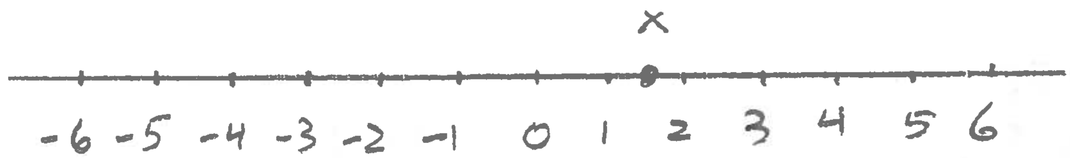
Chapter 4 supplemental notes



position x scalar

rate of change of $x = \frac{dx}{dt} = \vec{v}$ velocity (vector)

rate of change of velocity

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \vec{a}$$ acceleration (vector)

momentum $\vec{p} = m\vec{v}$ (vector)

Force = rate of change of momentum

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

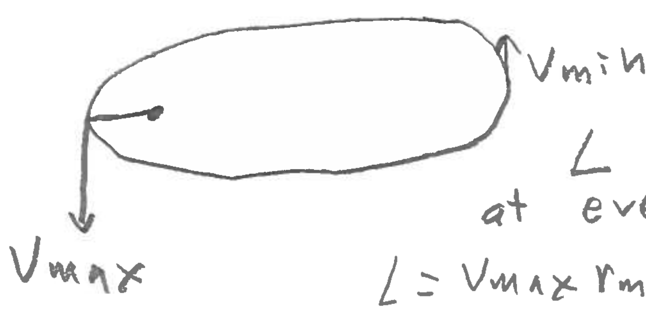
for constant mass $\vec{F} = m\vec{a}$

(Newton's second law)

Angular momentum $\vec{L} = m(\vec{v} \times \vec{r})$

if $\vec{v}, \vec{r}$ are perpendicular $L = mvr$

asteroid orbiting sun



L is the same at every point

$L = V_{max} r_{min} = V_{min} r_{max}$

Kinetic energy (KE) = $\frac{1}{2} m v^2$ (scalar)

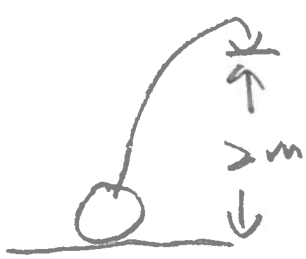
gravitational potential energy

$$U = mgh$$

on Earth $g = 9.8$ m/sec^2

depends on Earth's mass and radius

Example: a 7 kg bowling ball is on the floor. Put it on a shelf 2 meters above the floor.



$$U = 7\text{ kg}\left(9.8\ \frac{\text{m}}{\text{sec}^2}\right)(2\text{ m})$$

$$= 137.2 \text{ joules}$$

A freely falling object has downward velocity $= g t = 9.8\, t$

(in the absence of air resistance)

If ball rolls off shelf it will fall to the floor. Galileo found that

distance fallen $= \frac{1}{2} g t^2$

$$2\ m = \frac{1}{2}\left(9.8\ \frac{m}{sec^2}\right) t^2$$

$$t^2 = \left(\frac{4}{9.8}\right) \quad \text{or} \quad t = 0.6389\ sec$$

Kinetic energy of ball when it hits the floor is

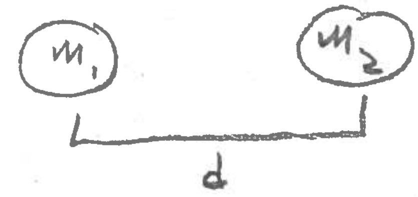
$$KE = \frac{1}{2} m v^2 = \frac{1}{2}(7\ kg)\left(\overbrace{9.8 \times 0.6389}^{6.261\ m/sec}\right)^2$$

$$= 137.2\ \text{joules}$$

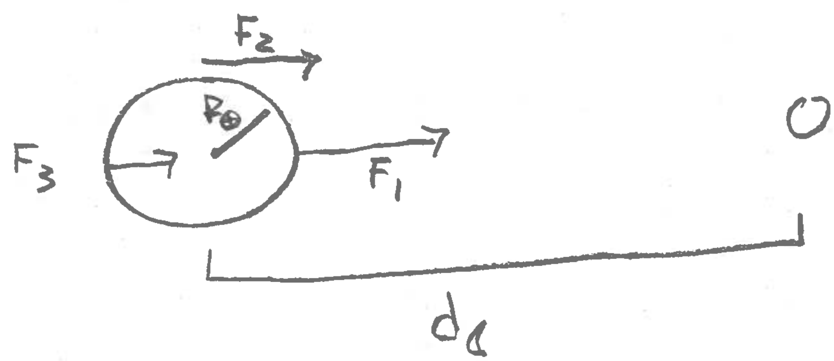
In the absence of wind resistance no energy is lost. The potential energy is converted to kinetic energy as the ball falls.

Newton's Law of Gravity

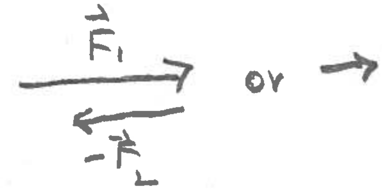
$$\vec{F} = \frac{G\, m_1\, m_2}{d^2}$$



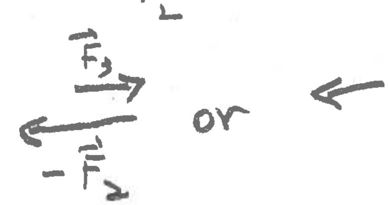
what is the gravitational attraction between the Moon and one cubic meter of water at various locations on Earth?



