

Searching for Lorentz Violation

Roland E. Allen and Seiichirou Yokoo

Physics Department, Texas A&M University, College Station, Texas 77843

Astrophysical, terrestrial, and space-based searches for Lorentz violation are very briefly reviewed. Such searches are motivated by the fact that all superunified theories (and other theories that attempt to include quantum gravity) have some potential for observable violations of Lorentz invariance. Another motivation is the exquisite sensitivity of certain well-designed experiments and observations to particular forms of Lorentz violation. We also review some new predictions of a specific Lorentz-violating theory: If a fundamental energy $\overline{m}c^2$ in this theory lies below the usual GZK cutoff E_{GZK} , the cutoff is shifted to infinite energy; i.e., it no longer exists. On the other hand, if $\overline{m}c^2$ lies above E_{GZK} , there is a high-energy branch of the fermion dispersion relation which provides an alternative mechanism for super-GZK cosmic-ray protons.

1. Introduction

During the past few years there has been increasingly widespread interest in possible violations of Lorentz invariance [1-29]. There are several motivations for this interest.

Theoretical: Every current candidate for a superunified theory contains some potential for Lorentz violation, and the same is true for more restricted theories which attempt to treat quantum gravity alone. (By a “superunified theory” we mean one which includes all known physical phenomena, and which is valid up to the Planck energy.) Theories with potential for Lorentz violation include superstring/M/brane theories, canonical and loop quantum gravity, noncommutative spacetime geometry, nontrivial spacetime topology, discrete spacetime structure at the Planck length, a variable speed of light or variable physical constants, various other *ad hoc* theories, including one that specifically addresses the GZK cutoff [2], and a fundamental theory which will be considered later in this paper [1]. Even in a theory which has Lorentz invariance at the most fundamental level, this symmetry can be spontaneously broken if some field acquires a vacuum expectation value which breaks rotational invariance or invariance under a boost. (It should be mentioned that cosmology already provides a preferred frame of reference – namely a comoving frame, in which the cosmic background radiation

does not have a dipole anisotropy – but this is not considered to be a breaking of Lorentz symmetry.) A second mechanism for Lorentz violation is the “quantum foam” of Hawking and Wheeler, originally envisioned in the context of canonical or path-integral quantization of Einstein gravity, but now generalized to other theories with quantum gravity. A third possibility is a theory in which Lorentz invariance is not postulated to be an exact fundamental symmetry, but instead emerges as a low-energy symmetry [1].

Experimental: Both terrestrial [3-14] and space-based [15-20] experiments have been designed with exquisite precision which would permit detection of even tiny deviations from certain aspects of Lorentz invariance. The systems include atoms, charged particles in traps, masers, cavity-stabilized oscillators, muons, neutrons, kaons, and other neutral mesons.

Observational: Particles traveling over cosmological distances from bright sources (including pulsars, supernovae, blazars, and gamma ray bursters) allow long-baseline tests which are again sensitive to even tiny deviations from particular forms of Lorentz violation [21-26].

Recall that Lorentz invariance in the context of general relativity means *local* Lorentz invariance, or an invariance of the action under rotations and boosts involving locally inertial frames of reference. There is clearly a connection with the equivalence principle, which can also be tested

in, e.g., space-based experiments. There is a close connection with CPT invariance as well: According to the CPT theorem, Lorentz invariance implies CPT invariance (with the supplementary assumptions of unitarity and locality). It follows that CPT violation implies Lorentz violation, although the reverse is not necessarily true. Finally, there is a connection to the spin-statistics theorem, which follows from Lorentz invariance and microcausality.

We know that P (in the 1950s) and CP (in the 1960s) have previously been found not to be inviolate symmetries, for reasons that are now understood in terms of the standard electroweak theory and the CKM matrix. Perhaps CPT and Lorentz symmetry are also not inviolate.

The most extensive theoretical program for systematizing potential forms of Lorentz violation and their experimental signals is that of Kostelecký and coworkers [3,4,9-18,20,26]. Their philosophy is to add small phenomenological Lorentz-violating terms to the Lagrangian of the Standard Model, and then interact with a wide variety of experiments that can detect such deviations from exact Lorentz or CPT invariance. The point of view of this group is rather conservative: The fundamental theory (e.g., string theory) is pictured as Lorentz-invariant, with Lorentz or CPT violation arising from some form of symmetry-breaking – for example, with a vector field or more general tensor field acquiring a vacuum expectation value. Their work has stimulated a considerable amount of experimental activity, with further experiments planned for both terrestrial and space-based laboratories.

