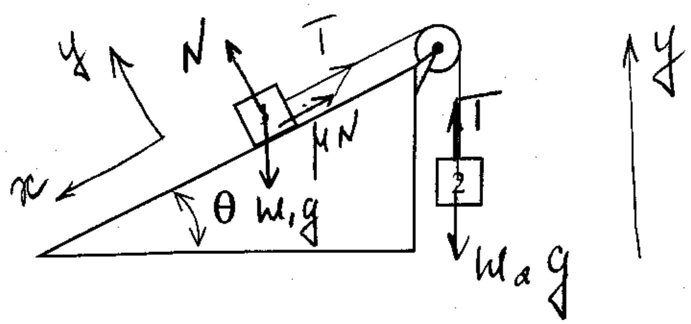
1. (25 points) Block 1, of mass m_1 is placed at rest on an inclined plane. **It is attached** by a massless, unstretchable string to block 2, of mass m_2. The **pulley is massless and** frictionless and just changes the direction of the tension in the **string. Assume the** variables are such that m_1 slides down the plane, starting at $t = 0$. **The coefficient of** friction between the plane and m_1 is the constant μ.

a. Draw the free body diagrams for block 1 and block 2.



b. Find the acceleration of block 1.

$$F_x = m a_x, \quad F_y = m a_y$$

$$\text{Block 1:} \begin{cases} m_1 g \sin\theta - T - \mu N = m_1 a_x \\ N - m_1 g \cos\theta = 0 \end{cases} \qquad a_x = a_y = a$$

$$\text{Block 2:} \quad T - m_2 g = m_2 a_y \quad (2)$$

$$m_1 g \sin\theta - T - \mu m_1 g \cos\theta = m_1 a_x \quad (1)$$

$$(1)+(2): \quad m_1 g \sin\theta - \mu m_1 g \cos\theta - m_2 g = (m_1 + m_2) a$$

$$a = \frac{m_1 g(\sin\theta - \mu\cos\theta) - m_2 g}{m_1 + m_2}$$

c. Find the velocity of block 1 as a function of time.

Since $a = \text{const}$

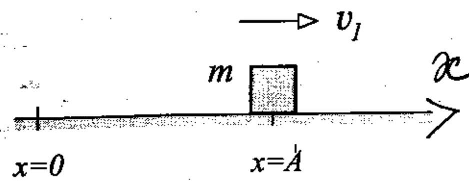
$v = at + C$

$v(0) = 0$

$v(t) = at$

2. (25 points) This is a one-dimensional problem. You need not concern yourself with the y direction. Do not spend very much time on algebra. Once you have one equation with one unknown you should stop!

An object of mass m is placed at the point $x = A$ on a horizontal table and given a velocity of magnitude v_1 to the right. The object is attracted to the origin by some mysterious force which has magnitude $\frac{\alpha}{x^2}$ where α is a constant. The coefficient of friction between the table and the object is μ.



1D problem

x

$$W^{net} = \int_{x_1}^{x_2} F^{net}\,dx = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$$

a. How far will the object go before it turns around and begins to move to the left?

$$\vec{F} = -\frac{\alpha}{x^2}\vec{i}\ ; \quad \vec{F}^{fr} = -\mu N\vec{i}\ (A \to x_s)\ ; \quad \vec{F}^{fr} = \mu N\vec{i}\ (x_s \to A)$$

$$W^F = \int_A^{x_s} -\frac{\alpha}{x^2}\,dx = \frac{\alpha}{x}\Big|_A^{x_s} = \frac{\alpha}{x_s} - \frac{\alpha}{A}$$

$$W^{fr} = \int_A^{x_s} -\mu N\,dx = -\mu mg\,(x_s - A) \quad (N = mg)$$

$$\frac{\alpha}{x_s} - \frac{\alpha}{A} - \mu mg\,(x_s - A) = -\frac{mv_1^2}{2}$$

$$\boxed{\frac{\alpha}{x_s} - \frac{\alpha}{A} - \mu mg\,(x_s - A) + \frac{mv_1^2}{2} = 0}$$

b. How fast will the object be going when it is again at the point $x = A$?

$$W^F = \int_A^A -\frac{\alpha}{x^2}\,dx = 0\ ; \quad W^{fr} = \int_A^{x_s} -\mu mg\,dx + \int_{x_s}^{A} \mu mg\,dx = -2\mu mg\,(x_s - A)$$

$$-2\mu mg\,(x_s - A) = \frac{mv_f^2}{2} - \frac{mv_1^2}{2}$$

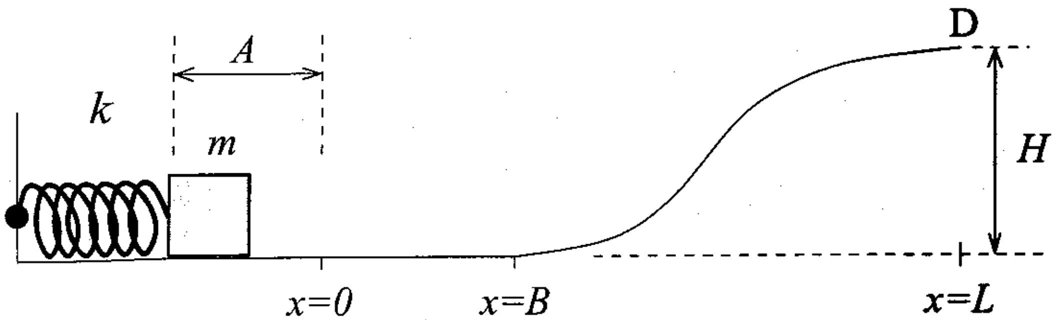
$$\boxed{v_f = \sqrt{\frac{2}{m}\left(\frac{mv_1^2}{2} - 2\mu mg\,(x_s - A)\right)}}$$

