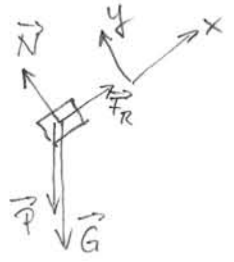
sec 801-803

P1 : Incline 8:00



$$y: \; ma_y = 0 = F_y =$$
$$= N - P\cos\theta - G\cos\theta$$
$$\Rightarrow N = (P+G)\cos\theta$$

$$x: \; ma_x = \mu N - P\sin\theta - G\sin\theta$$
$$= \mu(P+G)\cos\theta - (P+G)\sin\theta = F_x$$

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}m\underbrace{v_i^2}_{=0} = \int_0^{-L} \underbrace{F_{tot}}_{=F_x} dx$$

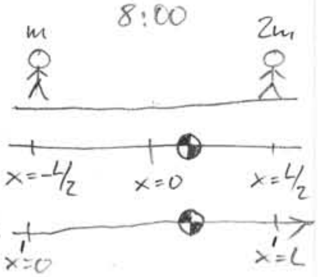
$$\Rightarrow \frac{1}{2}mv_f^2 = \int_0^{-L} \left[(P+G)(\mu\cos\theta - \sin\theta)\right] dx =$$
$$= \left[(P+G)(\mu\cos\theta - \sin\theta)\right] \cdot (-L)$$

$$\Rightarrow v_f = \sqrt{\frac{2L}{m}(P+G)(\sin\theta - \mu\cos\theta)}$$

Alternative: $a_x = \text{const} \Rightarrow v_f^2 = 2a_x \cdot (-L)$

$$\Rightarrow v^2 = \frac{2L}{m}(P+G)(\sin\theta - \mu\cos\theta)$$

P2: Two students on Ice 8:00



a)
$$X_{cm} = \frac{X_1 m_1 + X_2 m_2}{m_1 + m_2}$$
$$= \frac{-L/2\, m + L/2 \cdot 2m}{m + 2m}$$
$$= \frac{L/2}{3} = L/6$$

Or: $X'_{cm} = \frac{x'_1 m_1 + x'_2 m_2}{m_1 + m_2} = \frac{0 \cdot m + L \cdot 2m}{m + 2m} = \frac{2}{3} \cdot L$

b) will meet at center of mass as calc. in a)
Why: only int. forces! $\Rightarrow ma_{cm} = F_{ext} = 0$
$\Rightarrow v_{cm}$ does not change. However, initial
$v_{cm} = 0 \Rightarrow v_{cm} = 0$ for ever $\Rightarrow$ c.m. will
sit still forever $\Rightarrow$ students meet there,
because meeting point = c.m.

c) no ext. force $\Rightarrow \underbrace{\vec{P}_i}_{=0} = \vec{P}_f$

$$\Rightarrow 0 = mv_0 + 2mv_1 \Rightarrow v_1 = -\frac{v_0}{2}$$

P3: Trash compactor 8:00

sec 801-803

a) $F = -\frac{dU}{dx} \Rightarrow U = \int -F\,dx + \text{const}$

$\Rightarrow U(x) = \int -P + kx^2\,dx + \text{const} = -Px + \frac{k}{3}x^3 + \text{const}$

$\Rightarrow U(x) = -Px + \frac{k}{3}x^3 + C$

b) $E_i = E_f$

$\frac{1}{2}m\underbrace{v^2(x=0)}_{=0} + U(x=0) = \frac{1}{2}mv^2(x) + U(x)$

$0 + 0 + \cancel{C} = \underbrace{\frac{1}{2}mv^2(x)}_{KE(x)} - Px + \frac{k}{3}x^3 + \cancel{C}$

$\Rightarrow KE(x) = P\cdot x - \frac{k}{3}x^3$

c) Maximum $\Rightarrow \frac{d}{dx} KE(x) \overset{!}{=} 0$

$P - kx_{max}^2 \overset{!}{=} 0 \Rightarrow x_{max} = \sqrt{\frac{P}{k}}$

$KE_{max} = KE(x_{max}) = P\cdot\sqrt{\frac{P}{k}} - \frac{k}{3}\left(\sqrt{\frac{P}{k}}\right)^3$

