The cross-staff experiment: How angular sizes relate to linear sizes (due Wed., Feb 25)

Angles, angular sizes, and angular separation

The apparent positions and separations of objects in the sky are not determined by the linear distance between two objects, but by their *angular separation.* For example, the angular distance from the horizon to directly overhead, the zenith, is 90^o. Not only can we measure the angular separation between two objects, but we can also measure the *angular size* of an object such as the Moon, which is about 0.5^o.

Since the Moon or the Sun isn't always available to use as a basis to measure angular sizes, here are a few handy rules that you should remember. With your arm outstretched one finger-width is 2^o, one fist-width is 10^o, one hand span 20^o. This technique works because an angle depends on the ratio of the linear size d of the object to the distance (D) to the object. Since the ratio of finger size to arm size is approximately the same for different people, most people have a finger width of $\sim 2^o$ no matter what the length of their arms. It is still best if you calibrate your own body using the Moon and objects you can measure with your cross-staff.

When you start to measure angular sizes, you will soon realize that there is a relation between the angular size (a, in degrees) of an object and the distance to that object. If the linear size of an object (in meters, say) is much smaller than its distance in (in meters, once again), then the appropriate "small angle formula" is:

$$a \;=\; 57.3^o(d/D) \quad . \tag{1}$$

If angle a is greater than 10 degrees, then it becomes necessary to use a trigonometric formula:

$$a \;=\; 2 \times tan^{-1}(d/2D) \quad , \tag{2}$$

where $\tan^{-1}$ is the arc tangent function on your calculator.

The cross-staff

The cross-staff is a simple device, used for centuries before the invention of the telescope to determine the angular separation between two objects, or the angular size of a single object. If you are measuring a single object, the cross-staff can be used to determine its linear size or distance. The cross-staff consists of a yardstick and a crosspiece attached at a right angle. All you have to do is sight an object along the stick and slide the crosspiece back and forth (or change your own distance from the object) until the apparent or angular size of the crosspiece (or some portion of it) matches the apparent or angular size of the distance object.

If D' is the "working" length of the cross-staff and d' is the width of the crosspiece, then the angular size of the crosspiece is $a' = 57.3^o(d'/D')$. But this value of a' is also the angular size of a building, height of a tree, or whatever. Thus, if you measure the distance D to a building, you can derive its height d. In other words, the cross-staff works according to the principle of similar triangles:

$$\frac{d'}{D'} = \frac{d}{D} \tag{3}$$

Making the cross-staff

To construct your cross-staff, follow the instructions below carefully. You will need a good pair of scissors, a glue stick (or rubber cement), some thin cardboard, a box cutter, and a yardstick.

1. Download and print out the pattern page. It can be found here:
http://people.physics.tamu.edu/krisciunas/cross-staff-pattern.pdf

2. Cut out the pattern using the scissors.

3. Glue the pattern to a piece of thin cardboard (like the back of a pad of paper).

4. Place the cardboard on a thick stack of newspaper, a stack of corrogated cardboard, or a crummy board, and cut out the cross-staff using the box cutter. *Or:* cut out the cross-staff using your scissors.

5. Using a pencil, trace the width of your yardstick on the pattern, where you will cut two slots for the yardstick to pass through. Cut the two slots for the yardstick.

6. Fold the pattern as demonstrated in class and staple together.

Hint for use of the cross-staff: If you need the full span of the cross-staff to measure the angular size of some object or angular separation of two objects, place the graduated side of the cross piece facing the opposite end of the yardstick as your eye.

One thing to be careful about. The cross-staff pattern has what looks like a 4 inch ruler on it. Depending on the computer printer you use, this pattern may not be exactly 4 inches! You must measure it with a ruler to make sure what size it is.

To do

- This project can and should be done with one (1) partner. But if you want to work alone, that's OK too.

- Measure the angular size and the linear height of the Administration Building or the Eller Oceanography and Meteorology Building at the Texas A & M campus. These buildings are in the middle of the semicircle of Bizzell Street on the east end of campus. I recommend that you do this from the sidewalk on either side of New Main Drive, which runs from the Administration Building toward Texas Avenue. You should be at least 300 paces away when you make your measurements.

If you are pacing off the distance with your stride, you will have to *calibrate* your stride. This can be done using the yardage on a football field or using the distance from the 200 yard marker (blue) to the 100 yard marker (red) on one hole of the Texas A & M golf course. Or − you could mark off 30 feet on the ground with a ruler and figure out how many of your paces fit between one end and the other.

- Consider the bowl of the Big Dipper, which should be visible after 10 PM in the northeast around February 1st, at about 35 degrees above the horizon. Measure the angular size of the four sides of the Big Dipper (α to β, β to γ, γ to δ and δ to α). Since these angular distances are less than 10 degrees, you can use the small angle formula (Eqn. 1).

- Measure the angular size of the Moon when it very close to the horizon and when it is high in the sky. For this you will need to attach a short piece of a thin dowel rod to the cross piece of your cross staff to use as an occulting device for the Moon. Alternatively, you can take a piece of cardboard and punch a nice clean hole about 1/4 inch in size in the cardboard, attach this cardboard to the cross piece. In this case you adjust the cross piece until the Moon exactly fits in the hole.

What your report should contain

1. Purpose of the experiment.

2. Describe how you determined the height of the Administration building or the Eller O&M Building. Of course, since you're trying to determine the *height* of a building, you have to turn the cross staff 90 degrees one way or the other.

(a) Where were you standing when you made your measurements? What distance were you from the building when you measured its angular height? If the distance to the building was measured initially in paces, how did you transform that to feet or meters? (That is, do you know the length of your stride from marching band, some other kind of marching, golf, did you measure it with a baseline of known length, or did you just *assume* some number of feet/inches per stride?)

(b) Some portion of the "4 inch" ruler on the cross piece (d') at some number of inches down the yardstick (D') corresponds to angular extent of the building. For your measurements, what are d' and D'?

(c) Using the small angle formula, what was the angular size of the building from where you were standing?

(d) What is the linear height of the building? How might you estimate the uncertainty of your value?

3. How many inches (d') does your cross piece have to be down the yardstick (D') to measure the angular separation between the four stars that make up the bowl of the Big Dipper? Show how you obtained the dimensions (in degrees) of the four sides of this quadrilateral.

4. Explain how you used an occulting device of known thickness or hole of known diameter to measure the angular diameter of the Moon when it is low on the horizon and when it is high in the sky. How does your value compare to the mean angular diameter of the Moon (roughly 0.5 deg)? If the ellipticity of the Moon's orbit is 5 percent, do you think it would it be possible to measure the variation of the Moon's angular size with your equipment? Is the Moon basically the same angular size when it is low on the horizon as when it is high in the sky?