Exercise 14.3

$\vec{L}_i = \vec{L}_f$

$\vec{L}_i = \vec{L}_{i,man} + \vec{L}_{i,bullet} = 0 + 0$

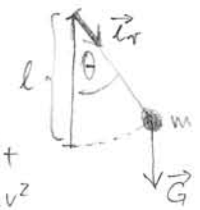
$\vec{L}_f = \vec{L}_{f,m} + \vec{L}_{f,b} = \underset{l/2}{r} \cdot M \underset{r\cdot\omega}{v_{\theta,m}} \vec{k} + \underset{l/2}{r} \cdot m \underset{v_b \sin\theta}{v_{\theta,b}} \vec{k}$

$\Rightarrow 0 = \frac{l}{2} M \frac{l}{2} \omega \vec{k} + \frac{l}{2} m v_b \sin\theta \vec{k} \quad | : \vec{k}$

$\Rightarrow \left(\frac{l}{2}\right)^2 M \omega + \frac{l}{2} m v_b \sin\theta \quad | \frac{l}{2}$

$\Rightarrow \omega = \frac{2}{l} \frac{m}{M} v_b \sin\theta$

Exercise 14.4



a) $E_i = E_f$

$\Rightarrow mg(l - l\cos\theta_0) = mg(l - l\cos\theta) + \frac{1}{2}mv^2$

Circle $\Rightarrow \vec{v} \perp \vec{i}_r \Rightarrow v = l\omega$

$\Rightarrow \frac{1}{2} m l^2 \omega^2 = mgl - mgl\cos\theta_0 - mgl + mgl\cos\theta$

$\Rightarrow \frac{1}{2} l \omega^2 = g(\cos\theta - \cos\theta_0) \Rightarrow \omega = \sqrt{\frac{2g}{l}(\cos\theta - \cos\theta_0)} \quad (1)$

b) $|\vec{L}| = m l^2 \omega = m \frac{l^2}{\sqrt{l}} \sqrt{2g(\cos\theta - \cos\theta_0)} = m l^{3/2} \sqrt{\dots} \quad (2)$

c) Find τ: $\vec{\tau} = \vec{r} \times \vec{G} \Rightarrow |\vec{\tau}| = l \cdot G_\theta = lmg\sin\theta \quad (3)$

Compare with: $\vec{\tau} = \frac{d\vec{L}}{dt}$ 2x chain rule

$\frac{d\vec{L}}{dt} \overset{(2)}{=} \frac{d}{dt}\left[m l^{3/2} \{2g(\cos\theta - \cos\theta_0)\}^{1/2}\right] =$

$= m l^{3/2} \cdot \frac{1}{2} \{2g(\cos\theta - \cos\theta_0)\}^{-1/2} \cdot 2g(-\sin\theta) \underbrace{\frac{d\theta}{dt}}_{=\omega} \overset{(1)}{=}$

$= m l^{3/2} \cdot \frac{1}{\cancel{2}} \{2g(\cos\theta - \cos\theta_0)\}^{-1/2} \cdot \cancel{2}g(-\sin\theta) \sqrt{\frac{2g}{l}(\cos\theta - \cos\theta_0)}$

$= m l^{3/2} \cancel{\{\dots\}^{-1/2}} \cdot g(-\sin\theta) \frac{1}{\sqrt{l}} \cancel{\{\dots\}^{+1/2}} =$

$= lmg(-\sin\theta) \Rightarrow \left|\frac{d\vec{L}}{dt}\right| = lmg\sin\theta = (3)$

Exercise 14.5

$(g = +9.8\ m/s^2)$

We know the kinematics: $v_x(t) = v_0\cos\theta \quad x(t) = v_0\cos\theta \cdot t$

$v_y(t) = -gt + v_0\sin\theta \quad y(t) = -\frac{1}{2}gt^2 + v_0\sin\theta \cdot t$

a) $\vec{\tau} = \vec{r} \times \vec{G} = \left[(v_0\cos\theta \cdot t)\vec{i} + (-\frac{1}{2}gt^2 + v_0\sin\theta \cdot t)\vec{j}\right] \times \left[-mg\vec{j}\right] = - mgv_0\cos\theta \cdot t \cdot \vec{k}$