So far there is no undisputed evidence for Lorentz violation, and the only solid results from both experiment and observation are strong constraints on particular ways in which this symmetry might be broken. As an example of an astrophysical constraint, we mention a recent paper by Stecker and Glashow [23], in which they conclude “We use the recent reanalysis of multi-TeV [up to 20 TeV] gamma-ray observations of [the blazar] Mrk 501 to constrain the Lorentz invariance breaking parameter involving the maximum electron velocity. Our limit is two orders of magnitude better than that obtained from the maxi-

imum observed cosmic-ray electron energy.” Their analysis involves the processes

$$\gamma + \gamma_{infrared} \rightarrow e^+ + e^- \quad \text{if } c_e > c_\gamma \quad (1)$$

which can lead to inconsistency with the observation of 20 TeV photons and

$$\gamma \rightarrow e^+ + e^- \quad \text{if } c_e < c_\gamma \quad (2)$$

which can lead to inconsistency with the observation of 50 TeV photons.

Another example of astrophysical constraints is the series of analyses by Jacobson et al. [21-24]. In Ref. 22, Jacobson, Liberati, Mattingly, and Stecker state “We strengthen the constraints on possible Lorentz symmetry violation (LV) of order E/M_{Planck} for electrons and photons in the framework of effective field theory (EFT). The new constraints use (i) the absence of vacuum birefringence in the recently observed polarization of MeV emission from a gamma ray burst and (ii) the absence of vacuum Čerenkov radiation from the synchrotron electrons in the Crab nebula, improving the previous bounds by eleven and four orders of magnitude respectively.”

Jacobson, Liberati, and Mattingly [21] have obtained a very strong constraint on a dispersion relation with a cubic term in the expression for E^2 :

$$E^2 = p^2 + p^3/M. \quad (3)$$

However, the constraint is less stringent for what may be the more natural form with a quartic term:

$$E^2 = p^2 + p^4/M^2. \quad (4)$$

Below we will derive the dispersion relation for a fundamental Lorentz-violating theory [1,28,29] and will find that it is easily consistent with these constraints, since it has a form quite different from either of those above.

Coleman and Glashow [2] proposed that the limiting velocity of protons, electrons, etc. may be very slightly different from the speed of light. (See also Ref. 24.) This is an *ad hoc* proposal, motivated by the apparent absence of a Greisen-Zatsepin-Kuz'min (GZK) cutoff: Ultrahigh energy cosmic ray protons colliding with the cosmic

microwave background radiation should produce pions,

$$p + \gamma_{cmb} \rightarrow p + \pi^0. \quad (5)$$

There should consequently be a cutoff in the spectrum of observed protons at about 50 EeV (or 5×10^7 TeV), if they were created in processes at distances of more than about 100 Mpc. But up to 300 EeV cosmic rays (presumably protons) appear to be observed, although this is not entirely certain [38], and there are also theoretical ideas for a closer origin [36].

We conclude by mentioning some reviews of terrestrial and space-based experiments.

Two reviews of atomic experiments to test both Lorentz and CPT symmetries, by Bluhm [14], describe the following: (1) Penning trap experiments with electrons and positrons, and with protons and antiprotons, which look for differences in frequencies or sidereal time variations; (2) clock comparison experiments, with clock frequencies typically those of hyperfine or Zeeman transitions; (3) hydrogen and antihydrogen experiments involving ground-state Zeeman hyperfine transitions (at Harvard) or 1S-2S transitions (proposed at CERN); (4) a spin-polarized torsion pendulum experiment (at the University of Washington); (5) muon and muonium experiments.

Two reviews by Russell [18] discuss clock-based experiments to test Lorentz and CPT invariance in space. Such experiments will probe the effects of variations in both orientation and velocity. Among the systems are H masers, laser-cooled Cs and Rb clocks, and superconducting microwave cavity oscillators. A number of specific space missions have been planned or proposed.

Finally, a review by Kostelecký [26] contains a discussion of experiments involving neutral meson (e.g. kaon) oscillations, a dual nuclear Zeeman He-Xe maser, and cosmological birefringence, in addition to the systems mentioned above.

