1A

1a) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$

$$\Rightarrow \begin{cases} F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0 \\ F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0 \end{cases}$$

sec 801-803

$$\Rightarrow \begin{cases} F_1 \cos\theta_1 - F_2 \sin\theta_2 - F_3 \cos\theta_3 + F_{4x} = 0 \\ -F_1 \sin\theta_1 - F_2 \cos\theta_2 + F_3 \sin\theta_3 + F_{4y} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} F_{4x} = -F_1 \cos\theta_1 + F_2 \sin\theta_2 + F_3 \cos\theta_3 \\ F_{4y} = +F_1 \sin\theta_1 + F_2 \cos\theta_2 - F_3 \sin\theta_3 \end{cases}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = \dots$$

$$\tan\theta_4 = \frac{F_{4y}}{F_{4x}}$$

1A

1b) $y(t) = c_1 t^4 + c_2 t^3 + \underset{?}{\underset{\uparrow}{k_1}} t + \underset{?}{\underset{\uparrow}{k_2}}$

$y(t=0) = H$

$v(t=0) = W$

$y(t=0) = k_2 = H \Rightarrow \boxed{k_2 = H}$

$v(t) = \frac{d}{dt} y(t) = 4c_1 t^3 + 3c_2 t^2 + k_1$

$v(t=0) = k \Rightarrow \boxed{k_1 = w}$

$a(t) = \frac{d}{dt} v(t)$

$a(t) = 12 c_1 t^2 + 6 c_2 t$

1A

2) $a_x(t) = \alpha t^2 \qquad x(t=0) = 0$

$a_y(t) = -\beta t \qquad y(t=0) = 0$

maximum? $V_y(t=T) = 0$

$V_y(t) = \int a_y(t)\,dt + V_{oy} = -\frac{1}{2}\beta t^2 + \overset{V_m \sin\theta}{\overset{\downarrow}{V_{oy}}}$

$V_y(t=T) = 0 \Rightarrow -\frac{1}{2}\beta T^2 + V_{oy} = 0$

$\Rightarrow T = \sqrt{\frac{2 V_{oy}}{\beta}}$

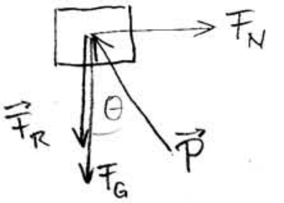
$V_x(t) = \int a_x(t)\,dt + V_{ox} = \frac{1}{3}\alpha t^3 + \overset{V_m \cos\theta}{\overset{\downarrow}{V_{ox}}}$

$x(t) = \int V_x(t)\,dt + X_0 = \frac{1}{12}\alpha t^4 + V_{ox} \cdot t + 0$

$$x(t=T) = \frac{\alpha}{12} \cdot \frac{4 V_m^2 \sin^2\theta}{\beta^2} + V_m \cos\theta \sqrt{\frac{2 V_m \sin\theta}{\beta}}$$

3a) 1A

① FBD ② Newton ③ Solve equations



sec 801-803

② $F_x = ma_x = 0$ (!) | $F_y = ma_y$ $+9.8\frac{m}{s^2}$

$P_x + F_{Nx} + F_{Gx} + F_{Rx} = 0$ | $P_y + F_{Ny} + F_{Gy} + F_{Ry} = ma_y$

$-P\sin\theta + F_N + 0 + 0 = 0$ | $P\cos\theta + 0 - mg - \mu F_N = ma_y$

$\Rightarrow F_N = P\sin\theta = -P_x$

$\Rightarrow ma_y = P\cos\theta - mg - \mu P\sin\theta$

$\Rightarrow a_y = -g + \frac{P}{m}(\cos\theta - \mu\sin\theta)$

$v(t) = a_y \cdot t$

3b) $\vec{P} = \overbrace{-c_1}^{P_x}\vec{i} + \overbrace{(mg + \alpha t)}^{P_y}\vec{j}$ 1A

$ma_y^* = P\cos\theta - mg - \mu P\sin\theta =$

$= P_y - mg + \mu P_x =$

$= (mg + \alpha t) - mg + \mu(-c_1) =$

$= \alpha t - \mu c_1$

Attention! $a(t) \geq 0$ for all times

$\Rightarrow a(t) = \begin{cases} 0 & \text{for } t \leq T \\ a^*(t) & \text{for } t > T \end{cases}$

calc. T: $\alpha T - \mu c_1 = 0 \Rightarrow T = \frac{\mu c_1}{\alpha}$

$\Rightarrow v(t) = 0$ for $t \leq T$

$v(t) = \int a\,dt + c_0 = \frac{\alpha}{2}t^2 - \mu c_1 t + c_0$

$v(t=T) = 0 = \frac{\alpha}{2}\frac{\mu^2 c_1^2}{\alpha^2} - \frac{\mu^2 c_1^2}{\alpha} + c_0 = -\frac{\mu^2 c_1^2}{2\alpha} + c_0$

$\Rightarrow c_0 = \mu^2 c_1^2 / 2\alpha$

1B

1a) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$

$\Rightarrow \begin{cases} F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0 \\ F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0 \end{cases}$

sec 810-812

$\Rightarrow \begin{cases} -F_1 \sin\theta_1 - F_2 \sin\theta_2 + F_3 \cos\theta_3 + F_{4x} = 0 \\ +F_1 \cos\theta_1 - F_2 \cos\theta_2 - F_3 \sin\theta_3 + F_{4y} = 0. \end{cases}$

