An experiment to determine the distance to the
Moon (without using a telescope)

Aristarchus of Samos (ca. 310-230 BC) made some very modern assertions that were counter to "common sense". He asserted that the rising and setting of the Sun, Moon, and stars was due to the rotation of the Earth. The lack of *parallax* of the stars is most easily explained if the Earth is stationary. But Aristarchus said that it just meant the stars were very, very far away. He placed the Sun at the center of the solar system. He also came up with a method of determining the distance to the Sun, using the quarter Moon.

Aristarchus' method of determining the relative distances from the earth to the moon and to the sun
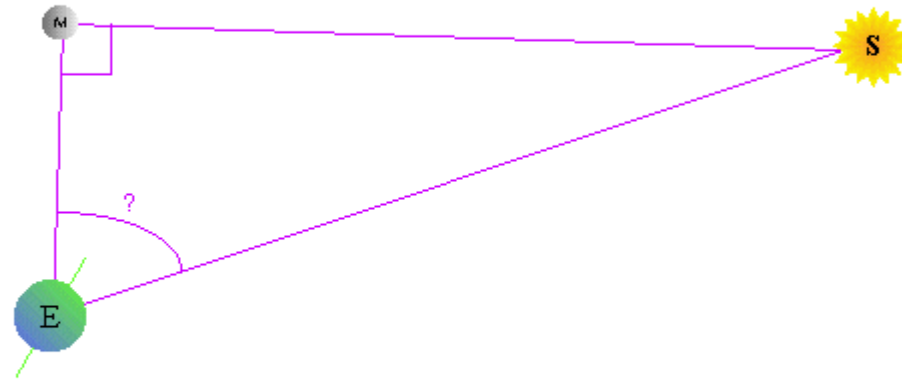
With the measurement of angle MES the ratio of EM and ES can be found.

He found that: MES = 87° so EM/ES = 1/19
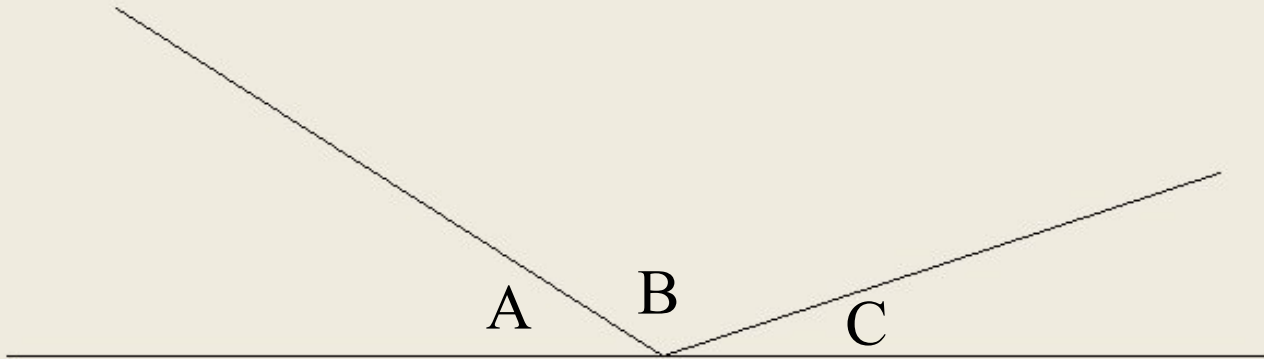actually : MES = 89° 51 and EM/ES = 1/400