Now let us turn to a specific Lorentz-violating theory [1] and some of its new predictions [27]. We begin with the action for a single initially massless Weyl fermion field [28], and with the coupling to gauge fields and variations in e_μ^α ne-

glected:

$$S_1 = \int d^4x \mathcal{L}_1 \quad (6)$$

$$\mathcal{L}_1 = \frac{1}{2} e \psi_1^\dagger \left(\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \sigma^\alpha \partial_\mu \right) \psi_1 + h.c.$$

Here M is a fundamental mass which is comparable to the Planck mass, $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric tensor, σ^k is a Pauli matrix, and σ^0 is the 2×2 identity matrix. Also, e_α^μ is the gravitational vierbein, which determines the gravitational metric tensor $g_{\mu\nu}$ through the relations

$$g_{\mu\nu} = \eta_{\alpha\beta} e_\mu^\alpha e_\nu^\beta, \quad e_\alpha^\mu e_\nu^\alpha = \delta_\nu^\mu. \quad (7)$$

A factor of $e^{-1/2}$ has been absorbed in ψ_1 , where

$$e = \det e_\mu^\alpha = (-\det g_{\mu\nu})^{1/2}. \quad (8)$$

Fundamental units are used, with $\hbar = c = 1$. Finally, “*h.c.*” means “Hermitian conjugate”, and \mathcal{L}_1 has been written in its more fundamental and manifestly Hermitian form. The action (6) is invariant under a rotation, but it is not invariant under a Lorentz boost because of the first term. (Recall that the transformation matrix $\Lambda_{1/2}$ is unitary for a rotation and not for a boost [30].) At low energies, however, this term is negligible and full Lorentz invariance is regained.

As before, we choose the directions of the spacetime coordinate axes to be such that all the e_α^μ are positive. If the term involving M is neglected, \mathcal{L}_1 has the form appropriate for a right-handed field. I.e., in order for S_1 to be invariant under local Lorentz transformations at low energy, all the fundamental fermionic fields ψ_1 must be taken to transform as right-handed spinors. This is the reverse of the usual convention in grand-unified theories, where they are all taken to be left-handed. However, we can convert ψ_1 to a left-handed field through the following well-known procedure [30,31,32], which is based on the fact that $(\sigma^2)^2 = 1$, $(\sigma^2)^\dagger = \sigma^2$, $(\sigma^2)^* = -\sigma^2$, and

$$\sigma^2 \sigma^k \sigma^2 = -(\sigma^k)^*. \quad (9)$$

Let

$$\psi_L = \sigma^2 \psi_1^* \quad \text{or} \quad \psi_1 = (\sigma^2 \psi_L)^* \quad (10)$$

and substitute into (6), using (in the fourth step below) the fact that Grassmann fields like ψ_L anticommute:

$$\begin{aligned}
& \mathcal{L}_1 \\
&= \frac{1}{2} e \left[(\sigma^2 \psi_L)^* \right]^\dagger \\
&\quad \times \left(\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \sigma^\alpha \partial_\mu \right) (\sigma^2 \psi_L)^* + h.c. \\
&= \frac{1}{2} e \left[\left(\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \sigma^\alpha \partial_\mu \right) (\sigma^2 \psi_L)^* \right]^\dagger \\
&\quad \times (\sigma^2 \psi_L)^* + h.c. \\
&= \frac{1}{2} e \left[\left(\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \sigma^\alpha \partial_\mu \right)^* (\sigma^2 \psi_L) \right]^T \\
&\quad \times (\sigma^2 \psi_L)^* + h.c. \\
&= -\frac{1}{2} e \left[(\sigma^2 \psi_L)^* \right]^T \\
&\quad \times \left[\left(\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu - i e_\alpha^\mu (\sigma^\alpha)^* \partial_\mu \right) (\sigma^2 \psi_L) \right] \\
&\hspace{15em} + h.c. \\
&= \frac{1}{2} e \psi_L^\dagger (\sigma^2)^\dagger \\
&\quad \times \left[\left(-\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu (\sigma^\alpha)^* \partial_\mu \right) (\sigma^2 \psi_L) \right] \\
&\hspace{15em} + h.c. \\
&= \frac{1}{2} e \psi_L^\dagger \\
&\quad \times \left[\left(-\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu (\sigma^2 \sigma^\alpha \sigma^2)^* \partial_\mu \right) \psi_L \right] \\
&\hspace{15em} + h.c. \\
&= \frac{1}{2} e \psi_L^\dagger \\
&\quad \times \left[\left(-\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \bar{\sigma}^\alpha \partial_\mu \right) \psi_L \right] + h.c. \tag{11}
\end{aligned}$$

where $\bar{\sigma}^0 = \sigma^0$ and $\bar{\sigma}^k = -\sigma^k$. Then ψ_L has the Lagrangian appropriate for a left-handed field (when the term containing M is neglected), and the definition (10) implies that it transforms as a left-handed field if ψ_1 is required to transform as a right-handed field [30,31,32].

