

## DARK MATTER EQUATION OF MOTION AND DENSITY PROFILES

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Cold dark matter simulations appear to disagree with the observations on small distance scales. Here we consider a modified description of CDM particles which is implied by a new fundamental theory: In this picture, the dark matter is composed of supersymmetric WIMPs, but they are scalar bosons with an unconventional equation of motion. The modified dynamics leads to much weaker gravitational binding, and therefore to a reduced tendency to form small clumps and central cusps.

Cold dark matter simulations provide a satisfactory description of the evolution of large-scale structure in the universe, but appear to disagree in two respects with the observations relevant to galactic halos<sup>1-13</sup>: First, the simulations demonstrate that standard CDM has too strong a tendency to form relatively small clumps. Second, the simulations yield cusps in the CDM density  $\rho(r)$  of the form<sup>2</sup>

$$\rho(r) \propto \frac{1}{(r/r_s)(1+r/r_s)^2} \propto r^{-1} \text{ as } r \rightarrow 0 \quad (1)$$

whereas analyses of the observational data seem to indicate a flattening of the density as  $r \rightarrow 0$ .

In this paper we consider a modified description of dark matter particles which is implied by a new fundamental theory<sup>14,15</sup>: The dark matter is composed of supersymmetric WIMPs, just as in more conventional models, but in the present picture these are scalar bosons with an unconventional equation of motion (rather than, e.g., neutralinos with standard relativistic or nonrelativistic dynamics). The modified dynamics leads to much weaker binding in a gravitational field, and therefore to a reduced tendency to form clumps and cusps. The compatibility of this modified dynamics with standard physics is discussed elsewhere<sup>14,15</sup>. The modifications are significant only for (i) fermions at extremely high energy and (ii) fundamental scalar bosons that have not yet been observed. The modified dynamics retains many of the features of Lorentz invariance, such as rotational invariance, CPT invariance, and the requirement that  $\omega = |\vec{p}|$  for massless particles. It also appears to be consistent with even the most sensitive experimental tests of Lorentz invariance.

To be more specific, in the present theory the dark matter is composed of fundamental scalar bosons with an R-parity of -1 and with the equation of motion

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi + i\overline{m} \sigma^\mu \partial_\mu \phi - m^2 \phi = 0 \quad (2)$$

where  $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  is the Minkowski metric tensor,  $m$  is the particle mass, and  $\overline{m}$  is an energy which is comparable to or larger than 1 TeV. For a plane-wave solution  $\phi \propto \exp(i\vec{p} \cdot \vec{x} - i\omega t)$  this becomes

$$[(\omega^2 - \vec{p}^2) + \overline{m}(\omega - \vec{\sigma} \cdot \vec{p}) - m^2] \phi = 0 \quad (3)$$

with the solutions

$$\omega = -\frac{1}{2}\bar{m} \pm \left[ \left( p \pm \frac{1}{2}\bar{m} \right)^2 + m^2 \right]^{1/2}, \quad p = |\vec{p}|. \quad (4)$$

The  $\pm$  signs are independent, but negative frequencies correspond to antiparticles in the usual way<sup>15</sup>. For simplicity, therefore, let us focus on the positive-frequency solution with particle velocity

$$v = \frac{\partial\omega}{\partial p} = \left[ 1 + m^2 \left( p + \frac{1}{2}\bar{m} \right)^{-2} \right]^{-1/2}. \quad (5)$$

The kinetic energy is

$$T = \int v(p) dp = m(\gamma - \gamma_0) \quad (6)$$

where

$$\gamma = \frac{1}{\sqrt{1-v^2}}, \quad \gamma_0 = \frac{1}{\sqrt{1-v_0^2}}, \quad v_0 = \left[ 1 + \left( \frac{2m}{\bar{m}} \right)^2 \right]^{-1/2}. \quad (7)$$

(As  $\bar{m} \rightarrow 0$ , we regain the standard result of special relativity in units with  $\hbar = c = 1$ :  $T = m(\gamma - 1)$ .) With an integration by parts, (6) can also be written in the form

$$T = pv + (\bar{m}/2)(v - v_0) + m(\gamma^{-1} - \gamma_0^{-1}). \quad (8)$$

Now consider a circular orbit of radius  $r$  about a mass  $M$ . The general formula for the centripetal force implies that

$$pv/r = GMm/r^2 \quad (9)$$

with  $V = -GMm/r$ , so

$$E = T + V = (\bar{m}/2)(v - v_0) + m(\gamma^{-1} - \gamma_0^{-1}). \quad (10)$$

As  $m/\bar{m} \rightarrow 0$ , we obtain  $v, v_0 \rightarrow 1$  and  $\gamma^{-1}, \gamma_0^{-1} \rightarrow 0$  (since  $v_0 \leq v \leq 1$ ). It follows that

$$E \rightarrow 0 \quad \text{as} \quad m/\bar{m} \rightarrow 0 \quad (11)$$

and the particles are only very weakly bound for  $m/\bar{m} \ll 1$ . This is qualitatively the same result as in special relativity when  $m \rightarrow 0$ .