Tidal force is $\vec{F}_1 - \vec{F}_2$



Tidal force is also $\vec{F}_3 - \vec{F}_2$



There is a bulge of the ocean on the Moon side of the Earth and on the opposite side

Tidal force is

$$F_1 - F_2 = \frac{G m_☾ m_{water}}{(d_☾ - R_⊕)^2} - \frac{G m_☾ m_{water}}{d_☾^2}$$

$$T_☾ = G m_☾ m_{water} \left[\frac{1}{(d_☾ - R_⊕)^2} - \frac{1}{d_☾^2} \right]$$

$$= G m_☾ m_{water} \left[\frac{d_☾^2}{(d_☾ - R_⊕)^2 d_☾^2} - \frac{(d_☾ - R_⊕)^2}{(d_☾ - R_⊕)^2 d_☾^2} \right]$$

$$= G m_☾ m_{water} \left[\frac{d_☾^2 - (d_☾^2 - 2 d_☾ R_⊕ + R_⊕^2)}{(d_☾ - R_⊕)^2 d_☾^2} \right]$$

$$= G m_☾ m_{water} \left[\frac{+2 d_☾ R_⊕ - R_⊕^2}{(d_☾ - R_⊕)^2 d_☾^2} \right]$$

This is exact so far. we want to simplify this using two approximations.

we know that the mean distance of the Moon is about 60 Earth radii.

The numerator is $\approx 120 R_⊕^2 - R_⊕^2 = 119 R_⊕^2$

Denominator is $(59 R_⊕)^2 (60 R_⊕)^2$

Our two approximations are that $119 R_\oplus^2 \approx 120 R_\oplus^2$

and that $59 R_\oplus \approx 60 R_\oplus^2$

So the tidal force is, approximately,

$$T_☾ \approx G m_☾ m_{water} \left[\frac{2 d_☾ R_\oplus}{(d_☾)^4} \right]$$

or $$\boxed{T_☾ \approx G m_☾ m_{water} \left[\frac{2 R_\oplus}{d_☾^3} \right]}$$

whereas the gravitational force between the Moon and Earth varies proportional to the $\left(\frac{1}{\text{distance}}\right)^2$,

The tidal force varies proportional to $\left(\frac{1}{\text{distance}}\right)^3$,

If we moved the Moon to half its present distance, the grav. force would be 4X stronger. The tidal force would be 8X stronger.

Analogously, we can write down an expression for the Sun's tidal force on the Earth's water

$$T_\odot \approx G m_\odot m_{water} \left[\frac{2 R_\oplus}{d_\odot^3} \right]$$

and

$$\frac{T_☾}{T_\odot} \approx \frac{G m_☾ m_{water} \left[\frac{2 R_\oplus}{d_☾^3} \right]}{G m_\odot m_{water} \left[\frac{2 R_\oplus}{d_\odot^3} \right]}$$

$$\boxed{\frac{T_☾}{T_\odot} = \left(\frac{m_☾}{m_\odot}\right) \left(\frac{d_\odot}{d_☾}\right)^3}$$

$m_☾ = 7.35 \times 10^{22}$ kg

$m_\odot = 2 \times 10^{30}$ kg

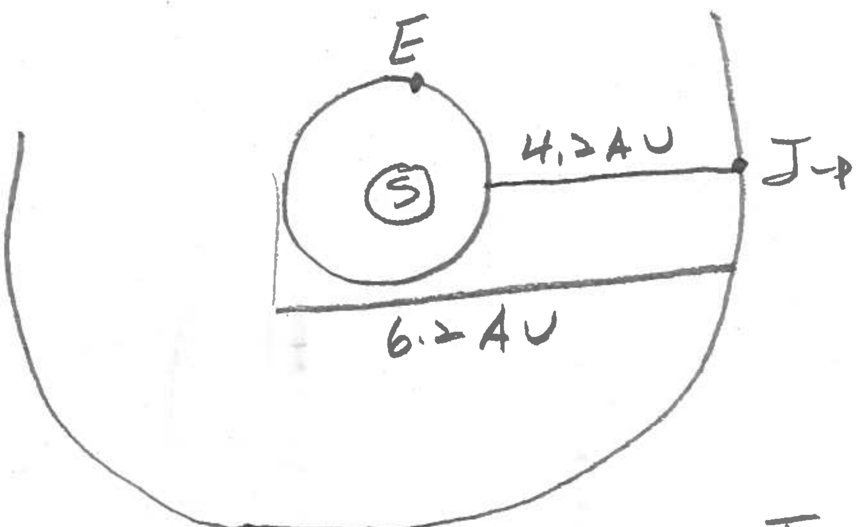
$d_\odot / d_☾ = 389$ on average

$$\frac{T_☾}{T_\odot} \approx 2.16$$

tidal force due to Moon is ~2.2 times stronger than tidal force due to Sun.

Tidal force on Earth due to Jupiter compared to Sun's tidal force on Earth is

$$\frac{T_{Jup}}{T_{\odot}} \approx \left(\frac{m_{Jup}}{m_{Sun}}\right)\left(\frac{d_{\odot}}{d_{Jup}}\right)^3$$



At 4.2 AU separation $\frac{T_{Jup}}{T_{\odot}} \approx \left(\frac{1}{1000}\right)\left(\frac{1}{4.2}\right)^3$

$$\approx \frac{1}{74000}$$

At 6.2 AU separation $\frac{T_{Jup}}{T_{\odot}} \approx \left(\frac{1}{1000}\right)\left(\frac{1}{6.2}\right)^3$

$$\approx \frac{1}{238,000}$$

Tidal forces from planets are <u>negligible</u>.