If ψ_L corresponds to a particle with a Dirac mass m , it is coupled through this mass to a right-handed field ψ_R . (The origin of this mass – i.e.,

the coupling to a Higgs field which acquires a vev – is not considered in the present paper.) The Lagrangian density for this pair of fields is then given by

$$\begin{aligned}
e^{-1} \mathcal{L} &= \psi_R^\dagger \left(\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \sigma^\alpha \partial_\mu \right) \psi_R \\
&\quad + \psi_L^\dagger \left(-\frac{1}{2M} \eta^{\mu\nu} \partial_\nu \partial_\mu + i e_\alpha^\mu \bar{\sigma}^\alpha \partial_\mu \right) \psi_L \\
&\quad - m \psi_L^\dagger \psi_L - m \psi_L^\dagger \psi_R \tag{12}
\end{aligned}$$

after an integration by parts to get the more familiar form. The resulting equations of motion can be written as

$$\begin{aligned}
& \left[\frac{1}{2M} (-e_\alpha^0 e_0^\alpha \partial_0 \partial_0 + e_\alpha^k e_l^\alpha \partial_l \partial_k) + i e_\alpha^\mu \sigma^\alpha \partial_\mu \right] \psi_R \\
&\hspace{15em} - m \psi_L = 0 \\
& \left[-\frac{1}{2M} (-e_\alpha^0 e_0^\alpha \partial_0 \partial_0 + e_\alpha^k e_l^\alpha \partial_l \partial_k) + i e_\alpha^\mu \bar{\sigma}^\alpha \partial_\mu \right] \psi_L \\
&\hspace{15em} - m \psi_R = 0
\end{aligned}$$

with $k, l = 1, 2, 3$. For simplicity, let us assume spatial isotropy and write

$$e_\alpha^k = \lambda \delta_\alpha^k, \quad e_k^\alpha = \lambda^{-1} \delta_k^\alpha = \lambda^{-2} e_\alpha^k \tag{13}$$

$$e_\alpha^0 = \lambda_0 \delta_\alpha^0, \quad e_0^\alpha = \lambda_0^{-1} \delta_0^\alpha = \lambda_0^{-2} e_\alpha^0. \tag{14}$$

After transforming to a locally inertial frame of reference, in which $e_\alpha^\mu = \delta_\alpha^\mu$, we have

$$\begin{aligned}
& [(-\beta \partial_0 \partial_0 + \alpha \partial_k \partial_k) + i (\sigma^0 \partial_0 + \sigma^k \partial_k)] \psi_R \\
&\hspace{15em} - m \psi_L = 0 \tag{15}
\end{aligned}$$

$$\begin{aligned}
& [(-\beta \partial_0 \partial_0 + \alpha \partial_k \partial_k) + i (\sigma^0 \partial_0 - \sigma^k \partial_k)] \psi_L \\
&\hspace{15em} - m \psi_R = 0 \tag{16}
\end{aligned}$$

where

$$\alpha = (2\lambda^2 M)^{-1}, \quad \beta = (2\lambda_0^2 M)^{-1}. \tag{17}$$

At fixed energy E and 3-momentum \vec{p} , these become

$$\vec{\sigma} \cdot \vec{p} \psi_R = [(\beta E^2 - \alpha p^2) + E] \psi_R - m \psi_L \tag{18}$$

$$\vec{\sigma} \cdot \vec{p} \psi_L = [(\beta E^2 - \alpha p^2) - E] \psi_L + m \psi_R \tag{19}$$

where p is the magnitude of \vec{p} , or, since $(\vec{\sigma} \cdot \vec{p})^2 = p^2$,

$$\left[(p^2 + m^2) - [(\beta E^2 - \alpha p^2) + E]^2 \right] \psi_R$$

$$= -2m (\beta E^2 - \alpha p^2) \psi_L \quad (20)$$

$$\begin{aligned} & \left[(p^2 + m^2) - [(\beta E^2 - \alpha p^2) - E]^2 \right] \psi_L \\ & = 2m (\beta E^2 - \alpha p^2) \psi_R. \end{aligned} \quad (21)$$

We then obtain

$$A_+ A_- = - [2m (\beta E^2 - \alpha p^2)]^2 \quad (22)$$

$$A_+ = (p^2 + m^2) - [(\beta E^2 - \alpha p^2) + E]^2 \quad (23)$$

$$A_- = (p^2 + m^2) - [(\beta E^2 - \alpha p^2) - E]^2 \quad (24)$$

and (discarding the unphysical root)

$$\begin{aligned} E^2 &= (p^2 + m^2) + (\beta E^2 - \alpha p^2) \\ & \times \left[2 (E^2 - m^2)^{1/2} - (\beta E^2 - \alpha p^2) \right]. \end{aligned} \quad (25)$$

If m^2 is neglected, (22)-(24) imply that the solutions are

$$E = \mp \frac{1}{2\beta} \pm \frac{1}{2\beta} [1 + 4\beta (\alpha p^2 \pm p)]^{1/2} \quad (26)$$

$$= \mp \frac{1}{2\beta} \pm \frac{1}{2\beta} [(1 \pm 2\beta p)^2 + 4\beta \gamma p^2]^{1/2} \quad (27)$$

where $\gamma = \alpha - \beta$ and the signs are independent.

