Newton, Einstein, and Gravity

"I have not been able to discover the cause of those properties of gravity from phenomena, and I *feign no hypotheses*...And to us it is enough that gravity does really exist, and act according to the law which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea."

-- Newton's *Principia,* 3rd edition (1726)

Kepler’s First Law states that the orbit of a planet is a(n) _____________ with the Sun at one focus.

a. circle
b. ellipse
c. parabola
d. hyperbola

Kepler’s Third Law may be written: $p^2 = a^3$. What is “a”?

a. awesomeness
b. acceleration
c. orbital semi-major axis size
d. azimuth
e. aphelion distance

So how does one solve problems with Kepler's 3rd Law? $p^2 = a^3$. If you're give the orbit size a in Astronomical Units, then cube it. If $a = 10$, then a^3 = 10 times 10 times 10 = 1000. What number times itself = 1000? Take the square root of 1000, which is a bit over 31.6. So the orbital period is about 31.6 years.

If you're given p, then square it. How to get a? Take the cube root of p^2.

An object that orbits the Sun is found to have a mean distance of 20 AU. What is the orbital period of this object?

a. 31.6 years
b. 63.2 years
c. 89.4 years
d. 164 years

Newton, Einstein, and Gravity

"I have not been able to discover the cause of those properties of gravity from phenomena, and I *feign no hypotheses*...And to us it is enough that gravity does really exist, and act according to the law which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea."

-- Newton's *Principia,* 3rd edition (1726)

Scalars and vectors

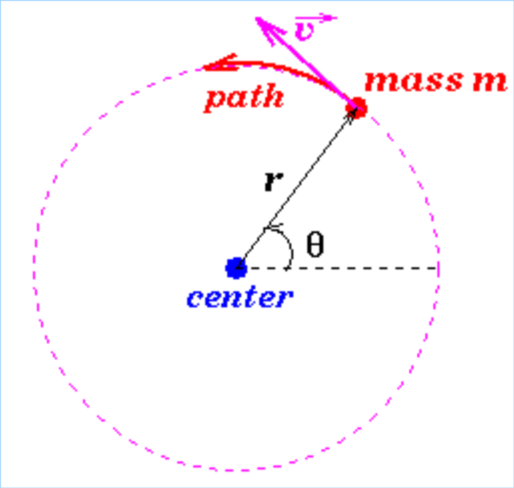
A physical quantity that does not depend on direction (such as mass or age) is called a **scalar**.

If the physical quantity is *directed*, it is called a **vector**. Examples are velocity, acceleration, and force.

For example, we all understand speed. Imagine if someone said, “I had an accident with another car, and we were both going 60 mph.” That would sound bad enough. But the next obvious question is, “Did one car bump the other from behind, or did they hit each other head on?” In the example of the rear end collision the *net velocity* could be close to 0, but in the case of the head on collision, the net velocity is 120 mph. You would much more easily survive the first, than the second.

So, velocity has a magnitude (called speed) *and* a direction.

Just as velocity is the directed change of position (measured in some unit of distance per unit time), the rate of change of velocity is called *acceleration*.
A car whose speed changes from 0 to 60 mph has accelerated. A planet moving in a perfectly circular orbit at constant speed is changing the *direction* of its velocity continuously, so is continuously accelerating.



As the book points out, your car actually has three accelerators – the gas pedal, the brake pedal, and the steering wheel. All three change the direction and/or speed of the car.

The gas pedal causes our speed to increase. This would be a positive value of acceleration.

The brakes cause our speed to decrease. This is negative acceleration, or deceleration.

The steering wheel causes us to change the direction of our velocity. Even if our speed is constant, the velocity is varying.

In order to be able to time falling bodies, Galileo constructed inclined planes which had tracks for balls to roll downhill. He placed musical strings perpendicular to the direction of the tracks and found that they had to be placed in intervals of 1, 4, 9, 16 ... units of length along the track for a rolling ball to cross the strings at equal intervals of time.

