

STANDARD SUPERSYMMETRY FROM A PLANCK-SCALE STATISTICAL THEORY

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We outline three new ideas in a program to obtain standard physics, including standard supersymmetry, from a Planck-scale statistical theory: (1) The initial spin 1/2 bosonic fields are transformed to spin 0 fields together with their auxiliary fields. (2) Time is defined by the progression of 3-geometries, just as originally proposed by DeWitt. (3) The initial (D-1)-dimensional “path integral” is converted from Euclidean to Lorentzian form by transformation of the fields in the integrand.

In earlier work it was shown that a fundamental statistical theory (at the Planck scale) can lead to many features of standard physics¹⁻³. In some respects, however, the results had nonstandard features which appear to present difficulties. For example, the primitive supersymmetry of the earlier papers is quite different from the standard formulation of supersymmetry which works so admirably in both protecting the masses of Higgs fields from quadratic divergences and predicting coupling constant unification at high energy. Also, the fact that the theory was originally formulated in Euclidean time seems physically unsatisfactory for reasons mentioned below. Here we introduce some refinements in the theory which eliminate these two problems. The ideas in the following sections respectively grew out of discussions of the first author with Seiichirou Yokoo (on the transformation of spin 1/2 to spin 0 fields) and Zorawar Wadiasingh (on the transformation of the path integral from Euclidean to Lorentzian form).

1. Transformation of Original Spin 1/2 Fields Yields Standard Supersymmetry

In Refs. 2 and 3, the action for a fundamental bosonic field was found to have the form

$$S_b = \int d^4x \psi_b^\dagger i\sigma^\mu \partial_\mu \psi_b \quad (1)$$

at energies that are far below the Planck energy m_P (with $\hbar = c = 1$) and in a locally inertial coordinate system. This is the conventional form of the action for fermions, described by 2-component Weyl spinors, but it is highly unconventional for bosons, because a boson described by ψ_b would have spin 1/2. We can, however, transform from the original 2-component field ψ_b to two 1-component complex fields ϕ and F by writing

$$\psi_b(x) = \psi^+(x) + \psi^-(x) \quad (2)$$

$$\psi^+(\vec{x}, t) = \sum_{\vec{p}, \omega} \phi(\vec{p}, \omega) u^+(\vec{p}) e^{i\vec{p}\cdot\vec{x}} e^{-i\omega t} (\omega + |\vec{p}|)^{1/2} \quad (3)$$

$$\psi^-(\vec{x}, t) = \sum_{\vec{p}, \omega} F(\vec{p}, \omega) u^-(\vec{p}) e^{i\vec{p}\cdot\vec{x}} e^{-i\omega t} (\omega + |\vec{p}|)^{-1/2} \quad (4)$$

with

$$\vec{\sigma} \cdot \vec{p} u^+(\vec{p}) = +|\vec{p}| u^+(\vec{p}) \quad , \quad \vec{\sigma} \cdot \vec{p} u^-(\vec{p}) = -|\vec{p}| u^-(\vec{p}) \quad (5)$$

$$\phi(\vec{p}, \omega) = \int d^4x \phi(\vec{x}, t) e^{-i\vec{p}\cdot\vec{x}} e^{i\omega t} \quad , \quad F(\vec{p}, \omega) = \int d^4x F(\vec{x}, t) e^{-i\vec{p}\cdot\vec{x}} e^{i\omega t} .$$

Substitution then gives

$$S_b = V^{-1} \sum_{\vec{p}, \omega} \left[\phi^*(\vec{p}, \omega) (\omega^2 - |\vec{p}|^2) \phi(\vec{p}, \omega) + F^*(\vec{p}, \omega) F(\vec{p}, \omega) \right] \quad (6)$$

$$= \int d^4x [-\partial^\mu \phi^*(x) \partial_\mu \phi(x) + F^*(x) F(x)] \quad (7)$$

where $\partial^\mu = \eta^{\mu\nu} \partial_\nu$, $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, and V is a 4-dimensional normalization volume. This is, of course, precisely the action for a massless scalar boson field ϕ and its auxiliary field F .

