1. (25 points)This is a one-dimensional problem. An object of mass m is acted upon by a force given by

$$F_x = -(c_1 x^2 - c_2 x)$$

where c_1 and c_2 are positive constants.

a. Determine whether or not this force is conservative.

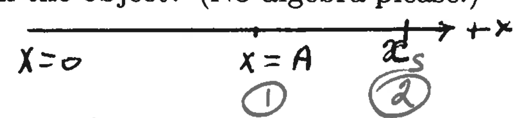
$$F_x = -\frac{dU}{dx}; \quad U = -\int F_x\,dx = -\int -(c_1 x^2 - c_2 x)\,dx =$$

$$= \boxed{c_1 \frac{x^3}{3} - c_2 \frac{x^2}{2} + C}$$

$$F_x = -\frac{\partial U}{\partial x} = -c_1 x^2 + c_2 x = -(c_1 x^2 - c_2 x)$$

$$F_y = -\frac{\partial U}{\partial y} = 0$$

b. If the object is placed at the point $x = A$ and given a velocity of magnitude v_1 in the positive x direction, at what point would the object stop moving in the positive x direction and begin moving in the negative x direction, assuming the above force is the only force acting on the object? (No algebra please!)



$$U_1 + KE_1 = U_2 + KE_2$$

$$\boxed{c_1 \frac{A^3}{3} - c_2 \frac{A^2}{2} + \frac{m v_1^2}{2} = c_1 \frac{x_s^3}{3} - c_2 \frac{x_s^2}{2}}$$

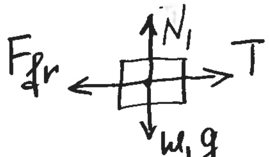
c. Find the object's kinetic energy as a function of x, given the conditions in b. above.

$$c_1 \frac{A^3}{3} - c_2 \frac{A^2}{2} + \frac{m v_1^2}{2} = c_1 \frac{x^3}{3} - c_2 \frac{x^2}{2} + \frac{m v^2}{2}$$

$$\boxed{\frac{m v^2}{2} = \frac{c_1 A^3}{3} - \frac{c_2 A^2}{2} + \frac{m v_1^2}{2} - c_1 \frac{x^3}{3} + c_2 \frac{x^2}{2}}$$

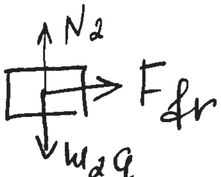
2. (25 points) A small block of mass, m_1, is placed on top of a larger block mass, m_2, and the top block is connected to a third block, m_3, by a massless, unstretchable string which goes over a frictionless pulley. (The pulley therefore changes the direction of the force exerted by the string but not it's magnitude.)The coefficient of friction between blocks is μ and the surface that the large block is on is frictionless. Assume the masses are such that the third block moves down and that the other blocks move together, in other words there is no slipping.

a. Isolate m_1, draw the free body diagram for it and apply Newton's Second Law.



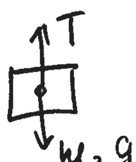
$$T - F_{fr} = m_1 a_{1x}$$
$$N_1 - m_1 g = 0$$

b. Isolate m_2, draw the free body diagram for it and apply Newton's Second Law.



$$F_{fr} = m_2 a_{2x}$$
$$N_2 - m_2 g = 0$$

c. Isolate m_3, draw the free body diagram for it and apply Newton's Second Law.



$$m_3 g - T = m_3 a_{3y} \qquad a_{1x} = a_{2x} = a_{3y} = a$$

e. What is the acceleration of m_3 ?

$$\begin{cases} T - F_{fr} = m_1 a & (1) \\ F_{fr} = m_2 a & (2) \\ m_3 g - T = m_3 a & (3) \end{cases}$$

$$(1)+(3): \quad m_3 g - F_{fr} = (m_1 + m_3) a$$
$$m_3 g - m_2 a = (m_1 + m_3) a$$
$$\boxed{a = \frac{m_3}{m_1 + m_2 + m_3} g}$$

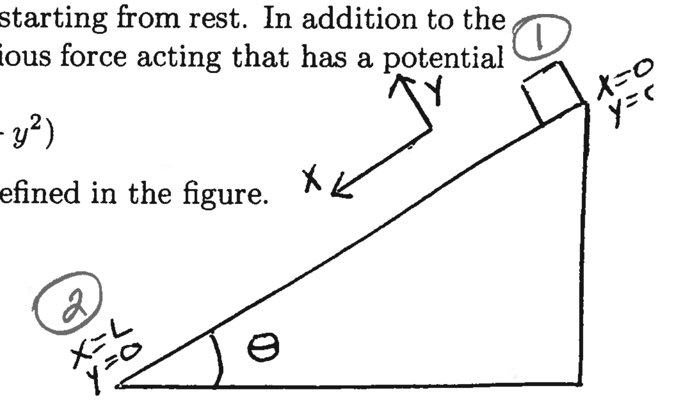
e. What is the largest value that the acceleration of m_3 can have before slipping occurs?

$$(2): \quad F_{fr} = \mu N_1; \quad \mu N_1 = m_2 a$$
$$N_1 = m_1 g$$
$$\mu m_1 g = m_2 a \implies \boxed{a = \mu g \frac{m_1}{m_2}}$$