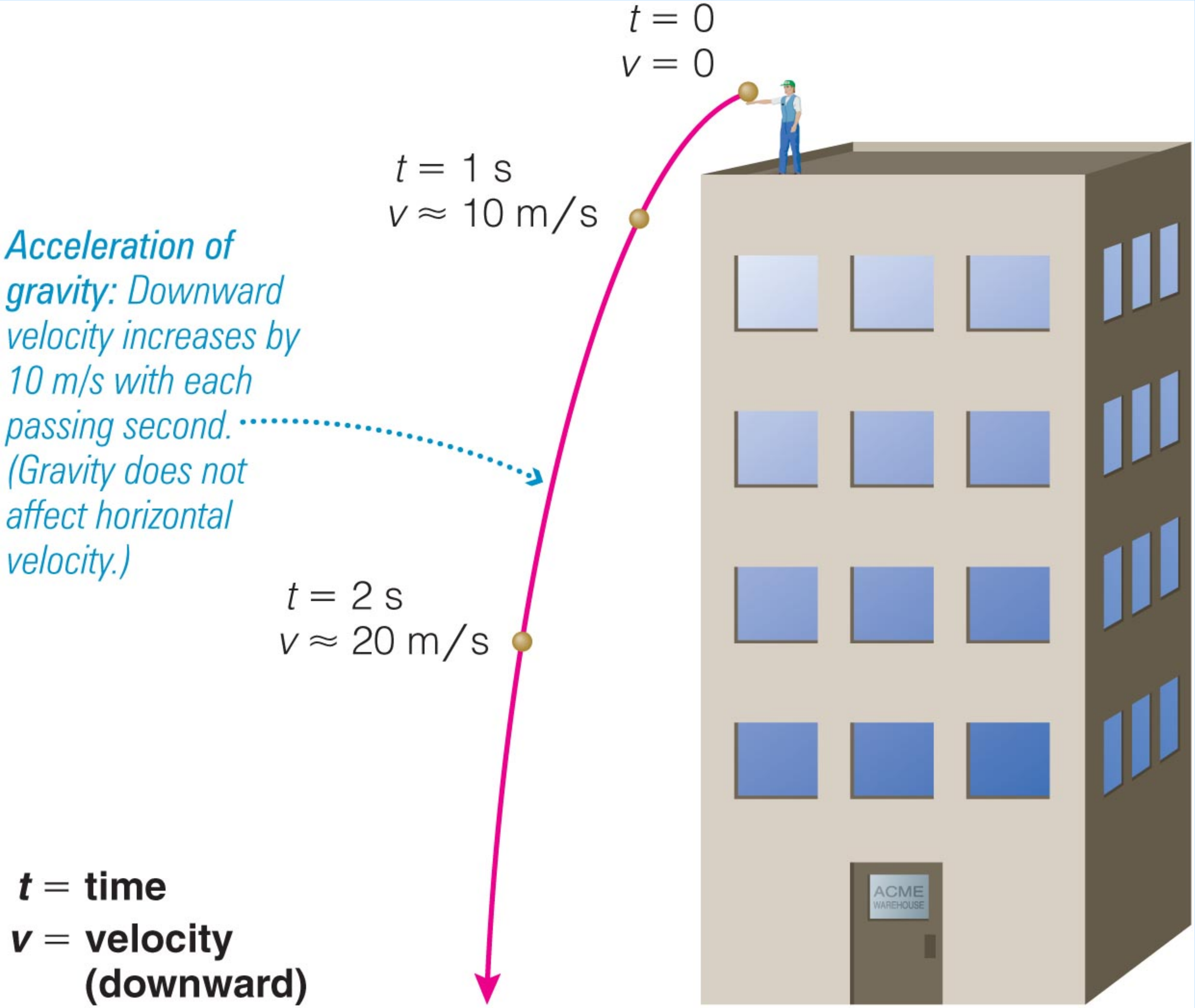
The velocity down the track increases proportional to the time (v = acceleration X time).

The distance travelled, d = (½) X accel. X time^2.

Which of the following is a *vector*?

a) acceleration
b) mass
c) speed
d) length

A ball in free fall is moving downwards a 9.8 m/sec after one second, 19.6 m/sec after two seconds, 29.4 m/sec after three seconds, etc., if this is happening at sea level on the Earth. This should be true for a wooden ball or an iron ball, independent of the size or weight. This is exactly the experiment suggested by Galileo that should be carried out the Leaning Tower of Pisa. The commonly held notion is that heavier objects should fall faster, an idea dating back to Aristotle. In 1971 Apollo 15 astronaut David Scott dropped a feather and hammer at the same time on the Moon. With no air resistance, there was no friction, and the two objects fell at the same rates.


$t = 0$
$v = 0$
$t = 1$ s
$v \approx 10$ m/s
Acceleration of gravity: Downward velocity increases by 10 m/s with each passing second. (Gravity does not affect horizontal velocity.)
$t = 2$ s
$v \approx 20$ m/s
ACME
WAREHOUSE
t = time
v = velocity (downward)

Galileo formulated a simple law of motion: “Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of acceleration or retardation are removed.”

Think of hitting a golf ball onto a very large frozen lake. The ball will just keep on rolling.

This was contrary to Aristotle's belief that motion can only continue if there is a force applied to the object.