$F_1 = m_1 g, \quad F_2 = m_2 \cdot g, \quad F_3 = m_3 \cdot g$

$\Rightarrow \begin{cases} -m_1 g \sin\theta_1 - m_2 g \sin\theta_2 + m_3 g \cos\theta_3 + F_{4x} = 0 \\ +m_1 g \cos\theta_1 - m_2 g \cos\theta_2 - m_3 g \sin\theta_3 + F_{4y} = 0 \end{cases}$

$\Rightarrow F_{4x} = +m_1 g \sin\theta_1 + m_2 g \sin\theta_2 - m_3 g \cos\theta_3$

$F_{4y} = -m_1 g \cos\theta_1 + m_2 g \cos\theta_2 + m_3 g \sin\theta_3$

$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = \ldots \quad ; \quad F_4 = m_4 g = \ldots$

$\tan\theta_4 = \dfrac{F_{4y}}{F_{4x}} = \ldots$

1B

1b) $x(t) = c_1 t^4 + c_2 t^3 + \underset{=?}{k_1} t + \underset{=?}{k_2}$

$x(t=0) = L$

$v(t=0) = W$

$x(t=0) = k_2 = L \quad \Rightarrow \boxed{k_2 = L}$

$v(t) = \dfrac{d}{dt} x(t) = 4c_1 t^3 + 3c_2 t^2 + k_1$

$v(t=0) = k_1 \quad \Rightarrow \quad \boxed{k_1 = W}$

$\boxed{a(t) = \dfrac{d}{dt} v(t) = 12 c_1 t^2 + 6 c_2 t}$

1B

2) $x(t=T) = C \qquad v_{0x} = v_0 \cos\theta$

$y(t=T) = D \qquad v_{0y} = v_0 \sin\theta$

$x(t=0) = 0 \qquad a_x = 0 \qquad v_0 = ?$

$y(t=0) = A \qquad a_y = -\beta t \qquad (T = ?)$

$x(t) = v_{0x} \cdot t$ because $a_x = 0$ & $x(0) = 0$

$x(t=T) = C \Rightarrow v_{0x} \cdot T = C \Rightarrow \boxed{T = \dfrac{C}{v_{0x}}}$

$v_y(t) = \int(-\beta t)\,dt + v_{0y} = -\frac{1}{2}\beta t^2 + v_{0y}$

$y(t) = \int\left(-\frac{1}{2}\beta t^2 + v_{0y}\right)dt + y_0 = -\frac{1}{6}\beta t^3 + v_{0y} t + A$

$y(t=T) = D \Rightarrow -\frac{1}{6}\beta \dfrac{C^3}{v_0^3 \cos^3\theta} + \dfrac{v_0 \sin\theta}{v_0 \cos\theta} \cdot C + A = D$

$\Rightarrow \dfrac{\beta}{6} \dfrac{C^3}{v_0^3 \cos^3\theta} = D - A - C \cdot \tan\theta$

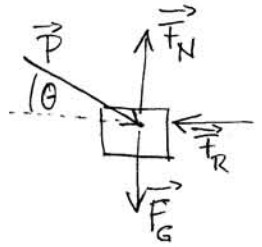
$\Rightarrow v_0^3 = \dfrac{\beta C^3}{6 \cos^3\theta} \dfrac{1}{D - A - C\tan\theta}$

$\Rightarrow v_0 = \dfrac{C}{\cos\theta} \sqrt[3]{\dfrac{\beta}{6(D - A - C\tan\theta)}}$

3a) 1B

① FBD ② Newton ③ Solve equations

①



sec 810-812

② $F_x = ma_x$ | $F_y = ma_y = 0\,(!)$

$P_x + F_{Nx} + F_{Gx} + F_{Rx} = ma_x$ | $P_y + F_{Ny} + F_{Gy} + F_{Ry} = 0$

$+P\cos\theta + 0 + 0 - \mu F_N = ma_x$ | $-P\sin\theta + F_N - mg + 0 = 0$

③ $\Rightarrow F_N = P\sin\theta + mg$

$P\cos\theta - \mu(P\sin\theta + mg) = ma_x$

$$\Rightarrow a_x = \frac{P}{m}(\cos\theta - \mu\sin\theta) - \mu g$$

$$\Rightarrow v(t) = \left[\frac{P}{m}(\cos\theta - \mu\sin\theta) - \mu g\right]\cdot t$$

because $a(t) = a(\cancel{t})$ and $v(t=0)=0$

3b) 1B

$$a = \frac{P}{m}(\cos\theta - \mu\sin\theta) - \mu g$$

$\theta = 0 \quad P = \alpha t$

$$\Rightarrow \overset{*}{a}(t) = \frac{\alpha t}{m} - \mu g$$

Attention! $a(t) \geq 0$ for all times

$\Rightarrow a(t) = 0$ for $t \leq T$

$a(t) = a^*(t)$ for $t > T$

calc. T: $\frac{\alpha T}{m} - \mu g = 0 \Rightarrow T = \frac{m\mu g}{\alpha}$

$\Rightarrow v(t) = 0$ for $t \leq T$

$$v(t) = \int a\,dt + C = \frac{\alpha t^2}{2m} - \mu g t + C$$

$$v(t=T) = 0 = \frac{\alpha m^2\mu^2 g^2}{2m\,\alpha^2} - \frac{m}{\alpha}\mu^2 g^2 + C \Rightarrow C = \ldots$$