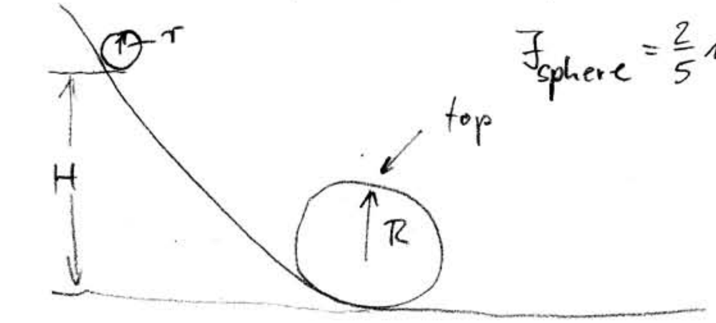
# Loop-the-loop with a sphere

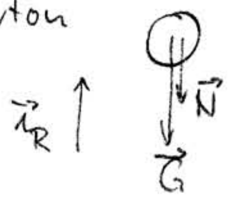



$J_{sphere} = \frac{2}{5} m r^2$

At what height should we start so that the sphere makes it?

„sphere makes it" $\Longleftrightarrow$ normal force $> 0$

Newton




$$F_R = m a_R = m\left[\underbrace{\cancel{\frac{d^2 R}{dt^2}}}_{R = const,\ 0} - R\omega_{top}^2\right]$$

$$-mg - N = -mR\omega_{top}^2$$

let's assume that sphere does not slip $\Rightarrow$ $v_{top} = \omega_{top} r$ (not $\omega R$ !!)

$$\Rightarrow \quad mg + N = mR\frac{v_{top}^2}{r^2} \qquad (1)$$

How do we get $v_{top}$? Energy cons.!

$$mg(H - 2R) = \frac{1}{2} m v_{top}^2 + \frac{1}{2} J \omega_{top}^2$$

$$= \frac{1}{2} m v_{top}^2 + \frac{1}{2} J \frac{v_{top}^2}{r^2} \qquad (2)$$

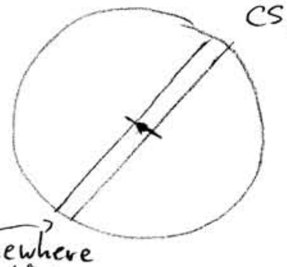
(1) for $N=0$ $\Rightarrow$ $\frac{v_{top}^2}{r^2} = \frac{g}{R}$ $\qquad v_{top}^2 = \frac{r^2}{R} g$

in (2) $\Rightarrow$ $mg(H - 2R) = \frac{1}{2} m \frac{r^2}{R} g + \frac{1}{2} J \frac{g}{R}$

$$\cancel{mg}(H - 2R) = \frac{1}{2}\cancel{mg}\frac{r^2}{R} + \frac{1}{\cancel{2}}\frac{\cancel{2}}{5}\cancel{m} r^2 \frac{\cancel{g}}{R}$$

$$\Rightarrow \boxed{H = \frac{3}{5}\frac{r^2}{R} + 2R}$$

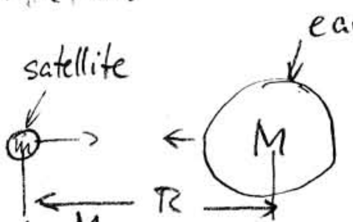
In order to get as quick as possible to the other end of earth, Aggies decide to build a tunnel straight through the entire earth. In order to get there one would just jump into the tunnel and eventually emerge at the „other end" of earth.




This looks like harmonic motion: If you don't hold to the walls of the tunnel, you would get back and forth forever (if there would be no friction).

It would be great help if we could squeeze the force acting on a guy in the tunnel into the $F = -kx$

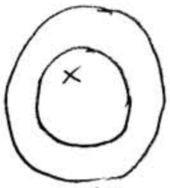
Before we start doing that a few facts about gravitation




$$F = -G \frac{mM}{R^2} \quad \text{with } G = 6.67\cdot 10^{-11} \frac{Nm^2}{kg^2}$$

(Newton's Law of Gravitation)

Another fact: If you are inside a a hollow sphere => no gravitation

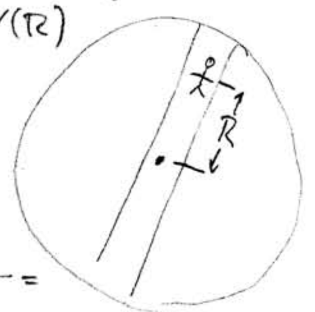


This means for our Aggie: Only that part of the mass of earth has to be taken for M which is closer to the center of earth than the Aggie

Assume uniform density of earth

$$\Rightarrow M(r<R) = \rho V(R)$$

$$M(R) = \rho \tfrac{4}{3} R^3 \pi$$




$$\Rightarrow F = -G \frac{m \rho \frac{4}{3} R^3 \pi}{R^3} =$$

$$= \underbrace{-\tfrac{4}{3} G m \rho \pi}_{\text{this plays the role of } k\,!} \cdot R$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\tfrac{4}{3} G \rho \pi}$$

mass of earth: $\rho = 6\cdot 10^{24}$ kg
radius of earth: $r_E = 6000$ km

$$\Rightarrow \rho = \frac{m}{\frac{4}{3} R^3 \pi} = 6.63 \cdot 10^3 \frac{kg}{m^3}$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$t$ for one-way trip through tunnel $\frac{T}{2}$

$$\Rightarrow t = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{\frac{4}{3} R^3 \pi}} = 40 \text{min}$$

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For the cracks:

tunnel between CS, Tx and Munich, Germany