$= \sqrt{\frac{P^3}{k}} - \frac{1}{3}\sqrt{\frac{P^3}{k}} = \frac{2}{3}\sqrt{\frac{P^3}{k}}$

P4: Mysterious Force on Bullet 8:00

a) $F \overset{!}{=} -\beta/x^2$; $F = -\frac{dU}{dx} \Rightarrow U = \int -F\,dx + C$

$\Rightarrow U(x) = -\frac{\beta}{x} + C$

Alternative:
$\Delta KE = \int_A^L F\,dx$

$E_i = E_f$

$\frac{1}{2}mv_0^2 + U(x=A) = 0 + U(x=L)$

$\frac{1}{2}mv_0^2 - \frac{\beta}{A} + \cancel{C} = -\frac{\beta}{L} + \cancel{C}$

$\Rightarrow \frac{L}{\beta} = \frac{1}{\frac{\beta}{A} - \frac{1}{2}mv_0^2} \Rightarrow L = \frac{\beta}{\frac{\beta}{A} - \frac{1}{2}mv_0^2}$

b) $E_i = E_f$

$\frac{1}{2}mv_0^2 + U(x=A) = \frac{1}{2}mv^2(x) + U(x)$

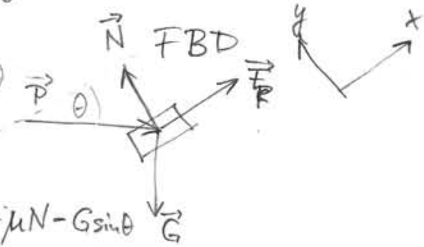
$\Rightarrow \frac{1}{2}mv_0^2 - \frac{\beta}{A} = \frac{1}{2}mv^2(x) - \frac{\beta}{x}$

$\Rightarrow \frac{1}{2}mv^2(x) = \frac{1}{2}mv_0^2 - \frac{\beta}{A} + \frac{\beta}{x}$

$\Rightarrow v(x) = \sqrt{v_0^2 - \frac{2\beta}{m}\left(\frac{1}{A} - \frac{1}{x}\right)}$

sec 810-812

P1: Incline 10:00



y: $F_y = 0 = N - P\sin\theta - G\cos\theta$

$\Rightarrow N = P\sin\theta + G\cos\theta$

x: $ma_x = F_x = P\cos\theta + \mu N - G\sin\theta$

$\Rightarrow a_x = \frac{1}{m}(P\cos\theta + \mu P\sin\theta + \mu G\cos\theta - G\sin\theta)$

WET: $\Delta KE = \int_0^{-L} F_{tot}\,dx = \int_0^{-L} F_x\,dx$

$\Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}m\underbrace{v_i^2}_{=0} = \int_0^{-L} P\cos\theta - G\sin\theta + \mu P\sin\theta + \mu G\cos\theta \; dx$

$\Rightarrow v_f = \sqrt{\frac{-2L}{m}(P\cos\theta - G\sin\theta + \mu P\sin\theta + \mu G\cos\theta)}$

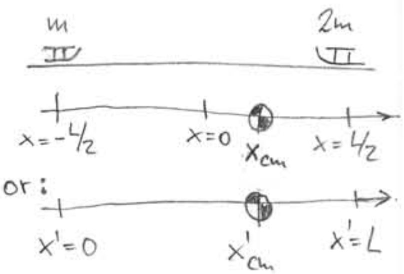
Alternative: $a_x = \text{const} \Rightarrow v_f^2 = 2a_x\cdot(-L)$

$\Rightarrow v_f^2 = \frac{-2L}{m}(P\cos\theta - G\sin\theta + \mu P\sin\theta + \mu G\cos\theta)$

P2: Children on Sleds 10:00

a) $x_{cm} = \frac{x_1m_1 + x_2m_2}{m_1+m_2}$

(5) $= \frac{-L/2\,m + L/2\cdot 2m}{m + 2m}$



$x_{cm} = \frac{L/2}{3} = L/6$

or: $x'_{cm} = \frac{x'_1m_1 + x'_2m_2}{m_1+m_2} = \frac{0m + L\cdot 2m}{3m}$

$\Rightarrow x'_{cm} = \frac{2}{3}L$

b) will meet at c.m. as calculated in a)!

