LORENTZ VIOLATION, with supersymmetry –
Proposed violations of Lorentz invariance in supersymmetric theories.

There are two scenarios in which supersymmetry holds as a fundamental symmetry of nature, but Lorentz invariance is violated at a level consistent with experiment and observation.

In the first [1-3], Lorentz invariance is assumed to be a fundamental symmetry at all energies, but to be spontaneously broken with, e.g., some superstring-derived field acquiring an expectation value $\langle \mathbf{A} \rangle$.

In the second, Lorentz invariance does not hold as a fundamental symmetry at the Planck scale, but instead emerges as a good low-energy symmetry [4].

In the first vein, Kostelecký and coworkers have explored many possible manifestations of Lorentz violation, and their experimental signatures. Very recently they have also constructed a Lorentz-violating extension of the Wess-Zumino model, with the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{WZ}} + \mathcal{L}_{\mathrm{Lorentz}},$$

where

$$\begin{aligned}\mathcal{L}_{\mathrm{Lorentz}} = {} & k_{\mu\nu}\partial^\mu A\,\partial^\nu A + k_{\mu\nu}\partial^\mu B\,\partial^\nu B \\ & +\tfrac{1}{2}k_{\mu\nu}k^{\mu}{}_{\rho}(\partial^\nu A\,\partial^\rho A + \partial^\nu B\,\partial^\rho B) \\ & +\tfrac{1}{2}ik_{\mu\nu}\overline{\psi}\gamma^\mu\partial^\nu\psi.\end{aligned}$$

In this equation, $k_{\mu\nu}$ is a dimensionless coefficient determining the magnitude of Lorentz violation.

This model is invariant up to a total derivative under the infinitesimal supersymmetry transformations

$$\begin{aligned}\delta A &= \overline{\epsilon}\psi \\ \delta B &= i\overline{\epsilon}\gamma_5\psi \\ \delta\psi &= -(i\partial\!\!\!/ + ik_{\mu\nu}\gamma^\mu\partial^\nu)(A + i\gamma_5 B)\epsilon \\ &\qquad\qquad +(F + i\gamma_5 G)\epsilon \\ \delta F &= -\overline{\epsilon}(i\partial\!\!\!/ + ik_{\mu\nu}\gamma^\mu\partial^\nu)\psi \\ \delta G &= \overline{\epsilon}(\gamma_5\partial\!\!\!/ + k_{\mu\nu}\gamma_5\gamma^\mu\partial^\nu)\psi\end{aligned}$$

where ϵ is a constant Majorana spinor. The presence of $k_{\mu\nu}$ implies the transformations are realized differently on particles with different orientations and boosts, as expected in a theory with Lorentz violation, and the usual Wess-Zumino transformations are recovered in the limit $k_{\mu\nu} \to 0$.

The commutator of two supersymmetry transformations yields a modified supersymmetry algebra

$$[\delta_1, \delta_2] = 2i\overline{\epsilon}_1\gamma^\mu\epsilon_2\partial_\mu + 2ik_{\mu\nu}\overline{\epsilon}_1\gamma^\mu\epsilon_2\partial^\nu$$

which involves the generator of translations. The fermion propagator is formally

$$\begin{aligned}iS_F(p) &= \frac{i}{p_\mu(\gamma^\mu + k_{\mu\nu}\gamma^\nu) - m} \\ &= i\frac{p_\mu(\gamma^\mu + k_{\mu\nu}\gamma^\nu) + m}{p^2 + 2p^\mu p^\nu k_{\mu\nu} + k_{\mu\rho}k^{\rho}{}_{\nu}p^\mu p^\nu}\end{aligned}$$

so fermion and boson propagators have the same structure, and one therefore anticipates that the usual successes of supersymmetry are preserved.

