy if the gases are the same.

d. What is the total change in the entropy if the gases are the same and $b = 1$? Can you explain the result?

Problem 5. *EXTRA PROBLEM, due the Final Exam. (+10 to Final)*

Consider a gas of electrons at temperature T. The concentration of electrons is small. There is no gravity.

a. The gas is s on top of infinitely large the positively charged plate with uniform charge σ per area. The number of electrons is N per area of the plate. Find the concentration as a function of distance to the plate.

b. The total number of electrons is N. Find the concentration of the electrons as a function of a distance from a uniformly and positively charged sphere of radius R and total charge Q

Homework 6. Due Feb. 27.

Problem 1. *Distribution function. 1983-Fall-SM-G-6*
A classical one dimensional particle, confined to the region $y \geq y_0$ is in a potential

$$V(y) = V_0 \log(y/y_0)$$

The statistical distribution is given by $\varrho(p,q) = Ae^{-E(p,q)/T}$, where A is a normalization parameter, T is a parameter which is called temperature, and $E(p,q)$ is the particle's energy.

a. Find the normalization constant A. Determine the critical temperature T_c above which the particle escapes to infinity (You need to figure out what it means.).

b. Write down the normalized positional distribution function $f(y)$ (i.e. the probability per unit distance to find the particle between y and $y + dy$) for this particle for $T < T_c$.

c. Find the average distance $\langle y \rangle$ for the particle. What happens if $0 < T_c/2 - T \ll T_c/2$

Problem 2. *Distribution, average, and fluctuations*
You through a dice N times count the number of times you have a "3". You repeat this activity many many times.

a. With what probability the number of 3s is n?

b. What is the average number of 3s?

c. What is the standard deviation (r.m.s. fluctuation)?

Problem 3. *Distribution function of an oscillator.*
A classical one dimensional oscillator ($V(x) = \frac{m\omega^2 x^2}{2}$) has a statistical distribution function $\varrho(p,q) = Ae^{-E(p,q)/T}$, where A is a normalization parameter, T is a parameter which is called temperature, and $E(p,q)$ is the oscillator's energy.

a. Find the normalization constant A.

b. Find the average coordinate of the particle.

c. Find the average momentum of the particle.

d. Find the r.m.s. fluctuations of the particle' coordinate and momentum.

e. Find the average energy of the particle. Find the heat capacity.

f. Find the distribution function for a quantity $f = f(p,x)$, where $f(p,x) = p - \omega m x$.

Problem 4. *EXTRA PROBLEM, due the end of the semester.*
In a huge cavity of temperature T there are two neutral atoms at large distance R from each other. Each atom has a magnetic susceptibility χ. Find the force between the two atoms. Neglect the retardation due to the finite speed of light and the electric dipole susceptibility.

Homework 7. Due March 6.

Problem 1. *Traveling frog.*
Consider a one dimensional frog. After every τ seconds it hops with probability $1/2$ one meter to the left and with probability $1/2$ one meter to the right. At $t = 0$ the frog is at $x = 0$.

1. Consider a function $p(x, t)$ – the probability for the frog to be at point x at time t. Find $p(x, t + \tau)$.
2. Consider a limit of large distances and long times. Find a differential equation for $p(x, t)$.
3. What is the initial condition for this equation?
4. Solve the equation.
5. What is the average coordinate of the frog? How does it change with time?
6. What is the average deviation of the frog's coordinate? How does it change with time?
7. Repeat all the steps for the situation when the frog hops with probability q to the left and probability $1 - q$ to the right.

Problem 2. *Electrons in wire*
A current I flows in the wire. Treat electrons as point like classical particles.

1. What is the probability that exactly n electrons cross through a wire cross-section in time T.
2. What is the average number of electrons which crossed a wire cross-section in time T?
3. What is the standard deviation of that number?

Homework 8. Due March 13.

Problem 1. *Statistical matrix*
A quantum oscillator of frequency ω and mass m is in a mixed state which is characterized by the following statistical matrix

$$w_{m,n} = \begin{cases} 0, & \text{for } n \neq m \\ Ae^{-\beta n}, & \text{for } n = m \end{cases}$$

1. Find A.
2. Find $\bar{x}$, and $\bar{p}$ where $\hat{x}$ is the coordinate and $\hat{p}$ is the momentum.
3. Find $\bar{x^2}$, and $\bar{p^2}$.
4. Find $\bar{E}$, the average energy.