Or $$\mu mg(A - x_s) + \int_{x_s}^{A} -\frac{\alpha}{x^2}\,dx = \frac{mv_f^2}{2}$$

$$\mu mg\,A - x_s + \frac{\alpha}{A} - \frac{\alpha}{x_s} = \frac{mv_f^2}{2}$$

$$\boxed{v_f = \sqrt{\frac{2}{m}\left(\mu mg\,(A - x_s) + \frac{\alpha}{A} - \frac{\alpha}{x_s}\right)}}$$

3. (25 points) A block of mass m is placed on a frictionless table where there is a spring, with spring constant k. The spring is not stretched or compressed at the point $x = 0$. The block is pushed to the left, so that the spring is compressed an amount A, and released from rest. Besides the normal force, gravity and the spring there is another force acting on the block given by $\vec{F}_1 = c_1\vec{i} + c_2\vec{j}$ where c_1 and c_2 are known constants, and $c_2 < mg$.



a. How fast will the block be going at $x = 0$?

$W^{F_1} = \int_{-A}^{0} c_1\,dx = c_1 A$; $W^{spring} = \int_{-A}^{0} -kx\,dx = -\frac{kx^2}{2}\Big|_{-A}^{0} = \frac{kA^2}{2}$

$c_1 A + \frac{kA^2}{2} = \frac{m v_f^2}{2}$; $v_f = \sqrt{\frac{2}{m}\left(c_1 A + \frac{kA^2}{2}\right)}$

b. How fast will the block be going at $x = B$? (Remember the spring is not attached to the block!)

$W^{F_1} = \int_{-A}^{B} c_1\,dx = c_1(B+A)$

$W^{spring} = \frac{kA^2}{2}$

$c_1(B+A) + \frac{kA^2}{2} = \frac{m v_f^2}{2} - 0$; $\boxed{v_f = \sqrt{\frac{2}{m}\left(c_1(B+A) + \frac{kA^2}{2}\right)}}$

c. Assuming the block makes it up the frictionless incline to the point D, how fast will it be going at the point D where $x = L$ and $y = H$?

$W^{Gr} = \int_0^H -mg\,dy = -mgH$; $W^{c_2} = \int_0^H c_2\,dy = c_2 H$

$W^{spring} = \int_{-A}^{0} -kx\,dx = -\frac{kx^2}{2}\Big|_{-A}^{0} = \frac{kA^2}{2}$ $\quad \int_{x_1}^{x} \vec{N}\cdot d\vec{r} = 0$

$W^{c_1} = \int_{-A}^{L} c_1\,dx = c_1(L+A)$

$c_1(L+A) + \frac{kA^2}{2} + c_2 H - mgH = \frac{m v_f^2}{2} - 0$

$v_f = \sqrt{\frac{2}{m}\left(c_1(L+A) + \frac{kA^2}{2} + c_2 H - mgH\right)}$

4. (25 points) This is a one-dimensional problem. You need not concern yourself with the y direction. An object of mass m is acted upon by a force given by

$$F_x = \frac{\alpha}{x^3} - \frac{\beta}{x^2}$$

where α and β are positive constants.

a. Verify that this force is conservative by finding a potential energy function.

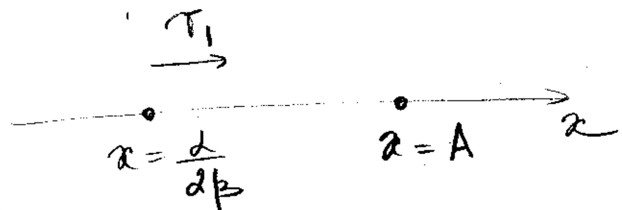
$$F = -\frac{dU}{dx}$$

$$U = -\int F_x\,dx = -\int\left(\frac{\alpha}{x^3} - \frac{\beta}{x^2}\right)dx = \frac{\alpha}{2x^2} - \frac{\beta}{x} + C$$

$$\boxed{U(x) = \frac{\alpha}{2x^2} - \frac{\beta}{x} + C}$$

$$F = -\frac{dU}{dx} = \frac{\alpha}{x^3} - \frac{\beta}{x^2}$$

b. If the object is placed at the point $x = \frac{\alpha}{2\beta}$ and given a velocity of magnitude v_1 in the positive x direction, what will its velocity be at the point $x = A$?



$$U\left(\frac{\alpha}{2\beta}\right) + \frac{m v^2\left(\frac{\alpha}{2\beta}\right)}{2} = U(A) + \frac{m v^2(A)}{2}$$

$$\frac{\alpha 4\beta^2}{2\alpha^2} - \frac{\beta 2\beta}{\alpha} + \frac{m v_1^2}{2} = \frac{\alpha}{2A^2} - \frac{\beta}{A} + \frac{m v^2(A)}{2}$$

$$\frac{2\beta^2}{\alpha} - \frac{2\beta^2}{\alpha} + \frac{m v_1^2}{2} = \frac{\alpha}{2A^2} - \frac{\beta}{A} + \frac{m v^2(A)}{2}$$

$$\boxed{v(A) = \sqrt{\frac{2}{m}\left(\frac{m v_1^2}{2} - \frac{\alpha}{2A^2} + \frac{\beta}{A}\right)}}$$