According to Aristarchus, angle MES = 87 degrees. If true, that would mean that the Sun is 19 times more distant than the Moon. It turns out that the Sun is actually about 389 times further than the Moon.
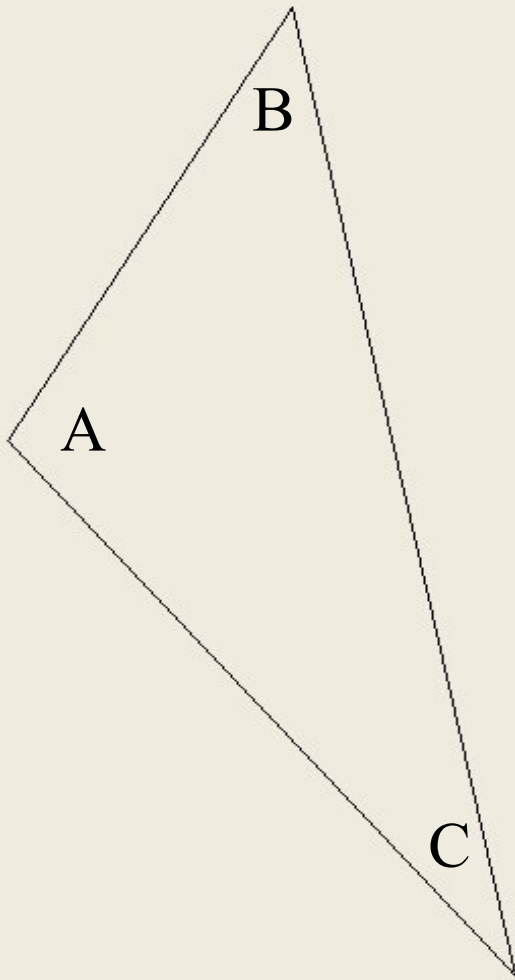
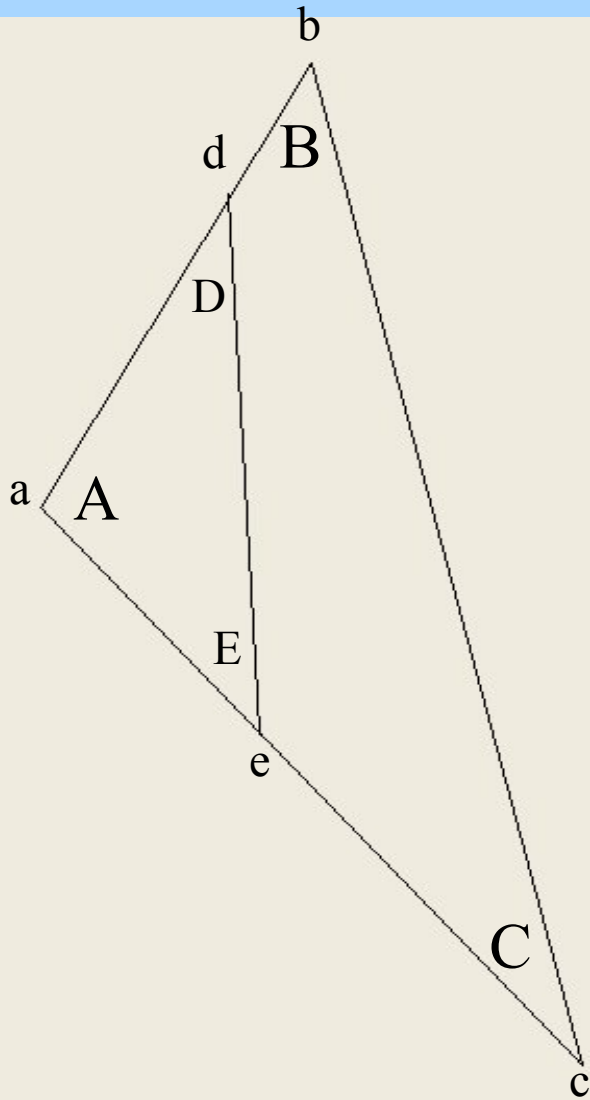What is the sum of the interior angles of this plane triangle?

A + B + C = ?

A. 90 degrees
B. 180 degrees
C. 270 degrees
D. 360 degrees

What is the sum of angles A, B, and C?
A. 90 degrees
B. 180 degrees
C. 270 degrees
D. 360 degrees

Since line *de* is not parallel to line *bc*, angle D is not equal to angle B, and angle C is not equal to angle E.