(5) Why? Only int. forces! $\Rightarrow ma_{cm} = F_{ext} = 0$

$\Rightarrow v_{cm}$ does not change. However, initial $v_{cm} = 0 \Rightarrow v_{cm} = 0$ for ever $\Rightarrow$ c.m. will sit still forever $\Rightarrow$ sleds will meet there, because meeting point = c.m.

c) no ext. force $\Rightarrow \underbrace{\vec{P}_i}_{=0} = \vec{P}_f$

(15)

$\Rightarrow 0 = mv_e + 2m(-v_0) \Rightarrow v_e = 2v_0$

P3 Engine: 10:00

sec 810-812

a) $F = -\frac{dU}{dx} \Rightarrow U(x) = \int -F\,dx + \text{const}$

$\Rightarrow U(x) = \int -F_0 + \alpha x^2\,dx + c$

$\Rightarrow U(x) = -F_0 x + \frac{\alpha}{3}x^3 + c$

b) $E_i = E_f$

$\frac{1}{2}m\underbrace{v^2(x=0)}_{0} + U(x=0) = \frac{1}{2}mv^2(x) + U(x)$

$0 + 0 + \cancel{c} = \underbrace{\frac{1}{2}mv^2(x)}_{KE(x)} - \frac{F_0}{x} + \frac{\alpha}{3}x^3 + \cancel{c}$

$\Rightarrow KE(x) = F_0 \cdot x - \frac{\alpha}{3}x^3$

c) Maximum $\Rightarrow \frac{d}{dx}KE(x) \overset{!}{=} 0$

$\Rightarrow F_0 - \alpha x^2 \overset{!}{=} 0 \Rightarrow x_{max} = \sqrt{\frac{F_0}{\alpha}}$

$KE_{max} = KE(x_{max}) = F_0 \cdot \sqrt{\frac{F_0}{\alpha}} - \frac{\alpha}{3}\left(\sqrt{\frac{F_0}{\alpha}}\right)^3$

$= \sqrt{\frac{F_0^3}{\alpha}} - \frac{1}{3}\sqrt{\frac{F_0^3}{\alpha}} = \frac{2}{3}\sqrt{\frac{F_0^3}{\alpha}}$

P4: Mysterious Force on Bullet 10:00

a) $F = +\beta/x^2$; $F = -\frac{dU}{dx} \Rightarrow U = \int -F\,dx + c$

$\overset{!}{\Rightarrow} U(x) = +\frac{\beta}{x} + c$

Alternative
$\Delta KE = \int_A^L F\,dx$

$E_i = E_f$

$\frac{1}{2}mv_0^2 + U(x=A) = 0 + U(x=L)$

$\frac{1}{2}mv_0^2 + \frac{\beta}{A} + \cancel{c} = +\frac{\beta}{L} + \cancel{c}$

$\Rightarrow \frac{L}{\beta} = \frac{1}{\frac{1}{2}mv_0^2 + \frac{\beta}{A}} \Rightarrow L = \frac{\beta}{\frac{1}{2}mv_0^2 + \beta/A}$

b) $E_i = E_f$

$\frac{1}{2}mv_0^2 + U(x=A) = \frac{1}{2}mv^2(x) + U(x)$

$\Rightarrow \frac{1}{2}mv_0^2 + \frac{\beta}{A} = \frac{1}{2}mv^2(x) + \frac{\beta}{x}$

$\Rightarrow \frac{1}{2}mv^2(x) = +\frac{1}{2}mv_0^2 + \frac{\beta}{A} - \frac{\beta}{x}$

$\Rightarrow v(x) = \sqrt{v_0^2 + \frac{2\beta}{m}\left(\frac{1}{A} - \frac{1}{x}\right)}$