3. A block of mass m slides down an inclined plane, starting from rest. In addition to the other forces acting on the body there is a mysterious force acting that has a potential energy function given by

$$U(x,y) = \beta(x^2 + y^2)$$

where β is an known constant and x and y are defined in the figure.



a. Find $\vec{F}_{mys}$ the mysterious force in terms of x,y, and β.

$$F_x = -\frac{\partial U}{\partial x} = -2\beta x$$

$$F_y = -\frac{\partial U}{\partial y} = -2\beta y$$

b. Find the velocity of the block when it reaches the bottom of the plane with no friction.

$$U_1 + KE_1 = U_2 + KE_2 \,; \quad mgL\sin\theta = \beta L^2 + \frac{mv^2}{2}$$

$$\boxed{v = \sqrt{\frac{2}{m}\left(mgL\sin\theta - \beta L^2\right)}}$$

Or, $W^{net} = \Delta KE$

$$W^{grav} = \int_0^L mg\sin\theta\, dx = mgL\sin\theta \qquad W^{F_y} = 0$$

$$W^{F_x} = \int_0^L -2\beta x\, dx = -\frac{2\beta L^2}{2} = -\beta L^2 \qquad mgL\sin\theta - \beta L^2 = \frac{mv^2}{2}$$

c. Find the velocity of the block when it reaches the bottom if there were a coefficient of friction between the block and the plane μ.

$$W^{non-cons} = \left[U_2 + KE_2\right] - \left[U_1 + KE_1\right]$$

$$W^{non-cons} = -\int_0^L \mu N\, dx = -\int_0^L (\mu mg\cos\theta + 2\beta \overset{=0}{y})\, dx =$$

$$F_y = ma_y \Rightarrow N - mg\cos\theta - 2\beta y = 0$$

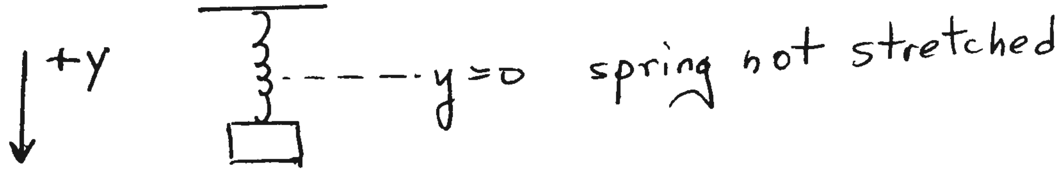
$$N = mg\cos\theta + 2\beta y$$

$$= -\mu mg\cos\theta\, L$$

$$-\mu mg\cos\theta\, L = \beta L^2 + \frac{mv^2}{2} - mgL\sin\theta$$

$$\boxed{v = \sqrt{\frac{2}{m}\left(mgL\sin\theta - \mu mgL\cos\theta - \beta L^2\right)}}$$

4. (25 points) In a famous Physics 218 experiment it was discovered that a real spring doesn't totally follow Hooke's Law. The force exerted by the spring is found to be a linear function of the amount stretched, as Hooke's Law says. However, instead of having $F_y = -ky$ where y is the amount stretched, the actual force exerted by the spring is approximately given by $F_y = -(ky + b)$. Here k and b are known constants. Thus if the spring is vertical there will be no stretching of the spring when a mass m is hung from it unless mg is greater than b.



a. Given this force, with b known, determine the equilibrium amount the spring will be stretched if a mass with $mg > b$ is suspended from it.(Call this y_{eq} as in the lab.)

$$F_y^{net} = 0$$

$$mg - (k y_{eq} + b) = 0 \qquad \boxed{y_{eq} = \frac{mg - b}{k}}$$

b. Find the maximum value of the spring's extension if such a mass were released from rest from $y = 0$, the point where the spring is not stretched. How is this distance related to y_{eq}.

$$U_1 + KE_1 = U_2 + KE_2$$

$$U = -\int F_y\,dy = -\int -(ky+b)\,dy = \boxed{\frac{ky^2}{2} + by + c}$$

$$U_1 = 0;\ KE_1 = 0;\ U_2 = \frac{ky_s^2}{2} + by_s - mgy_s;\ KE_2 = 0$$

$$0 = \frac{ky_s^2}{2} + by_s - mgy_s;\ y_s\left(\frac{ky_s}{2} + b - mg\right) = 0$$

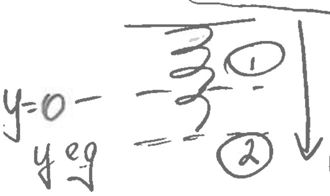
$$\boxed{y_s = \frac{2}{k}(mg - b) = 2y_{eq}}$$

(Or, $W^{net} = \Delta KE$. $W = \int_0^{y_s} (mg - ky - b)\,dy = 0$

$mgy_s - \frac{ky_s^2}{2} - by_s = 0$)

c. Find the maximum value of the mass's kinetic energy if it were released from rest from the point $y = 0$. (Not too much algebra please.)

$\boxed{U_{min},\ \frac{mv_{max}^2}{2} \text{ at } y_{eq}}$ $\qquad \boxed{U_{eq} = \frac{ky_{eq}^2}{2} + by_{eq} - mgy_{eq}}$



$$U_1 + KE_1 = U_{eq} + KE_{eq} \qquad \boxed{KE_{max} = -U_{eq}}$$

$$U_1 = 0;\ KE_1 = 0$$

$$U_{eq} + KE_{eq} = 0;$$