In an earlier Lorentz-violating supersymmetric theory [4], the propagator also has the same structure, and in fact exactly the same simple form

$$\left(p\!\!\!/ - m - \alpha p^\mu p_\mu + i\epsilon\right)^{-1}\ ,$$

for both fermions and bosons. In this theory, the set of bosonic and fermionic fields are initially arranged as an unconventional superfield with the form

$$\Phi = \begin{pmatrix}\Phi_b \\ \Phi_f\end{pmatrix},\ \Phi_b = \begin{pmatrix} z_b^1 \\ z_b^2 \\ \vdots \\ z_b^N\end{pmatrix},\ \Phi_f = \begin{pmatrix} z_f^1 \\ z_f^2 \\ \vdots \\ z_f^N\end{pmatrix}$$

where the components of Φ_b and Φ_f are respectively commuting and anticommuting variables. (The terms "superfield", "supersymmetry", "supergravity", etc. are here used in a generic sense [4].) The action is assumed to have the form

$$S = \int d^D x \left(\partial^M\Phi^\dagger\partial_M\Phi - \mu^2\Phi^\dagger\Phi + \tfrac{1}{2}b\left(\Phi^\dagger\Phi\right)^2\right).$$

Like the field and its action, a supersymmetry transformation also has the simplest imaginable form:

$$\begin{aligned}\Phi \to \Phi' &= U\Phi \\ U &= \begin{pmatrix} U_{bb} & U_{bf} \\ U_{fb} & U_{ff}\end{pmatrix}\ ,\ U^\dagger = U^{-1}.\end{aligned}$$

U_{bb} and U_{ff} consist of ordinary commuting variables, and U_{bf} and U_{fb} of anticommuting Grassmann variables. If U is constant, the supersymmetry transformation is global.

However, one can allow U to vary, and this leads to a new kind of supergravity theory which violates Lorentz invariance. (Recall that terms like "supergravity" are here taken to have a very broad meaning.) More specifically, Lorentz invariance is violated for (i) fermions at very high energy and (ii) fundamental bosons which have not yet been observed. However, many features of Lorentz invariance are preserved, including rotational invariance and the same velocity c for all massless particles at all energies. For these and other reasons, the theory appears to be consistent with all existing experimental and observational tests of Lorentz invariance.

With the supermatrix U allowed to vary, one can define gauge fields and the gravitational vierbein just as in Ref. 4, and the same procedure also leads to fermionic gauginos and gravitinos. The superpartners of the fundamental Higgs bosons are fermionic Higgsinos, and the most plausible candidate for dark matter is still the lightest neutralino. There is one new and remarkable result, however: Fundamental bosons can have spin 1/2. (See the detailed discussion of this feature in the third paper of Ref. 4. It is relevant that the spin-statistics theorem is a rather fragile result that requires perfect Lorentz symmetry. For example, both bosons and fermions are allowed to have any spin in nonrelativistic physics [5].) In this scenario, Higgs bosons have an R-parity of -1, and sfermions have an R-parity of $+1$, where the R-parity is defined as usual by

$$R = (-1)^{3(B-L)+2s} .$$

(Higgsinos still have spin 1/2, so they will have $R = -1$, and the lightest such neutralino remains a stable dark matter candidate. Also, Standard Model fermions still have $R = +1$.) This result implies that a Higgs boson can only be created in conjunction with another particle having $R = -1$, whereas a sfermion can be created as a single particle. I.e., Higgs bosons are much harder to observe, and sfermions much easier to observe, than in the Standard Model or standard supersymmetric models.

BIBLIOGRAPHY

[1] D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **58**, 116002 (1998); hep-ph/9809521.

[2] *CPT and Lorentz Symmetry*, edited by V. A. Kostelecký (Singapore, World Scientific, 1999); *Proceedings of the Second Meeting on CPT and Lorentz Symmetry*, edited by V. A. Kostelecký (Singapore, World Scientific, 2002).

[3] M.S. Berger and V. Alan Kostelecký, *Phys. Rev.*, **D65**, 091701 (2002).

[4] R. E. Allen, in the first volume of Ref. 2, and hep-ph/9902228; R. E. Allen, *Int. J. Mod. Phys. A* **12**, 2385 (1997), and hep-th/9612041; R. E. Allen, in *Beyond the Desert 2002*, edited by H. V. Klapdor-Kleingrothaus et al. (Institute of Physics, Bristol, 2003), hep-th/0008032.

[5] A. S. Wightman, http://ejde.math.swt.edu, *Electron. J. Diff. Eqns. Conf.* **04**, 207 (2000).