Problem 2. *N independent particles*
There is a system consisting of N independent particles. Each particle can have only one of the two energy levels 0 and $+\epsilon_0$.

a. Find the stat. weight $\Delta\Gamma$ of a state with the total energy $E = M\epsilon_0$, $(M = 0, \ldots, N)$.

b. Calculate the entropy of the system as a function of its energy.

c. Find the relation between the temperature and the energy of the system.

d. How does the energy of the system depend on the temperature for $T \ll \epsilon_0$ and $T \gg \epsilon_0$?

Problem 3. *An oscillator.*
N classical particles with mass m are in the $3D$ harmonic oscillator trap potential $V = \frac{m\omega^2 r^2}{2}$.

a. Calculate the number of states (volume of phase space) with the total energy $E < E_0$.

b. Then using $w(E) = Ae^{-E/T}$, calculate the average energy of the system. (Do not forget normalization)

c. Calculate the heat capacity of the system.

d. Calculate the entropy of the system.

Problem 4. *A spin* 1/2
A spin 1/2 is in a magnetic field $\mathcal{H}$ pointing in z direction. The spin is at equilibrium with heat bath at temperature T.

a. Calculate the average components of the spin.

b. Calculate the average square of the components.

c. Calculate the r.m.s. fluctuation of the components.

Homework 9. Due March 20.

Problem 1. *N spins 1/2*
N non-interacting spins 1/2 are in a magnetic field $\mathcal{H}$ pointing in z direction. They are in equilibrium with heat bath at temperature T.

a. Calculate the free energy of the system?

b. What is the heat capacity of the system?

c. What is the magnetization $\mathcal{M}$ (total spin) of the system?

d. What is the magnetic susceptibility of the system? (magnetic susceptibility is defined as $\chi = (\partial\mathcal{M}/\partial\mathcal{H})_T$)

Plot the answers for the last three questions.

Problem 2. *2001-Fall-SM-U-3*
Consider a system of N independent harmonic oscillators with the same frequency ω. The system is at, temperature T.

1. Show that the partition function of the system is
$$Q_N = \left[2\sinh\left(\frac{\hbar\omega}{2k_BT}\right)\right]^{-N}.$$
2. Using this result, obtain the internal energy U of the system as a function of T and N.
3. Show that the heat capacity is
$$C = Nk_B\frac{e^{\hbar\omega/k_BT}}{\left(e^{\hbar\omega/k_BT}-1\right)^2}\left(\frac{\hbar\omega}{k_BT}\right)^2$$
and it approaches to 0 as $T \to 0$.
4. Determine its Helmholtz free energy F.
5. Determine its entropy S.

Problem 3. *Quantum oscillator*
A quantum particle of charge q is in the potential $V(x) = \frac{m\omega^2}{2}x^2$ in $1D$ at temperature T.

a. Find the heat capacity of this system.

b. Find the electric dipole susceptibility of the system. ($d = -(\partial F/\partial\mathcal{E})_{V,N,T}$, $\chi = (\partial d/\partial\mathcal{E})_{V,N,T}$, where $\mathcal{E}$ is the electric field.)

c. For the oscillator state ψ_n in the presence of electric field $\mathcal{E}$ calculate $x_n = \langle\psi_n|\hat{x}|\psi_n\rangle$, and then average displacement $\bar{x} = \sum_n x_n w_n$.

Homework 10. Fluctuations. Due March 27

Problem 1. *String.*
A $3D$ string of length L has a temperature dependent tension $f(T)$.

a. What is the average value of the amplitude a_k^α of the kth harmonic of a small deformation of the string?

b. What is $\langle a_k^\alpha a_{k'}^\beta \rangle$ for two harmonics k and k'?

c. Calculate $\left\langle e^{\vec{n}\vec{a}_k} \right\rangle$.

Problem 2. *Fluctuations*
a. Find $\langle (\Delta P)^2 \rangle$, $\langle (\Delta S)^2 \rangle$, and $\langle \Delta S \Delta P \rangle$.

b. Find $\langle (\Delta E)^2 \rangle$.

c. Find $\langle (\Delta W)^2 \rangle$.

d. Find $\langle \Delta P \Delta T \rangle$.

e. Find $\langle \Delta P \Delta V \rangle$.

f. Find $\langle \Delta S \Delta T \rangle$.