The various solutions lead to interesting possibilities for new physics which will be considered in detail elsewhere. For the moment, however, consider only the normal branch, for which the first sign is $-$ and the last two signs are both $+$. The velocity is then

$$\begin{aligned} v &= \partial E / \partial p \quad (28) \\ &= \left[(1 + 2\beta p)^2 + 4\beta \gamma p^2 \right]^{-1/2} (1 + 2\beta p + 2\gamma p) \\ &= \left[1 + 4\gamma \frac{p + \alpha p^2}{1 + 4\beta p + 4\beta \alpha p^2} \right]^{1/2}. \end{aligned} \quad (29)$$

It follows that

$$v > 1 \quad \text{if } \alpha > \beta \quad \text{and} \quad v < 1 \quad \text{if } \alpha < \beta. \quad (30)$$

As we will find below, the first possibility would imply vacuum Čerenkov radiation, and the second pair production in vacuum, so the only plausible possibility is

$$\alpha = \beta \quad \text{which implies that} \quad v = 1. \quad (31)$$

(In the present paper we do not try to explain the origin of this condition, but simply accept it as

a phenomenological constraint on a cosmological scale, far from local gravitational sources.) Then (27) becomes

$$\begin{aligned} E &= \frac{\bar{m}}{2} \left[\mp 1 \pm \left(1 \pm \frac{2}{\bar{m}} p \right) \right] \quad (32) \\ &= p, -p, -\bar{m} + p, -\bar{m} - p, \bar{m} + p, \bar{m} - p, p, -p \end{aligned}$$

where

$$\bar{m} = \beta^{-1}. \quad (33)$$

All massless particles thus travel at the speed of light $c = 1$. As usual, the destruction operators for negative energies are reinterpreted as creation operators for antiparticles with positive energies [28]. The implications of negative group velocities for particles and antiparticles will be considered elsewhere, and the existence of very high-energy branches in the dispersion relation will be discussed below.

For a nonzero mass, but with $\beta = \alpha$, (25) gives

$$\begin{aligned} E^2 &= (p^2 + m^2) + \alpha (E^2 - p^2) \\ & \times \left[2 (E^2 - m^2)^{1/2} - \alpha (E^2 - p^2) \right]. \end{aligned} \quad (34)$$

We are primarily interested in particles with large energy, for which m^2 (or more precisely m^2/p^2) can be treated as a perturbation:

$$E^2 = [E^2]_{m^2=0} + [\partial E^2 / \partial m^2]_{m^2=0} m^2. \quad (35)$$

From (34) we obtain

$$\begin{aligned} \partial E^2 / \partial m^2 &= [1 - \alpha (E^2 - p^2) |E|^{-1}] \\ & \times \left[1 + \alpha [\alpha (E^2 - p^2) - 2|E| \right. \\ & \left. + (E^2 - p^2) (\alpha - |E|^{-1}) \right]^{-1} \end{aligned} \quad (36)$$

when $\partial E^2 / \partial m^2$ is evaluated at $m = 0$. For the solutions with $E^2 = p^2$ (when $m = 0$), this becomes

$$\left[\frac{\partial E^2}{\partial m^2} \right]_{m^2=0} = [1 - 2\alpha p]^{-1} \quad (37)$$

or

$$E^2 = p^2 + \frac{m^2}{1 - 2\alpha p} \quad (38)$$

to lowest order in m^2/p^2 , which reproduces the usual result $E^2 = p^2 + m^2$ as $\alpha p \rightarrow 0$. The particle

velocity is then $v = \partial E / \partial p = (\partial E^2 / \partial p) / (2E)$.
or

$$v = \left[1 + \frac{\alpha m^2}{p(1-2\alpha p)^2} \right] \times \left[1 + \frac{m^2}{p^2(1-2\alpha p)} \right]^{-1/2} \quad (39)$$

$$\approx 1 - \frac{m^2}{2p^2} \frac{1-4\alpha p}{(1-2\alpha p)^2} \quad (40)$$

$$= 1 - \frac{m^2}{2p^2} \left(1 - \frac{1}{((2\alpha p)^{-1} - 1)^2} \right) \quad (41)$$

so that

$$v \rightarrow 1 \text{ as } p \rightarrow \infty \quad (42)$$

and

$$v < 1 \text{ for } p < \bar{m}/4. \quad (43)$$

Furthermore, it is easy to see that particles with $p > \bar{m}/4$ will be superluminal by only an extremely small amount except when p lies in a narrow range of energies near $p = \bar{m}/2$ (i.e., $\alpha p = 1/2$): Letting $\alpha p = 1/2 + \delta$ in (41), we obtain

$$v - 1 \approx \frac{1}{2} \frac{m^2}{\bar{m}^2} \frac{1}{\delta^2}. \quad (44)$$