But $A + B + C = 180$ deg. Also, $A + D + E = 180$ deg.
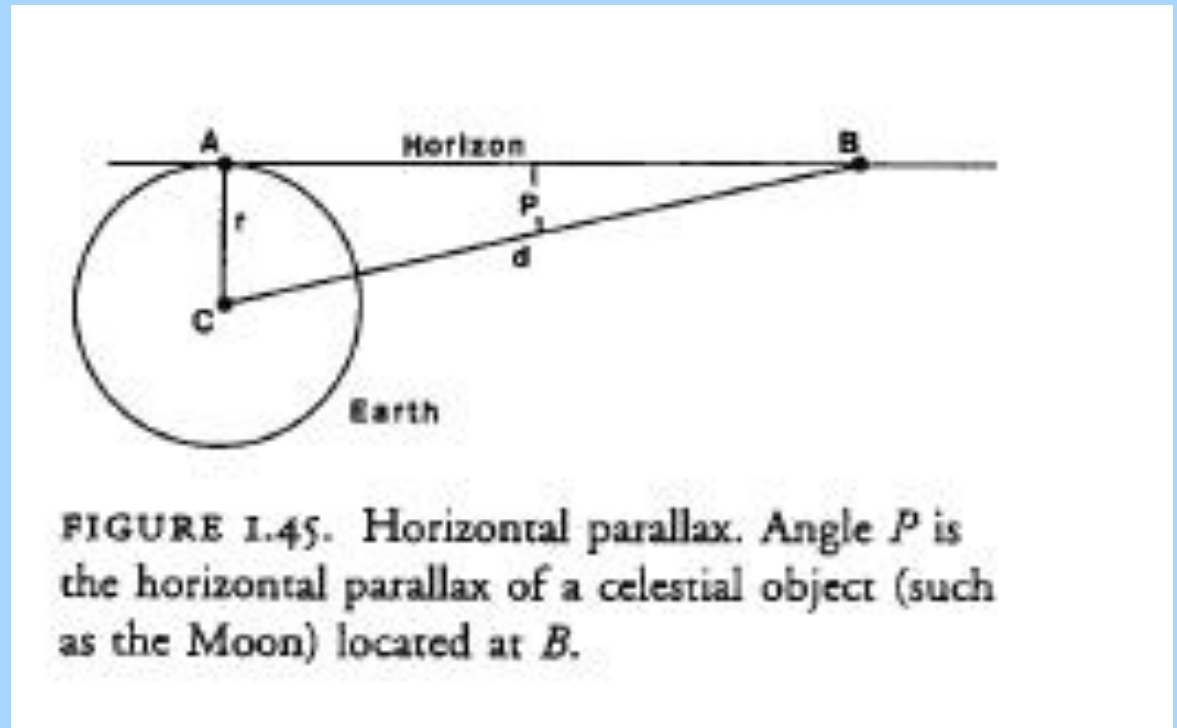
So $B + C = 180 - A$, and $D + E = 180 - A$.

Therefore, angle B + angle C = angle D + angle E, or $B + C = D + E$.

What is the minimum amount of information one needs to show how far away the Moon is compared to the size of the Earth? Aristarchus used information from lunar eclipses. But first we must define what we mean by the *horizontal parallax* of an object such as the Moon or Sun.

R_Earth/d = sin (P),

so

d = R_Earth/sin(P)

FIGURE 1.45. Horizontal parallax. Angle P is the horizontal parallax of a celestial object (such as the Moon) located at B.

Evans, *The History and Practice of Ancient Astronomy*, p. 69.

It has been known since ancients times that a total solar eclipse lasts at most only a few minutes. Thus, the angular size of the Sun is quite comparable to the angular size of the Moon.