Problem 3. *Two masses*
In $1D$ two equal masses m are connected with the walls and with each other by springs with spring coefficients k. The springs are unstretched. The coordinate of the first mass is x_1, the coordinate of the second is x_2. The temperature is T. Find

a. $\langle (\Delta x_1)^2 \rangle$

b. $\langle (\Delta x_2)^2 \rangle$

c. $\langle \Delta x_1 \Delta x_2 \rangle$

d. What will happen if we allow the masses to fluctuate in $3D$?

Problem 4. *N masses*
In $1D$ N equal masses m are connected with the walls and with each other by springs with spring coefficients k. The springs are unstretched. The coordinates of the masses are x_i, $i = 1 \dots N$. The temperature is T. Find

a. $\langle (\Delta x_i)^2 \rangle$

b. $\langle \Delta x_i \Delta x_j \rangle$

Homework 11. Maxwell and Ising. Due April 3.

Problem 1. *Averages.*
For a gas in $3D$

1. Find $\left\langle \frac{1}{v} \right\rangle$, where v is the magnitude of the particle's velocity. Express the result through $\frac{1}{\langle v \rangle}$.

2. Find $\left\langle \frac{1}{\vec{v}^2} \right\rangle$, where $\vec{v}$ is the particle's velocity. Express the result through $\frac{1}{\langle \vec{v}^2 \rangle}$.

3. Find $\left\langle e^{\vec{v}\cdot\vec{l}} \right\rangle$, where $\vec{v}$ is the particle's velocity, and $\vec{l}$ is some arbitrary vector.

4. Find $\left\langle \log\left(\frac{m\vec{v}^2}{2T} \right) \right\rangle$, where v is the particle's velocity.

Problem 2. *Small hole.*
A vessel with an ideal gas is held at constant temperature T. There is a small whole in the wall of the vessel. Calculate how the density of the gas is changing with time if outside of the vessel is vacuum.

Problem 3. *1992-Spring-SM-U-1*
An ensemble of non-interacting pairs of Ising spins is in a magnetic field h and at temperature T. Each spin variable s_i^z can only take on values $s^z = \pm 1$. The two spins within each pair interact according to the Hamiltonian

$$H = -J s_1^z s_2^z - \mu_B h \left(s_1^z + s_2^z \right), \qquad \text{with } J > 0$$

1. Enumerate the possible states of a single pair and compute their corresponding energies.

2. Derive an expression for the average value of a spin, $\langle s_i^z \rangle$, $(i = 1, 2)$ as a function of J, T and h.

3. Given the above model, determine whether there exists a temperature T_c for which $\langle s_i^z \rangle$ can be non-zero at $h = 0$. Evaluate T_c.

Problem 4. *Ising chain.*
Ising chain, or one dimensional Ising model is the following. There is a $1D$ chain with sites enumerated by $i = 1 \ldots N$. in each site there is Ising variable $\sigma_i = \pm 1$. The Hamiltonian of the chain is given by

$$H = -J \sum_{i=1}^{N-1} \sigma_i \sigma_{i+1}.$$

Calculate specific heat of this chain. Is there a phase transition?

Homework 12. Due April 10

Problem 1. *Oscillator. An exercise.*

For one $1D$ particle of charge q at temperature T in the potential $V(x) = \frac{m\omega^2}{2}x^2$ calculate the zero field electric dipole susceptibility using perturbation theory.

a. In classical case.

b. In quantum case.

Problem 2. *Oscillator. Anharmonicity.*

For the classical $1D$ harmonic oscillator $V(x) = \frac{m\omega^2}{2}x^2$, temperature T charge q and small anharmonicity term $V_a = \beta x^4$:

a. Calculate the first anharmonic correction to the heat capacity.

b. Calculate the first anharmonic correction to the zero field electric dipole susceptibility.

Homework 13. Due April 17

Problem 1. *Occupation numbers.*
Calculate the fluctuations of the occupation numbers for

a. Fermi gas.

b. Bose gas.

c. Classical gas.