For example, if m is ~ 1 GeV and \bar{m} were $\sim 10^{10}$ TeV, then $\delta \sim 10^{-4}$ would imply that $(v - 1) \sim 10^{-18}$, and the deviation falls like $1/\delta^2$. However, it should also be emphasized that superluminal velocities of any size are not a violation of causality in the present theory, because all signals still propagate forward in time in the initial (preferred) frame of reference.

For the solutions with $E^2 = (\bar{m} + p)^2$ (when $m = 0$), we obtain

$$\begin{aligned} \frac{\partial E^2}{\partial m^2} &= \left[1 - (1 + 2\alpha p)(1 + \alpha p)^{-1} \right] \\ &\times \left[1 + \left[(1 + 2\alpha p) - 2(1 + \alpha p) \right. \right. \\ &\left. \left. + (1 + 2\alpha p) \left(1 - (1 + \alpha p)^{-1} \right) \right] \right]^{-1} \quad (45) \\ &= \left[1 - \frac{(1 + 2\alpha p)}{(1 + \alpha p)} \right] \left[(1 + 2\alpha p) - \frac{(1 + 2\alpha p)}{(1 + \alpha p)} \right]^{-1} \end{aligned}$$

$$= -\frac{1}{1 + 2\alpha p} \quad (46)$$

since $\bar{m} = \alpha^{-1}$. We then have

$$E^2 = (\bar{m} + p)^2 - \frac{1}{1 + 2\alpha p} m^2 \quad (47)$$

again to lowest order in m^2/p^2 , and

$$v = \left[(\bar{m} + p) + \frac{\alpha}{(1 + 2\alpha p)^2} m^2 \right] \times \left[(\bar{m} + p)^2 - \frac{1}{(1 + 2\alpha p)} m^2 \right]^{-1/2} \quad (48)$$

$$\approx 1 + \frac{(3 + 4\alpha p)}{2(1 + \alpha p)^2} \frac{m^2}{(1 + 2\alpha p)^2 \bar{m}^2} \quad (49)$$

so

$$v \rightarrow 1 \text{ as } p \rightarrow \infty \quad (50)$$

and

$$v \rightarrow 1 + \frac{3}{2} \frac{m^2}{\bar{m}^2} \equiv v_0 \text{ as } p \rightarrow 0. \quad (51)$$

These particles are then slightly superluminal. For example, if m is ~ 1 GeV and \bar{m} were $\sim 10^{10}$ TeV, then $v_0 - 1$ would be $\sim 10^{-26}$. Again, however, a superluminal velocity of any size in the present theory does not imply a violation of causality.

Now let us turn to the GZK cutoff, [2,33-38] which results from collision of a charged fermion with a blackbody photon. The incoming photon has energy ω and momentum $(-\omega \cos \theta, -\omega \sin \theta, 0)$ in units with $\hbar = c = 1$. The incoming fermion has mass m_a , energy E , and momentum $(p, 0, 0)$. The outgoing fermion has mass m_b , energy $E + \omega$, and momentum $(p - \omega \cos \theta, -\omega \sin \theta, 0)$. If ω is small (as it is for a blackbody photon), it is valid to use

$$\Delta E = \frac{\partial E}{\partial p_x} \Delta p_x + \frac{\partial E}{\partial p_y} \Delta p_y + \frac{\partial E}{\partial m^2} \Delta m^2 \quad (52)$$

with $\partial E / \partial p_k = v p_k / p$ and $v = \partial E / \partial p$, so that

$$1 + v \cos \theta = \frac{\partial E}{\partial m^2} \frac{\Delta m^2}{\omega} \quad (53)$$

and the threshold is for a head-on collision. Consider the normal branch of the dispersion relation, described by (37), (38), and (41). With $\partial E / \partial m^2 = \partial E^2 / \partial m^2 / (2E)$, (53) becomes

$$2(1 + v \cos \theta)(1 - 2\alpha p)p = \Delta m^2 / \omega \quad (54)$$

where m^2 has been neglected in comparison to p^2 . This quadratic equation in p has a solution only if

$$2(1 + v \cos \theta) > 8\alpha \Delta m^2 / \omega \quad (55)$$

or

$$\bar{m} > 8(\Delta m^2 / 4\omega) \quad (56)$$

since again $\alpha^{-1} = \bar{m}$.

