The cross-staff experiment: How angular sizes relate to actual sizes (due Wed., Feb 20)[1]

Angles, angular sizes, and angular separation

The apparent positions and separations of objects in the sky are not determined by the linear distance between two objects, but by their *angular separation*. For example, the angular distance from the horizon to directly overhead, the zenith, is 90^o. Not only can we measure the angular separation between two objects, but we can also measure the *angular size* of an object such as the Moon, which is about 0.5^o.

Since the Moon or the Sun isn't always available to use as a basis to measure angular sizes, here are a few handy rules that you should remember. With your arm outstretched one finger-width is 2^o, one fist-width is 10^o, one hand span 20^o. This technique works because an angle depends on the ratio of the linear size d of the object to the distance (D) to the object. Since the ratio of finger size to arm size is approximately the same for different people, most people have a finger width of $\sim 2^o$ no matter what the length of their arms. It is still best if you calibrate your own body using the Moon and objects you can measure with your cross-staff.

When you start to measure angular sizes, you will soon realize that there is a relation between the angular size (in degrees) of an object and the distance to that object. If the linear size of an object (in meters, say) is much smaller than its distance in (in meters, once again), then the appropriate "small angle formula" is:

$$a = 57.3^o(d/D) \ .$$

[1] Based on a lab written by Prof. W. Sullivan, University of Washington. If we are thwarted by bad weather, perhaps the due date will be extended.

The cross-staff

The cross-staff is a simple device, used for centuries before the invention of the telescope to determine the angular separation between two objects, or the angular size of a single object. If you are measuring a single object, the cross-staff can be used to determine its linear size or distance. The cross-staff consists of a yardstick and a crosspiece attached at a right angle. All you have to do is sight an object along the stick and slide the crosspiece back and forth (or change your own distance from the object) until the apparent or angular size of the crosspiece (or some portion of it) matches the apparent or angular size of the distance object.

If D' is the "working" length of the cross-staff and d' is the width of the crosspiece, then the angular size of the crosspiece is $a' = 57.3^o(d'/D')$. But this value of a' is also the angular size of a building, height of a tree, or whatever. Thus, if you measure the distance D to a building, you can derive its height d. In other words, the cross-staff works according to the principle of similar triangles:

$$\frac{d'}{D'} = \frac{d}{D}$$

To do

• This project can and should be done with one (1) partner. But if you want to work alone, that's OK too.

• Measure the angular size and the height of the Richardson Petroleum Engineering Building at the Texas A & M campus. If you are pacing off the distance with your stride, you will have to *calibrate* your stride. This can be done using the yardage on a football field or using the distance from the 200 yard marker (blue) to the 100 yard marker (red) on one hole of the Texas A & M golf course. Or − you could mark off 30 feet on the ground with a ruler and figure out how many of your paces fit between one end and the other.

• In the winter/spring of 2008 the planet Mars is easily visible as a bright orange starry object in or near the constellations Taurus, Auriga, and Orion. First find Mars and also the stars Betelgeuse (α Orionis) and Aldebaran (α Tauri). Determine the angular distance between those two stars using the cross staff. Then determine the angular distance of Mars from each of those two stars. This will delineate a very specific triangle in the sky, which will locate the position of Mars.

Making the cross-staff

To construct your cross-staff, follow the instructions below carefully. You will need a good pair of scissors, a glue stick (or rubber cement), some thin cardboard, a box cutter, and a yardstick.

1. Download and print out the pattern page. It is also the last page of this handout.
http://faculty.physics.tamu.edu/krisciunas/physics306/cut-out.pdf

2. Cut out the pattern using the scissors.

3. Glue the pattern to a piece of thin cardboard (like the back of a pad of paper).

4. Place the cardboard on a thick stack of newspaper, a stack of corrogated cardboard, or a crummy board, and cut out the cross-staff using the box cutter. *Or:* cut out the cross-staff using your scissors.

5. Using a pencil, trace the width of your yardstick on the pattern, where you will cut two slots for the yardstick to pass through. Cut the two slots for the yardstick.

6. Fold the pattern as demonstrated in class and staple together.

Hint for use of the cross-staff: If you need the full 4 inch span of the cross-staff to measure the angular size of some object or angular separation of two objects, place the graduated side of the cross piece facing the opposite end of the yardstick as your eye.

One thing to be careful about. The cross-staff pattern has what looks like a 4 inch ruler on it. Depending on the computer printer you use, this pattern may not be exactly 4 inches! You must measure it with a ruler to make sure what size it is.

What your report should contain

1. Purpose of the experiment.

2. Describe how you determined the height of the Richardson building. Of course, since you're trying to determine the *height* of the building, you have to turn the cross staff 90 degrees one way or the other.

(a) Where were you standing when you made your measurements? What distance were you from the building when you measured its angular height? If the distance to the building was measured initially in paces, how did you transform that to feet or meters? (That is, do you know the length of your stride from marching band, some other kind of marching, golf, did you measure it with a baseline of known length, or did you just *assume* some number of feet/inches per stride?)

(b) Some portion of the "4 inch" ruler on the cross piece (d') at some number of inches down the yardstick (D') corresponds to angular extent of the Richardson building. For your measurements, what are d' and D'?

(c) Using the small angle formula, what is the angular size of the building?

(d) What is the linear height of the building? How might you estimate the uncertainty of your value?

3. (a) Using the cross staff determine the angular separation (in degrees) of Betelgeuse and Aldebaran. (Alternately, one *could* use other *identified* stars closer to Mars on the sky to make a smaller triangle.) Show how your value was determined (d' inches of the cross piece scale at D' inches from your eye). Accurate coordinates for these two stars show that the angular distance between Betelgeuse and Aldebaran is 21.4 degrees. How close did you get to this value?

(b) What is the angular distance from Mars to Betelgeuse? What was the date and time of your observations? (The date and time are *very* important to record, because Mars moves from day to day.)

(c) What is the angular distance from Mars to Aldebaran on the same night at approximately the same time?

(d) Make a drawing of the constellations near Mars (Orion, Taurus, Auriga, and Gemini) and indicate the position of Mars. Label the angular sizes of the three sides of your astronomical triangle (Mars-Betelgeuse-Aldebaran).