Problem 2. *1999-Fall-SM-G-5.*
N spin 1/2 fermions with positive charge $+e$ placed in a magnetic field with field strength B in the $\hat{z}$-direction. The single particle energy levels are Landau levels, are characterized by two quantum numbers and can be written as

$$\epsilon_n(p_z) = (n + 1/2)\hbar\omega_c + \frac{p_z^2}{2m}$$

where p_z $(-\infty \le p_z \le \infty)$ is the continuous projection of the momentum on the $\hat{z}$ direction and non-negative integer $n = 0, 1, 2 \ldots$ is associated with the motion in $x - y$ plane. Here m is the mass of the particle and $\omega_c = eB/mc$ is the cyclotron frequency.

The degeneracy of each level is given by

$$g\left[\epsilon_n(p_z)\right] = \frac{m\omega_c}{2\pi\hbar} A,$$

where $A = V/L$ is the area of the system in the $x - y$ plane, V is the volume of the system, and L is the length of the system in the $\hat{z}$ direction.

Assuming the BOLTZMANN statistics in canonical ensemble is valid, determine

a. The equation of state.

b. The magnetization of the gas.

c. What condition on the chemical potential must be true so that the use of the Boltzmann statistics is justified?

Problem 3. *Dielectric constant of ideal gas.*
Consider an ideal classical gas of rigid dipolar molecules in an electric field E. The dipole moment of each molecule is μ. Calculate the linear dielectric constant ϵ of the gas as a function of temperature T and density $\rho = N/V$.

Homework 14. Due April 24

Problem 1. *1984-Spring-SM-G-4.*
There are three quantum states of energies 0, ϵ, and 2ϵ. Consider a system of two indistinguishable non-interacting particles which can occupy these states. The system is coupled with the heat bath at temperature T.

a. Calculate the free energy of the system in case the particles are Bosons.

b. Calculate the free energy of the system in case the particles are Fermions.

c. What is the ration of occupation probability of the highest energy state of the system to the lowest energy state in each of these cases?

Problem 2. *Degenerate electron gas.*

a. Find the temperature dependence of the chemical potential of the D dimensional gas of fermions at small temperatures.

b. Are you sure your answer is correct for $D = 2$?

c. What is the condition for the temperature to be small enough?

d. Estimate this temperature for an electron gas in a typical metal.

Problem 3.
The identical particles of the D-dimensional non interacting gas have dispersion relation $\epsilon_p = A|p|^\alpha$.

a. Calculate the density of states $\nu_D(\epsilon)$.

b. Calculate how PV depends on the Energy of the gas.

c. In case the particles are Fermions calculate the Fermi momentum p_F and Fermi energy ϵ_F of the gas as functions of particle density.

Problem 4. *Mean speed.*

a. Find the mean speed of the fermions in an ideal D-dimensional fermi gas at $T = 0$ (the dispersion is $\epsilon_p = p^2/2m$) in terms of v_F – Fermi velocity (velocity at ϵ_F).

b. Find the average kinetic energy of a fermion in terms of v_F.

Problem 5. *Electrons in metals.*
Find how the heat capacity of the D dimensional Fermi gas with $\epsilon(p) = \vec{p}^2/2m$ depends on the density at small temperatures.

Homework 15. Extra fun! Due May 2

Problem 1. *Chemical potential of the Bose gas.*
Find how the chemical potential of a D-dimensional Bose gas depends on temperature when the chemical potential is small.

Problem 2. *1993-Spring-SM-G-3*
Consider a non interacting gas of He^4 (spin$=0$) confined to a film (i.e., confined in $\hat{z}$ direction, but infinite in the x and y directions). Because of the confinement, the p_z component of the momentum is quantized into discrete levels, $n = 0, 1, \ldots$, so that the energy spectrum is

$$\epsilon_n(\mathbf{p}) = \frac{\mathbf{p}^2}{2m} + \epsilon_n,$$

where $\mathbf{p}$ is the two-dimensional (p_x, p_y) momentum, and $0 = \epsilon_0 < \epsilon_1 < \ldots \epsilon_n < \ldots$.

a. Write down the density of states for the system.

b. Write down the expression which determines the density of He^4 atoms n as a function of a chemical potential μ and temperature T. Evaluate integrals explicitly.

c. As $T \to 0$, i.e. $T \ll \epsilon_1$ find the limiting value of μ and how the specific heat depends on T in this limit.

d. Determine $\mu(N, V, T)$ in the limit $|\mu| \gg T$ (classical limit) assuming $\epsilon_n = \epsilon n$.