If \bar{m} is lower than eight times the standard GZK cutoff energy, therefore, the present theory implies that the GZK cutoff is eliminated. The reason for this is that the $(1 - 2p/\bar{m})$ factor in (38) and (54) tends to push the cutoff up to higher energies even if \bar{m} is large, and completely eliminates it if \bar{m} falls below $2\Delta m^2/\omega$.

Now consider the high-energy branch of (46), (47), and (49), for which $E = \bar{m} + p$ when the mass is neglected. According to (47) and (51), particles on this branch travel at essentially the speed of light and have an enormous energy

$$E = \sqrt{\bar{m}^2 - m^2} \approx \bar{m} \quad (57)$$

even if they have lost essentially all their momentum. If such a particle collides with another particle, it can undergo a transition to the lower branch, with the two particles recoiling in opposite directions to conserve momentum. Either of them can then enter the Earth's atmosphere with extraordinary energy comparable to \bar{m} .

If \bar{m} is larger than the standard GZK cutoff energy, therefore, the present theory provides an alternative mechanism for cosmic ray particles above the GZK cutoff. Namely, a particle on the very high-energy branch (47) can travel cosmological distances without losing more energy, once it has fallen to the minimum energy \bar{m} for this branch, and can then undergo a collision relatively near the Earth which releases this energy.

Finally, let us return to the standard astrophysical threat to a Lorentz-violating theory, that it may lead to disagreement with the observations of high-energy matter particles or photons, including prediction of new processes in the vacuum which are not observed. An example is vacuum Čerenkov radiation. Conservation of energy and

momentum implies that

$$-\omega = \Delta E = \frac{\partial E}{\partial p_x} \Delta p_x + \frac{\partial E}{\partial p_y} \Delta p_y = \frac{\partial E}{\partial p} (-\omega \cos \theta)$$

so this process can occur if

$$v = 1/\cos \theta \geq 1. \quad (58)$$

If we were to have $\beta < \alpha$, the particle velocity at high momentum would be greater than the velocity of light, and there would be a radiation of photons in vacuum which is in conflict with observation [2].

Next consider the process $photon \rightarrow e^+e^-$, which will occur if

$$2E(p) = \omega = 2p \cos \theta. \quad (59)$$

The normal branch for $E(p)$ corresponds to the choice of signs $-, +, +$ in (27). For 20 or 50 TeV photons, it is reasonable to assume $\alpha p, \beta p \ll 1$, and keep only the terms of first and second order in α and β . Then (27) gives $E(p) \approx p + \gamma p^2$. When the mass term in $E(p)^2$ is also treated only to lowest order in α and β , it is simply m^2 . (E.g., see (38).) For a massive particle, therefore, $E(p)$ becomes

$$E(p) \approx \left[(p + \gamma p^2)^2 + m^2 \right]^{1/2} \approx p + \gamma p^2 + m^2/2p$$

and the condition for vacuum pair production is

$$1 + \gamma p + m^2/2p^2 = \cos \theta. \quad (60)$$

For $\gamma < 0$ this will have a solution if

$$p^3 > m^2/2|\gamma|. \quad (61)$$

Since observations indicate that 20 TeV photons do not decay in vacuum, $|\gamma|^{-1}$ must lie above the Planck energy.

If $\beta = \alpha$, or $\gamma = 0$, the unphysical processes considered above do not occur. More broadly, since many features of Lorentz invariance are retained in the present theory (including rotational invariance and the same velocity c for all massless particles) it appears that the theory is consistent with experiment and observation. The theory is also fundamental, rather than *ad hoc*, and it leads to various new predictions. Here we have emphasized one feature: the behavior of fermions at extremely high energy, and the possible implications for the GZK cutoff.