With the fermionic action left in its original form, we now have the standard supersymmetric action for each pair of susy partners:

$$S_{fb} = \int d^4x \left[\psi_f^\dagger i\sigma^\mu \partial_\mu \psi_f - \partial^\mu \phi^*(x) \partial_\mu \phi(x) + F^*(x) F(x) \right] . \quad (8)$$

There is a major point that will be discussed at length elsewhere, in a more complete treatment of the present theory: The above transformation works only for $\omega + |\vec{p}| \geq 0$, since otherwise the sign of the integrand would be reversed. However, a stable vacuum already requires $\omega \geq 0$, so we must define time for would-be negative-frequency fields in such a way that this condition is satisfied.

2. Time is Defined by Progression of 3-Geometries in External Space

In our earlier work, the time coordinate x^0 was initially defined in exactly the same way as each spatial coordinate x^k , so x^0 was initially a Euclidean variable. For reasons given in the following section, however, this does not seem to be as physically reasonable as a picture in which time is Lorentzian when it is first defined. In this section, therefore, we move to a new picture in which the initial “path integral” Z_E still has the Euclidean form

$$Z_E = \int \mathcal{D}(\text{Re } \phi) \mathcal{D}(\text{Im } \phi) e^{-S}, \quad S = \int d^{D-1}x \phi^*(\vec{x}) A \phi(\vec{x}) \quad (9)$$

but there is *initially no time*. We are then confronted with the well-known situation in canonical quantum gravity⁴, where the “wavefunction of the universe” is a functional of only 3-geometries, with no time dependence. Roughly speaking, cosmological time is then defined by the cosmic scale factor R (except that there can be different branches for the state of the universe, corresponding to, e.g., expansion and contraction, as well as different initial conditions). More precisely, the progression of time is locally defined by the progression of local 3-geometries.

An analogy is a stationary state for a proton with coordinates \vec{X} passing a hydrogen atom with coordinates \vec{x} . The time-independent Schrödinger equation can be written

$$\left(-\frac{\hbar^2}{2m_p} \nabla_p^2 + H_e \right) \Psi(\vec{X}) \psi(\vec{x}, \vec{X}) = E \Psi(\vec{X}) \psi(\vec{x}, \vec{X}) \quad (10)$$

with Ψ required to satisfy

$$-\frac{\hbar^2}{2m_p} \nabla_p^2 \Psi(\vec{X}) = E \Psi(\vec{X}). \quad (11)$$

Then the equation for ψ is

$$\left(-\frac{\hbar^2}{m_p} \Psi^{-1} \vec{\nabla}_p \Psi \cdot \vec{\nabla}_p - \frac{\hbar^2}{2m_p} \nabla_p^2 + H_e \right) \psi(\vec{x}, \vec{X}) = 0. \quad (12)$$

The first term involves a local proton velocity

$$\vec{v}_p = \hbar \vec{\nabla}_p \theta / m_p, \quad \Psi = |\Psi| e^{i\theta}. \quad (13)$$

For a state in which the proton is moving rapidly, with

$$\Psi = \Psi_0 e^{i\vec{P} \cdot \vec{X} / \hbar}, \quad (14)$$

and in which $(\hbar^2/2m_p) \nabla_p^2 \psi$ is relatively small, we obtain

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = H_e \psi(\vec{x}, t) \quad , \quad \frac{\partial}{\partial t} \equiv \frac{\vec{P}}{m_p} \cdot \vec{\nabla}_p. \quad (15)$$