$\vec{j} \times \vec{j} = 0, \ \vec{i} \times \vec{j} = \vec{k}$

b) $\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} =$

$= m\left[(v_0\cos\theta \cdot t)\vec{i} + (-\frac{1}{2}gt^2 + v_0\sin\theta \cdot t)\vec{j}\right] \times \left[(v_0\cos\theta)\vec{i} + (-gt + v_0\sin\theta)\vec{j}\right]$

$= m(v_0\cos\theta \cdot t)(-gt + v_0\sin\theta)\underbrace{\vec{i} \times \vec{j}}_{=\vec{k}} + m(-\frac{1}{2}gt^2 + v_0\sin\theta \cdot t)(v_0\cos\theta)\underbrace{\vec{j} \times \vec{i}}_{=-\vec{k}} =$

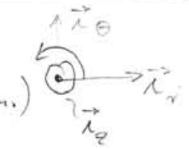
$= \left[-mgv_0\cos\theta \cdot t^2 + \cancel{mv_0^2\cos\theta\sin\theta \cdot t} + \frac{1}{2}mgv_0\cos\theta t^2 - \cancel{mv_0^2\cos\theta\sin\theta \cdot t}\right] \cdot \vec{k}$

$= -\frac{1}{2}mgv_0\cos\theta \cdot t^2 \cdot \vec{k} = \vec{L}$

c) $\frac{d\vec{L}}{dt} = -mgv_0\cos\theta \cdot t \cdot \vec{k} = \vec{\tau}$

see a)

Exercise 14.6



$\vec{L}_f = \vec{L}(t) = \vec{L}_i$ (Conserv. of AngMom.)

$\vec{L}_i = \vec{L}_{1,i} + \vec{L}_{2,i} = \vec{r}_{1,i} \times \vec{p}_{1,i} + \vec{r}_{2,i} \times \vec{p}_{2,i}$

$\vec{L}_{i,1} = \vec{r}_{1,i} \times \vec{p}_{1,i} = r_{i,1}\vec{i}_r \times p_{\theta i,1}\vec{i}_\theta = \frac{B}{4} \cdot m_1 \underset{r_{i,1}\cdot\omega}{v_{\theta i,1}} \underbrace{\vec{i}_r \times \vec{i}_\theta}_{=\vec{e}_z}$, $r_{i,1} = B/4$

$= \left(\frac{B}{4}\right)^2 m_1 \omega \vec{e}_z$

in the same way: $\vec{L}_{i,2} = \left(\frac{B}{4}\right)^2 m_2 \omega$

$\vec{L}_f = \vec{L}_1(t) + \vec{L}_2(t) = \underbrace{\left(\frac{B}{4} - at^2\right)^2}_{=r_1^2(t)} m_1 \omega \vec{i}_z + r_2^2(t) m_2 \omega \vec{e}_z =$

Cons. of Ang. Mom. $\Rightarrow$

$\left(\frac{B}{4}\right)^2 m_1 \cancel{\omega \vec{i}_z} + \left(\frac{B}{4}\right)^2 m_2 \cancel{\omega \vec{i}_z} = \left(\frac{B}{4} - at^2\right)^2 m_1 \cancel{\omega \vec{i}_z} + r_2^2(t) m_2 \cancel{\omega \vec{i}_z}$

$\Rightarrow \cancel{\left(\frac{B}{4}\right)^2 m_1} + \left(\frac{B}{4}\right)^2 m_2 = \cancel{\left(\frac{B}{4}\right)^2 m_1} - \frac{B}{2}at^2 m_1 + (at^2)^2 m_1 + r_2^2(t) m_2$

$\Rightarrow \left(\frac{B}{4}\right)^2 m_2 + \frac{B}{2}at^2 m_1 - (at^2)^2 m_1 = r_2^2(t) m_2$

$\Rightarrow r^2(t) = \left(\frac{B}{4}\right)^2 + \frac{m_1}{m_2} at^2 \left[\frac{B}{2} - at^2\right]$

$\Rightarrow r(t) = \sqrt{\left(\frac{B}{4}\right)^2 + \frac{m_1}{m_2} at^2 \left[\frac{B}{2} - at^2\right]}$