Figure 1 shows the binding energy  $-E$  as a function of  $r$  with  $\bar{m}/m$  taken to be 10. In the same figure we show  $-E(r)$  with the particles obeying the standard dynamics of special relativity. (Newtonian gravity was used, with the gravitational mass taken to be the rest mass. A graph for nonrelativistic dynamics would be indistinguishable from the dashed line of Fig. 1, since relativistic corrections are not important for  $r \gg 2GM$ .) The binding is orders of magnitude weaker in the present theory.

The slope in Fig. 1 for the standard dynamics is -1, since

$$-E(r) = -(T + V) = -\frac{1}{2}V = \frac{1}{2} \frac{GM}{r} m \propto \frac{1}{r} \quad \text{for} \quad r \gg 2GM. \quad (12)$$

The slope for the dynamics of the present theory is -2, indicating that  $-E(r) \propto 1/r^2$ . One can, in fact, obtain a simple expression for  $E(r)$ , by expanding (5) in powers of  $p$ ,

Figure 1. Binding energy  $-E(r)$ , with the equation of motion (2) and  $\bar{m}/m$  taken to be 10 (solid line) compared with the dynamics of standard special relativity (dashed line). The binding is much weaker in the present theory.

substituting the resulting expression into (9), inverting to find  $p$  as a function of  $1/r$ , and then substituting into (10). The final relation is

$$-E(r) = \frac{1}{2} \left( \frac{2m}{\bar{m}} \right)^3 \left( \frac{GM}{r} \right)^2 m \propto \frac{1}{r^2} \quad \text{for } r \gg 2GM. \quad (13)$$

Let us now consider a simplistic model in which

$$\rho(r) = \frac{a}{r^\alpha} \quad \text{so that} \quad M(r) = \frac{4\pi a}{3-\alpha} r^{3-\alpha} \quad \text{and} \quad F(r) = -\frac{dV(r)}{dr} = -\frac{GM(r)m}{r^2}. \quad (14)$$

The virial theorem states that

$$\langle \vec{p} \cdot \vec{v} \rangle = - \langle \vec{F} \cdot \vec{r} \rangle = \langle GM(r)m/r \rangle. \quad (15)$$

On the other hand, the equipartition theorem implies that

$$\langle \vec{p} \cdot \vec{v} \rangle = \langle p^k \partial H / \partial p^k \rangle = 3\tau \quad (16)$$

where  $H$  is the classical Hamiltonian and  $\tau$  is the temperature in units with  $k_B = 1$ . Then an isothermal model corresponds to  $\langle \vec{p} \cdot \vec{v} \rangle = \text{constant}$ , or

$$M(r) \propto r \quad , \quad V(r) \propto \log r + \text{constant} \quad , \quad \rho(r) \propto r^{-2}. \quad (17)$$

This is, however, exactly the same qualitative result that we would have obtained with standard nonrelativistic or relativistic dynamics. Furthermore, it is easy to see that a complete solution for an isothermal model with a Boltzmann distribution<sup>16,17</sup> also gives (17) at large  $r$ , which is again the same result as one obtains with standard dynamics.

The results of (11) or (13) and Fig. 1 lead us to conclude that the present theory predicts much weaker gravitational binding than conventional cold dark matter models. As a result, there is a greatly reduced tendency for particles to be bound at small values of  $r$ , and one does not expect the formation of cusps or clumps on small distance scales. On the other hand, it is plausible that the successes of standard CDM models on large distance scales may be preserved in the present theory, since an isothermal model yields (17) for either standard dynamics or the modified dynamics represented by (5). We intend to test these conclusions with simulations.

The present theory has implications for terrestrial dark matter searches<sup>18</sup>: Since the velocity  $v$  in (9) or (15) is  $\sim c$  rather than  $10^{-3}c$ , the momentum  $p$  is reduced by a factor of roughly  $10^{-3}$  compared to the expectation for conventional cold dark matter.

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