REFERENCES

1. R. E. Allen, Int. J. Mod. Phys. A 12 (1997) 2385, hep-th/9612041.
2. S. Coleman and S. L. Glashow, Phys. Lett. B 405 (1997) 249; Phys. Rev. D 59 (1999) 116008, hep-ph/9812418.
3. R. Bluhm, V. A. Kostelecký, N. Russell, Phys. Rev. D 57 (1998) 3932, hep-ph/9809543.
4. D. Colladay and V. A. Kostelecký, Phys. Rev. D 58 (1998) 116002; hep-ph/9809521.
5. D. F. Phillips, M. A. Humphrey, E. M. Mattison, R. E. Stoner, R. F. C. Vessot, R. L. Walsworth, Phys. Rev. D 63 (2001) 111101, physics/0008230.
6. V. W. Hughes, M. G. Perdekamp, D. Kawall, W. Liu, K. Jungmann, G. zu Putlitz, Phys. Rev. Lett. 87 (2001) 111804, hep-ex/0106103.
7. P. Wolf, S. Bize, A. Clairon, A. N. Luiten, G. Santarelli, and M. E. Tobar, Phys. Rev. Lett. 90 (2003) 060402, gr-qc/0210049.
8. J. A. Lipa, J. A. Nissen, S. Wang, D. A. Stricker, and D. Avaloff, Phys. Rev. Lett. 90 (2003) 060403, physics/0302093.
9. V. A. Kostelecký and C. Lane, Phys. Rev. D 60 (1999) 116010, hep-ph/9908504.
10. R. Bluhm, V. A. Kostelecký, and C. Lane, Phys. Rev. Lett. 84 (2000) 1098, hep-ph/9912451.
11. R. Bluhm and V. A. Kostelecký, Phys. Rev. Lett. 84 (2000) 1381, hep-ph/9912542.
12. D. Bear, R. E. Stoner, R. L. Walsworth, V. A. Kostelecký, C. Lane, Phys. Rev. Lett. 85 (2000) 5038, physics/0007049.
13. V. A. Kostelecký, Phys. Rev. D 64 (2001) 076001, hep-ph/0104120.
14. R. Bluhm, hep-ph/9904508 and hep-ph/0111323.
15. R. Bluhm, V. A. Kostelecký, C. Lane, and N. Russell, Phys. Rev. Lett. 88 (2002) 090801, hep-ph/0111141.
16. V. A. Kostelecký and M. Mewes, Phys. Rev. D 66 (2002) 056005, hep-ph/0205211.
17. R. Bluhm, V. A. Kostelecký, C. Lane, and N. Russell, Phys. Rev. D 68 (2003) 125008, hep-ph/0306190.
18. N. Russell, hep-ph/0111320 and hep-ph/0208065.
19. G. Amelino-Camelia, Gen. Rel. Grav. 36 (2004) 539, astro-ph/0309174; Phys. Rev. D 62 (2000) 024015, gr-qc/9903080.
20. V. A. Kostelecký and M. Mewes, Phys. Rev. Lett. 87 (2001) 251304, hep-ph/0111026.
21. T. Jacobson, S. Liberati, and D. Mattingly, Phys. Rev. D 67 (2003) 124011, hep-ph/0209264; Nature 424 (2003) 1019, astro-ph/0212190.
22. T. Jacobson, S. Liberati, D. Mattingly, and F. W. Stecker, astro-ph/0309681.
23. F. W. Stecker and S. L. Glashow, Astropart. Phys. 16 (2001) 97, astro-ph/0102226.
24. F. W. Stecker, astro-ph/0309027.
25. G. Amelino-Camelia, J. Ellis, N. E. Mavroumatos, D. V. Nanopoulos, and S. Sarkar, Nature 393 (1998) 763.
26. V. A. Kostelecký, hep-ph/0005280.
27. R. E. Allen, to be published.
28. R. E. Allen, in *Proceedings of Beyond the Desert 2002*, edited by H. V. Klapdor-Kleingrothaus, (IOP, London, 2003), hep-th/0008032.
29. R. E. Allen, in *Proceedings of Beyond the Desert 2003*, edited by H. V. Klapdor-Kleingrothaus, hep-th/0310039.
30. M. E. Peskin and D. V. Schroeder, *Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Massachusetts, 1995), p. 44.
31. P. Ramond, *Field Theory: A Modern Primer* (Addison-Wesley, Redwood City, California, 1990), pp.16-20.
32. G. G. Ross, *Grand Unified Theories* (Benjamin, Menlo Park, 1984).
33. K. Greisen, Phys. Rev. Lett. 16, 748 (1966); G.T. Zatsepin and V.A. Kuz'min, JETP Lett. 41, 78 (1966) [Pisma Zh. Eksp. Teor. Fiz. 4, 114 (1966)].
34. M. Nagano and A. A. Watson, Rev. Mod. Phys. 72 (2000) 689.
35. P. Bhattacharjee and G. Sigl, Phys. Rept. 327 (2000) 109.
36. T. J. Weiler, AIP Conf. Proc. 579 (2001) 58, hep-ph/0103023.
37. L. Anchordoqui, T. Paul, S. Reucroft and J. Swain, hep-ph/0206072.
38. J. N. Bahcall and E. Waxman, Physics Letters B 556 (2003) 1, hep-ph/0206217.