One then has an “internal time” defined within a stationary state⁵. Simi-

larly, one can define time as a progression of 3-geometries, just as proposed 40 years ago by DeWitt, whose formulation of canonical quantum gravity (following the classical canonical decomposition of Arnowitt, Deser, and Misner, and the work of Dirac, Wheeler, and others) involves the local canonical momentum operator

$$\pi^{kl}(\vec{x}) = -i \frac{\delta}{\delta g_{kl}(\vec{x})}, \quad (16)$$

which corresponds to the proton momentum operator $-i \hbar \vec{\nabla}_p$ in the analogy above. After introducing the 3-dimensional metric tensor in the way described in Refs. 1-3, and the gravitational action in a way that will be described in a more complete treatment, we move from the original path-integral quantization to canonical quantization, with a state

$$\Psi_{\text{total}} = \Psi_{\text{gravity}}[g_{kl}(\vec{x})] \Psi_{\text{otherfields}}[\phi_{\text{otherfields}}(\vec{x}), g_{kl}(\vec{x})] \quad (17)$$

and time is defined essentially in the same way as in the analogy.

3. Transformation of 3-Dimensional “Path Integral”

Changes Euclidean Factor e^{-S} to Lorentzian Factor e^{iS}

A Euclidean path integral with the form of (9), but with time included, is formally transformed into a Lorentzian path integral

$$Z_L^D = \int \mathcal{D}(\text{Re } \phi_L) \mathcal{D}(\text{Im } \phi_L) e^{iS_L^D} \quad , \quad S_L^D = \int d^D x \mathcal{L}_L \quad (18)$$

through an inverse Wick rotation $x_E^0 = t_E \rightarrow ix_L^0 = it_L$. S_L^D has the usual form of a classical action, and it leads to the usual description of quantized fields via path-integral quantization. In other words, the standard equations of physics follow from S_L^D , and are therefore formulated in Lorentzian time. The Euclidean formulation, in either coordinate or momentum space, is ordinarily regarded as a mere mathematical tool which can simplify calculations and make them better defined.

Hawking, on the other hand, has suggested that Euclidean spacetime may actually be more fundamental than Lorentzian spacetime. In his well-known popular book, he says⁶ “So maybe what we call imaginary time is

really more basic, and what we call real is just an idea that we invent to help us describe what we think the universe is like.” And in a more technical paper he states⁷ “In fact one could take the attitude that quantum theory and indeed the whole of physics is really defined in the Euclidean region and that it is simply a consequence of our perception that we interpret it in the Lorentzian regime.”

However, there is a fundamental problem with this point of view, because the factor $e^{iS_L^D}$ in the Lorentzian formulation results in interference effects, whereas the factor $e^{-S_E^D}$ in the Euclidean formulation does not. Also, a formal transformation from t_E to t_L mixes all of the supposedly more fundamental Euclidean times in the single Lorentzian time that we actually experience. Finally, it appears difficult to formulate a mathematically well-founded and physically well-motivated transformation of a general path integral from Euclidean to Lorentzian spacetime.

Here we adopt a very different point of view: (1) Nature is fundamentally statistical, essentially as proposed in Refs. 1-3, but the initial path integral (or partition function) does not contain the time as a fundamental coordinate. Instead time is defined by the local 3-space geometry (or more generally, (D-1)-space geometry). (2) It is, however, still necessary to transform from the Euclidean form (9), with e^{-S} , to the Lorentzian form (18), with e^{iS} (but also with no time coordinate, so that $D \rightarrow D - 1$ in (18)), and this is our goal in the present section.

Consider a single complex scalar field ϕ with a 3-dimensional “Euclidean path integral”

$$Z_E = \int \mathcal{D}(\text{Re } \phi) \mathcal{D}(\text{Im } \phi) e^{-S} \quad , \quad S = \int d^3x \phi^*(\vec{x}) A \phi(\vec{x}). \quad (19)$$

In a discrete picture, the operator A is replaced by a matrix with elements $A(\vec{x}, \vec{x}')$:

$$S = \sum_{x, x'} \phi^*(\vec{x}) A(\vec{x}, \vec{x}') \phi(\vec{x}'). \quad (20)$$

