

Small angles and their measurement (due Friday, September 18)

Angles, angular sizes, and angular separation

The apparent positions and separations of objects in the sky are not determined by the linear distance between two objects, but by their *angular separation*. For example, the angular distance from the horizon to directly overhead, the zenith, is 90° . Not only can we measure the angular separation between two objects, but we can also measure the *angular size* of an object such as the Moon, which is about 0.5° .

Since the Moon or the Sun isn't always available to use as a basis to measure angular sizes, here are a few handy rules that you should remember. With your arm outstretched one finger-width is 2° , one fist-width is 10° , one hand span 20° . This technique works because an angle depends on the ratio of the linear size d of the object to the distance (D) to the object. Since the ratio of finger size to arm size is approximately the same for different people, most people have a finger width of $\sim 2^\circ$ no matter what the length of their arms. It is still best if you calibrate your own body using the Moon and objects you can measure with your cross-staff.

When you start to measure angular sizes, you will soon realize that there is a relation between the angular size (a , in degrees) of an object and the distance to that object. If the linear size of an object (in meters, say) is much smaller than its distance (in meters, once again), then the appropriate "small angle formula" is:

$$a(\text{deg}) = 57.3^\circ(d/D) . \quad (1)$$

If angle a is smaller than one degree, we might express it in terms of arc minutes. Since 1 degree equals 60 arc minutes,

$$a(\text{arcmin}) = 3438'(d/D) . \quad (2)$$

If angle a is greater than 10 degrees, then it becomes necessary to use a trigonometric formula:

$$a(\text{deg}) = 2 \times \tan^{-1}(d/2D) , \quad (3)$$

where \tan^{-1} is the arc tangent function on your calculator.

Similar triangles

Consider plane triangles ABC and AEF below. Since BC is parallel to EF, these two triangles are shaped exactly the same. The interior angles of one triangle are identical to the corresponding interior angles of the other. Also, the lengths of the sides are proportional. In other words, $BC/EF = AB/AE = AC/AF$.

The cross-staff

The cross-staff is a simple device, used for centuries before the invention of the telescope to determine the angular separation between two objects, or the angular size of a single object. If you are measuring a single object, the cross-staff can be used to determine its linear size or distance. The cross-staff consists of a yardstick and a crosspiece attached at a right angle. All you have to do is sight an object along the stick and slide the crosspiece back and forth (or change your own distance from the object) until the apparent or angular size of the crosspiece (or some portion of it) matches the apparent or angular size of the distance object.

If D' is the “working length” of the cross-staff and d' is the width of the crosspiece, then the angular size of the crosspiece is $a' = 57.3^\circ(d'/D')$. But this value of a' is also the angular size of a building, height of a tree, or whatever. Thus, if you measure the distance D to a building, you can derive its height d . In other words, the cross-staff works according to the principle of similar triangles:

$$\frac{d'}{D'} = \frac{d}{D} \quad (4)$$

Making the cross-staff

To construct your cross-staff, follow the instructions below carefully. You will need a good pair of scissors, a glue stick (or rubber cement), some thin cardboard, a box cutter, and a yardstick. The yardstick ideally shows centimeters and millimeters along one edge.

1. Use the last page of this handout or download and print out the pattern page from web address: faculty.physics.tamu.edu/krisciunas/cut-out.pdf

2. Cut out the pattern using the scissors.

3. Glue the pattern to a piece of thin cardboard (like the back of a pad of paper).

4. Place the cardboard on a thick stack of newspaper, a stack of corrugated cardboard, or a crummy board, and cut out the cross-staff using the box cutter. *Or:* cut out the cross-staff using your scissors.

5. Using a pencil, trace the width of your yardstick on the pattern, where you will cut two slots for the yardstick to pass through. Cut the two slots for the yardstick.

6. Fold the pattern as demonstrated in class and staple together.

7. Finally, since we are going to measure something less than a degree in size, take a small piece of cardboard and a hole punch and make a nice, neat hole about 1/4 inch in diameter in the small piece of cardboard. Tape this piece of cardboard to the cross piece such that the hole is neither to the left or right of the center of the yardstick as you slide the cross piece back and forth.

To do

- This project can and should be done with one (1) partner. But if you want to work alone, that's OK too.
- Using a millimeter ruler, as accurately as you can measure the diameter (d'), in millimeters, of your 1/4 inch hole that will be used for sighting.
- Cut out a nice neat circular disk of diameter $d = 91$ mm. A pattern can be found on the next to last page.
- Tape the 91 mm disk to a door or wall at your eye level, and measure off 10 meters (=

10,000 millimeters = 32 feet 9.7 inches) from the location of the disk. Put a piece of tape on the floor at distance $D = 10$ meters.

- Using Equation 4, estimate how many millimeters along the ruler (D') the sighting hole must be to match the angular size of the 91 mm disk which is at a distance of 10 meters. Round off this number to the nearest millimeter.
- Now, put your toes just behind the 10 meter line, put one end of the yardstick up against your cheekbone under your better eye and slide the cross staff to the position where the angular size of the sighting hole matches, as accurately as possible, the angular size of the 91 mm circle 10 meters away. How many mm down the yardstick did the sighting hole have to be?
- Using Equation 2, calculate the angular size of the 91 mm circle at a distance of 10 meters. Round this off to the nearest 0.1 arc minute. We'll call this the *true* angular size of the circle.
- Calculate the angular size (in arc minutes) of the sighting hole at the best-measured distance down the yardstick to match the angular size of the circle on the wall 10 meters away. We'll call this *measured* angular size of the circle.
- Take the ratio of these two angular sizes ("true" divided by "measured"). We'll call this "parameter C". How close is the number to 1.000?
- Now go outside on a reasonably clear night when the Moon is up and measure the angular size of the Moon using the same 1/4 inch sighting hole and yardstick. How close to 31 arc minutes did you get?
- If you multiply your measured angular size of the Moon by "parameter C", what is your corrected value of the angular size of the Moon?

To turn in

A scientific report, however brief, contains complete sentences and describes what you did and what it means. The numbers presented should have a sensible number of significant figures. If the diameter of the 91 mm disk is known to ± 0.5 mm, then how can its angular size at a distance of 10 meters be given to the nearest 0.0000001 arc minute?

You need to give the diameter of your sighting hole to the nearest 0.1 mm if you can. You need to give the distance down the yardstick, for measurements of the 91 mm circle and

the Moon, to the nearest 1 mm. Angles in arc minutes can be given to the nearest 0.1 unit.

Were your sightings done with the left eye or right eye? How good is your vision? 20/20 or 20/40 or worse? Do you have a bit of astigmatism, even in your better eye? Were you wearing glasses or contacts?

You need to write some commentary on the ability of an observer without a telescope to measure the angular size of the Moon. What are the challenges and limitations?