1. (25 points)A car is speeding along a straight road with velocity of magnitude v_1 when the driver sees a policeman on a motorcycle in the rear view mirror. At that instant the policeman is a distance L behind the car. The driver tries to outrun the cop by accelerating with a constant acceleration a_1. The cop, who was also going at v_1 engages a turbo and accelerates with an acceleration βt^3 where β is a known constant and t is the time after the car starts to accelerate.

a. What is the car's position as a function of time, calling the point where it starts to accelerate the origin?

$$x_{car}(t) = \frac{1}{2} a_1 t^2 + v_1 t$$

b. What is the motorcycle's position as a function of time?.

$$v_M(t) = \int \beta t^3 dt = \frac{\beta t^4}{4} + v_1$$

$$x_M(t) = \int \left(\frac{\beta t^4}{4} + v_1\right) dt = \frac{\beta t^5}{20} + v_1 t - L$$

c. Obtain an equation for the time at which the cop will catch the speeder. **DO NOT SOLVE!**

$$\frac{\beta t^{*5}}{20} + v_1 t^* - L = \frac{1}{2} a_1 t^{*2} + v_1 t^*$$

$$\text{or} \quad x_M(t^*) = x_{car}(t^*)$$

2. (25 points) An object is at the point $x = L$ and $y = H$ at the time $t = 0$. It has velocity given by

$$\vec{v}(t) = c_1 t\vec{i} + c_2 t^3 \vec{j}$$

where c_1 and c_2 are known constants.

a. Find the object's position and acceleration as functions of time.

$$v_x(t) = c_1 t$$

$$v_y(t) = c_2 t^3$$

$$\boxed{\begin{aligned} a_x(t) &= \frac{d v_x(t)}{dt} = c_1 \\ a_y(t) &= \frac{d v_y(t)}{dt} = 3 c_2 t^2 \\ x(t) &= \int c_1 t \, dt = \frac{c_1 t^2}{2} + L \\ y(t) &= \int c_2 t^3 dt = \frac{c_2 t^4}{4} + H \end{aligned}}$$

b. Find the angle that the object's position vector makes with the x axis at the time $t = 1 second.$

$$x(1s) = \frac{c_1}{2} + L$$

$$y(1s) = \frac{c_2}{4} + H$$

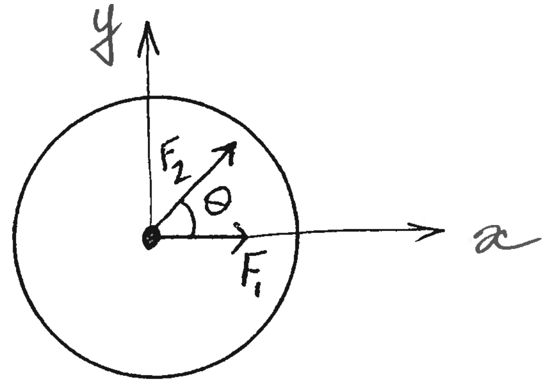
$$\boxed{\tan\theta = \frac{\frac{c_2}{4} + H}{\frac{c_1}{2} + L}}$$

3. (25 points)An object of mass m is at rest at $t = 0$ on a frictionless table at a point defined to be the origin. Two forces, in the plane of the table, are applied to it as shown. Both the forces are fixed in the directions shown with magnitudes given by

$$F_1 = c_1 \qquad \textit{and} \qquad F_2 = c_2 t^2,$$

where c_1 and c_2 are known constants. Find the object's position as a function of time.

$$F_x = m a_x$$
$$F_y = m a_y$$



$$\begin{cases} F_1 + F_2 \cos\theta = m a_x \\ F_2 \sin\theta = m a_y \end{cases}$$

$$\begin{cases} c_1 + c_2 t^2 \cos\theta = m a_x \\ c_2 t^2 \sin\theta = m a_y \end{cases}$$

$$a_x = \frac{c_1 + c_2 t^2 \cos\theta}{m}; \qquad a_y = \frac{c_2 t^2 \sin\theta}{m}$$

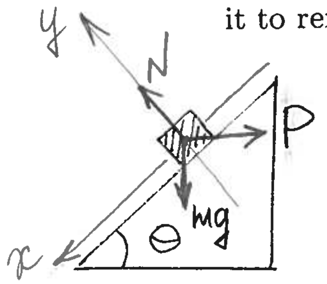
$$v_x(t) = \int \frac{c_1 + c_2 t^2 \cos\theta}{m}\, dt = \frac{c_1}{m} t + \frac{c_2}{m} \cos\theta \frac{t^3}{3}$$

$$v_y(t) = \int \frac{c_2 t^2 \sin\theta}{m}\, dt = \frac{c_2 \sin\theta}{m} \frac{t^3}{3}$$

$$\boxed{\begin{aligned} x(t) &= \frac{c_1 t^2}{2m} + \frac{c_2}{12m} \cos\theta\, t^4 \\ y(t) &= \frac{c_2 \sin\theta}{12m} t^4 \end{aligned}}$$

4. (25 points)A block of mass m starts at rest at the position shown on an inclined plane. (The angle θ is known.)

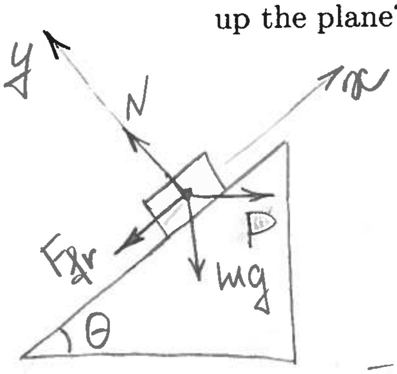
a. If there is no friction, what horizontal force must be applied to the block in order for it to remain at rest?



$$F_x = ma_x$$
$$F_y = ma_y$$

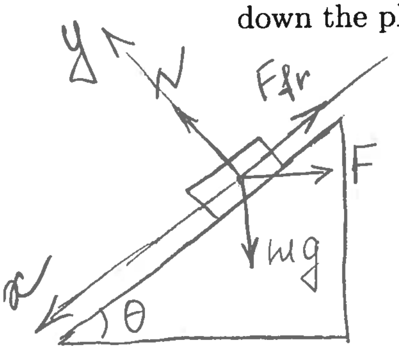
$$mg\sin\theta - P\cos\theta = 0$$
$$\boxed{P = mg\tan\theta}$$

b. If there were a coefficient of friction μ between the block and the plane, what is the smallest horizontal force that could be applied to the block and have the block move up the plane? (Essentially zero acceleration up the plane.)



$$\begin{cases} -mg\sin\theta + P\cos\theta - \mu N = 0 \\ N - P\sin\theta - mg\cos\theta = 0 \end{cases}$$
$$N = P\sin\theta + mg\cos\theta$$
$$-mg\sin\theta + P\cos\theta - \mu(P\sin\theta + mg\cos\theta) = 0$$
$$P = \frac{mg\sin\theta + \mu mg\cos\theta}{\cos\theta - \mu\sin\theta}$$

c. If there were a coefficient of friction μ between the block and the plane, what is the largest horizontal force that could be applied to the block and have the block move down the plane? (Essentially zero acceleration down the plane.)



$$mg\sin\theta - \mu N - P\cos\theta = 0$$
$$N - mg\cos\theta - P\sin\theta = 0$$
$$mg\sin\theta - \mu(mg\cos\theta + P\sin\theta) - P\cos\theta = 0$$
$$P = \frac{mg\sin\theta - \mu mg\cos\theta}{\mu\sin\theta + \cos\theta}$$