A can be diagonalized to $A(\vec{k}, \vec{k}') = a(\vec{k}) \delta_{\vec{k}, \vec{k}'}$. Then

$$Z_E \equiv \left[\prod_{\vec{x}} \int_{-\infty}^{\infty} d(\text{Re } \phi(\vec{x})) \int_{-\infty}^{\infty} d(\text{Im } \phi(\vec{x})) \right] \exp \left(- \sum_{\vec{x}, \vec{x}'} \phi^*(\vec{x}) A(\vec{x}, \vec{x}') \phi(\vec{x}') \right)$$

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becomes⁸

$$Z_E = \left[\prod_{\vec{k}} \int_{-\infty}^{\infty} d\text{Re } \phi(\vec{k}) \int_{-\infty}^{\infty} d\text{Im } \phi(\vec{k}) \right] \exp \left(- \sum_{\vec{k}} \phi^*(\vec{k}) a(\vec{k}) \phi(\vec{k}) \right). \quad (21)$$

The Gaussian integrals over $\text{Re } \phi(\vec{k})$ and $\text{Im } \phi(\vec{k})$ may be evaluated as usual at each \vec{k} to give

$$Z_E = \prod_{\vec{k}} \frac{\pi}{a(\vec{k})} = \frac{\prod_{\vec{k}} \pi}{\det A}. \quad (22)$$

Here, and in the earlier papers, two representations of the path integral are taken to be physically equivalent if they give the same result for all operators A (including those which produce zero except for arbitrarily restricted regions of space and sets of fields). For example, we might define a path integral Z' with fields ϕ' and $\bar{\phi}'$ which are treated as independent and which each vary along the real axis. It is then appropriate to include the formal Jacobian, with a value of $1/2$, which would correspond to a transformation from $\text{Re } \phi$ and $\text{Im } \phi$ to $\phi' = \text{Re } \phi + i \text{Im } \phi$ and $\bar{\phi}' = i(\text{Re } \phi - i \text{Im } \phi)$. Since

$$\begin{aligned} Z' &\equiv \left[\prod_{\vec{k}} \frac{1}{2} \int_{-\infty}^{\infty} d(\phi'(\vec{k})) \int_{-\infty}^{\infty} d(\bar{\phi}'(\vec{k})) \right] \exp \left(\sum_{\vec{k}} i \bar{\phi}'(\vec{k}) a(\vec{k}) \phi'(\vec{k}) \right) \\ &= \prod_{\vec{k}} \frac{1}{2} \int_{-\infty}^{\infty} d(\phi'(\vec{k})) \int_{-\infty}^{\infty} d(\bar{\phi}'(\vec{k})) \exp \left(i \bar{\phi}'(\vec{k}) a(\vec{k}) \phi'(\vec{k}) \right) \\ &= \prod_{\vec{k}} \frac{1}{2a(\vec{k})} \int_{-\infty}^{\infty} d(a(\vec{k}) \phi'(\vec{k})) 2\pi \delta(a(\vec{k}) \phi'(\vec{k})) \end{aligned} \quad (23)$$

$$= \prod_{\vec{k}} \frac{\pi}{a(\vec{k})} \quad (24)$$

$$= Z_E \quad (25)$$

for any operator A , we regard Z_E and Z' as being physically equivalent.

Now let us define a ‘‘Lorentzian path integral’’ Z_L by

$$\begin{aligned} Z_L &= \int \mathcal{D}(\text{Re } \phi) \mathcal{D}(\text{Im } \phi) e^{iS} \quad (26) \\ &\equiv \left[\prod_{\vec{x}} \frac{1}{i} \int_{-\infty}^{\infty} d(\text{Re } \phi(\vec{x})) \int_{-\infty}^{\infty} d(\text{Im } \phi(\vec{x})) \right] \exp \left(i \sum_{\vec{x}, \vec{x}'} \phi^*(\vec{x}) A(\vec{x}, \vec{x}') \phi(\vec{x}') \right). \end{aligned}$$

