COSMOLOGICAL MODELS, with supersymmetry –

Models which address cosmological problems with various features of supersymmetry. These include the following.

- New sources of CP violation which may be relevant to baryogenesis, as discussed in the article on mSUGRA.
- An excellent candidate for dark matter, as discussed in the article on "DARK MATTER, supersymmetry and supergravity models".
- New scalar fields which may have naturally flat potentials, or which otherwise may provide more convincing scenarios for inflation.

Regarding the last of these topics, first consider the general ideas behind inflationary models [1,2]. Einstein's field equations lead to the Friedmann equations

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + 3p\right) + \frac{1}{3}\Lambda_0$$
$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2} + \frac{1}{3}\Lambda_0$$

where R is the cosmic scale factor, G the gravitational constant, ρ the mass density, p the pressure, Λ_0 the cosmological constant, H the Hubble parameter, and k the curvature parameter. Natural units are used, with $\hbar = c = k_B = 1$, and in the following it is assumed that $k = \Lambda_0 = 0$.

A scalar inflaton field ϕ is postulated to exist in the very early universe. Its stress-energy tensor gives the energy density and pressure

$$\rho = \dot{\phi}^2/2 + V\left(\phi\right)$$
$$p = \dot{\phi}^2/2 - V\left(\phi\right)$$

and the semiclassical equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} = -\frac{\partial V}{\partial \phi}.$$

The second term in this last equation represents damping due to the expansion of the universe (which reduces the kinetic energy by red-shifting the momentum $\dot{\phi}$). The third term is added to represent the decay rate Γ^{-1} for $\phi \to$ other particles. For a very flat positive potential, there will be a period of "slow roll", when the first and third terms can be neglected, and $\dot{\phi}^2$ is also small. During this period, we have

$$\dot{\phi} \approx -\frac{1}{3H}\frac{\partial V}{\partial \phi}\ ,$$

$$\rho + p = \dot{\phi}^2 \ll \rho \approx V\ ,$$

and $\ddot{R}/R = \left(\dot{R}/R\right)^2 = \Lambda/3$, where $\Lambda = 8\pi G\rho$ is the effective cosmological constant due to V. Then R inflates exponentially: $R\left(t\right) = R_0 e^{Ht}$.

There are quantum fluctuations whose size is set by the Gibbons-Hawking temperature T_{GH} associated with the de Sitter space event horizon:

$$\delta\phi \sim T_{GH} = H/2\pi.$$

These fluctuations in ϕ give rise to density inhomogeneities:

$$\delta\rho \approx \frac{\partial V}{\partial \phi}\delta\phi.$$

Each perturbation is effectively frozen in as it is carried through the horizon by the rapid expansion of the universe. Detailed analysis [3] shows that one can equate the value of the quantity $\delta\rho/\left(\rho + p\right)$ when the perturbation eventually crosses back inside the horizon to the value it had when it first crossed outside the horizon during inflation. For the second crossing, one has $p = \rho/3$ for the case of a radiation-dominated universe and $p = 0$ for the case of a matter-dominated universe. For the crossing during inflation, on the other hand, one has $\rho + p = \dot{\phi}^2$. It follows that

$$\left(\frac{\delta\rho}{\rho}\right)_{HOR} \sim \frac{\partial V/\partial\phi}{\dot{\phi}^2}\delta\phi$$
$$\sim \frac{\partial V/\partial\phi}{\dot{\phi}}\frac{H/2\pi}{\left(\partial V/\partial\phi\right)/\left(3H\right)}$$
$$\sim H^2/\dot{\phi}.$$

Since H and $\dot{\phi}$ are nearly constant during inflation, the fluctuations $\left(\delta\rho/\rho\right)_{HOR}$ have the Harrison-Zel'dovich scale invariance which has been observed in cosmic microwave background measurements as temperature fluctuations.

This scale invariance results from the exponential expansion during the de Sitter phase, and is a robust feature of inflationary models. However, the above result for $\delta\rho/\rho$ permits estimates of the size of these fluctuations in specific models.

An inflationary model with supersymmetry was proposed by Holman, Ramond, and Ross [4]. They postulated a superpotential having the simple form

$$W = \left(\Delta^2/M\right)\left(\phi - M\right)^2$$

where $M = M_P/\sqrt{8\pi}$. The resulting scalar potential is [2]

$$\begin{aligned} V(\phi) &= \exp\left(|\phi|^2/M^2\right)(|\partial W/\partial\phi \\ &\qquad\qquad +\phi^* W/M^2|^2 - 3|W|^2/M^2) \\ &= \Delta^4\left(1 - 4\phi^3/M^3 + 6.5\phi^4/M^4 + ...\right). \end{aligned}$$

A value of $\Delta \sim 3\times10^{-5}M$ gives density perturbations of a reasonable size with an interesting energy scale of about 10^{14} GeV, and there are other pleasant features [1].

A set of supersymmetric models with hybrid inflation was proposed by Randall, Soljacic, and Guth [5]. These models involve superpotentials with, e. g., the form

$$W = \frac{\phi^2\psi^2}{2M'}$$

and they have testable consequences. For example, there is a large spike in the density perturbation spectrum at present wavelengths of about 1 Mpc or less.

The topics listed above are, of course, only a sample of the applications of supersymmetry in cosmology, but some other cosmological implications of supersymmetry, supergravity, and superstring theory are mentioned elsewhere in this volume.

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