It is relatively easy to measure the angular size of the Moon at night. To the nearest tenth of a degree the angular size of the Moon is 0.5 deg.
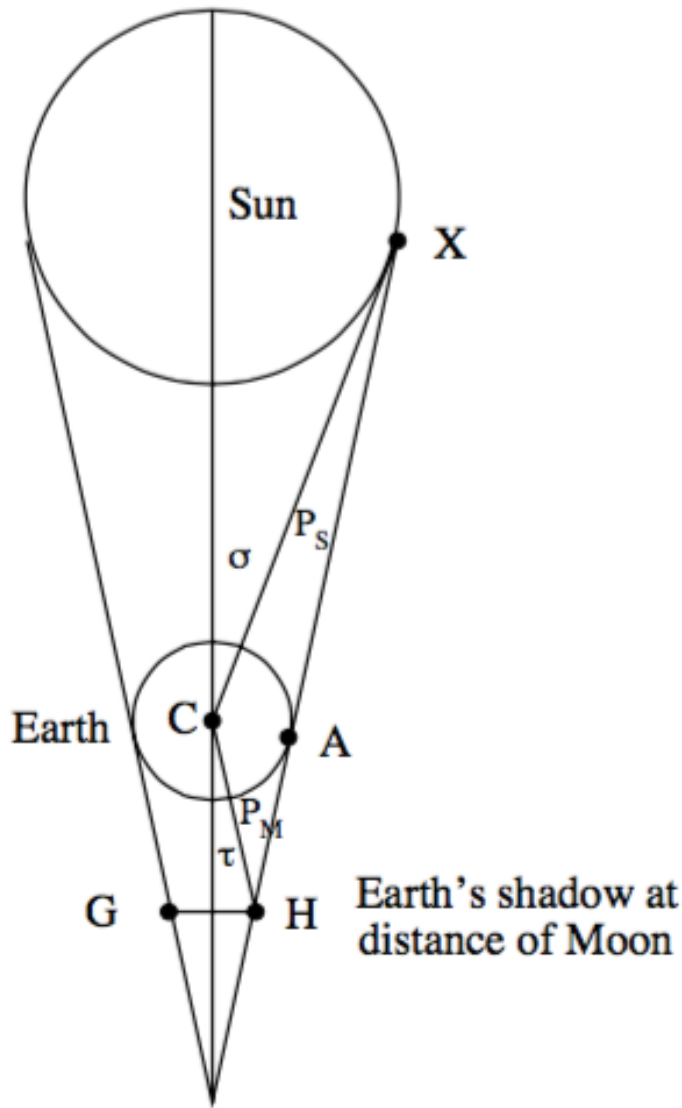
Here, σ is the angular radius of the Sun ~ 0.25 deg, τ is the angular radius of the Earth's shadow at the distance of the Moon.

$$\sigma + \tau = 180° - \text{angle } XCH.$$

The three interior angles of triangle XCH must add to 180° as well. So $P_M + P_S = 180° - \text{angle } XCH$. This gives: $\sigma + \tau = P_M + P_S$.

See: Evans, *The History and Practice of Ancient Astronomy*, p. 69

It's hard to imagine the shadow of the Earth, even during a lunar eclipse, but we could time how long the total part of the eclipse lasts and use the motion of the Moon to determine how big the shadow is.

Ptolemy (whom we shall meet shortly), says in the *Almagest*, V, 14, that the shadow of the Earth is 2 and 3/5 times as big as the full Moon. We can confirm this with lunar eclipse photos and a compass.

From six images of the lunar eclipse of June 15, 2011, we find that the Earth's shadow was 2.56 +/- 0.03 times the angular diameter of the Moon.

In an article in the *American Journal of Physics* in 2010 I give a mean measured angular diameter of the Moon of 31.18 arc minutes. So $\tau = 2.56 \times (31.18/2) = 39.91$ arcmin. Two measurements of the angular diameter of the Sun on May 1, 2010, and June 11, 2011 gave a mean value of 30.8 +/- 0.9 arcmin. So $\sigma = (30.8/2) = 15.4$ arcmin. Simultaneous measurements of asteroid 1996 HW1 from Socorro, New Mexico, and Ojai, California, on July 24, 2008, allowed us to determine a value of the length of the Astronomical Unit (the average distance of the Earth to the Sun) of 1.59×10^8 km. The solar parallax was therefore 0.14 arcmin, far smaller than Aristarchus assumed.

We derive a lunar parallax of

$P_M = \sigma + \tau - P_S = 15.4 + 39.91 - 0.14 \sim 55.17$ arcmin.

55.17 arcmin / 60 arcmin per deg = 0.9195 deg.

An estimate of the distance to the Moon on June 15, 2011, is $1/\sin(0.9195$ deg$) \sim 62.3$ times the radius of the Earth. This is pretty close to the true mean value of 60.27 R_{Earth}. The actual true range is from 55.9 to 63.8 Earth radii.

From observations of the shadow of the Sun using a *gnomon* in South Bend, Indiana, and College Station, Texas, we obtained a value of the Earth's radius of 6290 km. (The true value is 6378 km.) Our value of the radius of the Earth and our value of the distance to the Moon of 62.3 R_{earth} gives a distance to the Moon of

$D_{Moon} = 62.3\ R_{earth} = 62.3 \times 6290\ km = 3.92 \times 10^5\ km$

The angular diameter of the Moon is

$\theta = 31.18\ arcmin\ /\ [(180/\pi)\ 60] = 9.07 \times 10^{-3}\ radians$

The radius of the Moon is $\theta\ D_{Moon}/2 = 1778\ km$. The true value is 1738 km.