Diagonalization of A gives

$$Z_L = \left[\prod_{\vec{k}} \frac{1}{i} \int_{-\infty}^{\infty} d\text{Re } \phi(\vec{k}) \int_{-\infty}^{\infty} d\text{Im } \phi(\vec{k}) \right] \exp \left(i \sum_{\vec{k}} \phi^*(\vec{k}) a(\vec{k}) \phi(\vec{k}) \right)$$

$$= \prod_{\vec{k}} \frac{1}{i} \frac{i\pi}{a(\vec{k})} \quad (27)$$

$$= Z_E. \quad (28)$$

Then Z_E can be replaced by Z_L , which involves the original operator A and the original spatial coordinates \vec{x} , but a different form for the integrand. This replacement is possible because time is introduced only after Z is in Lorentzian form.

The transformation from Z_E to Z_L can be regarded as a transformation of the fields in the integrand, with the lines along which $\text{Re } \phi$ and $\text{Im } \phi$ are integrated each being rotated by 45° in the complex plane⁹.

4. Outline of Broad Program: From a Planck-Scale Statistical Theory to Standard Physics with Supersymmetry

The ideas above are part of a broad program to obtain standard physics, including supersymmetry, from a description at the Planck scale which is purely statistical. The major steps in the complete program are as follows:

(1) The fundamental statistical picture gives a $D-1$ ‘‘Euclidean action’’ for bosons only (and with no time yet):

$$Z_b^{D-1} = \int \mathcal{D}(\text{Re } \phi) \mathcal{D}(\text{Im } \phi) e^{-S_b} \quad , \quad S_b = \int d^{D-1}x \mathcal{L}_b^{D-1}. \quad (29)$$

(2) Random fluctuations then give a ‘‘Euclidean action’’ with bosons, fermions, and a primitive supersymmetry:

$$Z_E^{D-1} = \int \mathcal{D}(\text{Re } \phi) \mathcal{D}(\text{Im } \phi) \mathcal{D}(\text{Re } \psi) \mathcal{D}(\text{Im } \psi) e^{-S} \quad , \quad S = \int d^{D-1}x \mathcal{L}^{D-1}. \quad (30)$$

(3) Transformation of the integrand in the ‘‘path integral’’ changes the ‘‘Euclidean factor’’ e^{-S} to the ‘‘Lorentzian factor’’ e^{iS} :

$$Z_L^{D-1} = \int \mathcal{D}(\text{Re } \phi) \mathcal{D}(\text{Im } \phi) \mathcal{D}(\text{Re } \psi) \mathcal{D}(\text{Im } \psi) e^{iS} \quad , \quad S = \int d^{D-1}x \mathcal{L}^{D-1}. \quad (31)$$

(4) The 3-dimensional gravitational metric tensor g_{kl} and $SO(N)$ gauge fields A_k (and their initial, primitive supersymmetric partners) result from

rotations of the vacuum state vector, in both 3-dimensional external space and $D - 4$ dimensional internal space.

(5) Time is defined by the progression of 3-geometries in external space.

(6) The Einstein-Hilbert action for the gravitational field (as well as the cosmological constant), the Maxwell-Yang-Mills action for the gauge fields, and the analogous terms for the gaugino and gravitino fields are assumed to arise from a response of the vacuum that is analogous to the diamagnetic response of electrons.

(7) The gravitational field is approximately quantized via first a path-integral formulation and then the canonical formulation of Ref. 4.

(8) Heisenberg equations of motion are then obtained for all fields.

(9) Transformation of the initial spin 1/2 bosonic fields, followed by definition of standard gaugino and gravitino fields, gives standard supersymmetry.

(10) One finally obtains an effective action which is the same as that of standard physics with supersymmetry, except that particle masses, Yukawa couplings, and self-interactions are assumed to arise from supersymmetry breaking and radiative corrections.

A more complete treatment will be given in a much longer paper.

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