When a bowling ball is falling from some height
to the floor it is

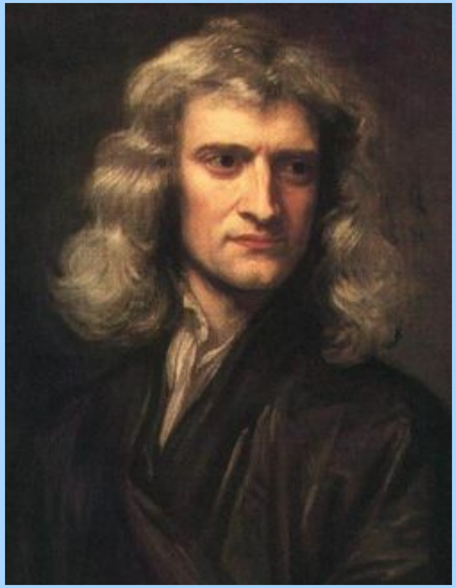
a) converting gravitational potential energy to kinetic energy
b) converting kinetic energy to gravitational potential energy
c) increasing its mass
d) increasing its acceleration

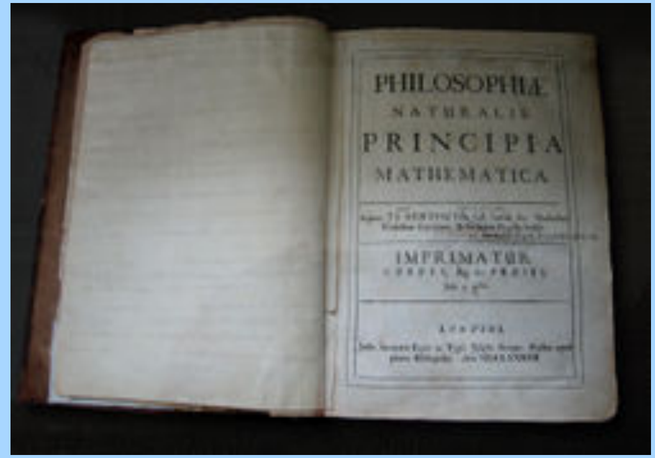
Table 5-1 | Newton's Three Laws of Motion

I. A body continues at rest or in uniform motion in a straight line unless acted upon by some force.

II. The acceleration of a body is inversely proportional to its mass, directly proportional to the force, and in the same direction as the force.

III. To every action, there is an equal and opposite reaction.





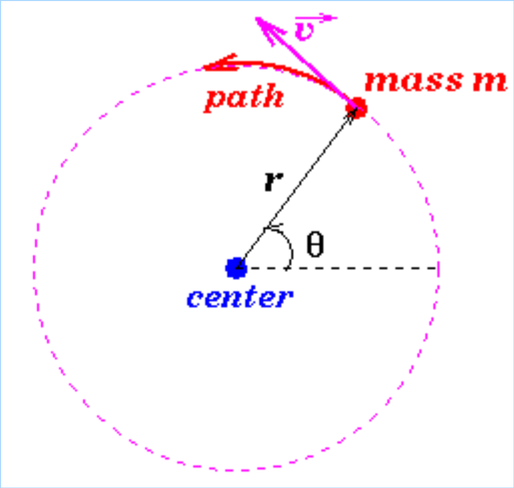
Isaac Newton (1642-1727) invented the reflecting telescope. He and Gottfried Wilhelm Leibniz (1646-1716) independently invented calculus. Newton's great work *Mathematical Principles of Natural Philosophy* was first published in1687.

A body in motion will have a tendency to keep moving because it has **momentum**. Momentum is a vector and is equal to the product of mass and velocity ($\mathbf{p} = m\mathbf{v}$). One of the rules of simples physics is the conservation of momentum.

For example, if you are running at 7 m/sec and collide with a 260 pound linebacker running towards you at 7 m/sec, if you weighed less than 260 you would be knocked backwards. If you weighed more than 260, you would knock the linebacker over, even if you weren't very muscular.

Newton's Second Law is often written as **F** = m**a**, or force equals mass times acceleration.

To be more exact, force is the rate of change of momentum. If the mass is constant, the equation above applies. But if you have a rocket using up fuel, the mass will be changing, not just the velocity.



For an object in uniform, circular motion, the force is directed towards the center of motion.

Newton knew from Kepler that the orbit of a planet was an ellipse with the Sun at one focus. He was able to show that the path of a planet will be an ellipse only if the force of attraction varies inversely with the square of the distance. It also varies proportionally to the masses of the two objects. Here is his Law of Gravity:

$$\mathbf{F} = - G\, Mm/r^2 \qquad .$$

Here M is the mass of the Sun, m is the mass of a planet, and r is the distance between them. There is a minus sign because the force is attractive.

A body (such as a comet or planet) is moving under the the influence of a force that decreases proportional to the square of the distance. The orbital path can be

a) A circle
b) An ellipse
c) A parabola
d) A hyperbola
e) All of the above

The parameter G is Newton's constant, and this law is also known as the law of *universal* gravitation. All masses in the universe attract all other masses. Thus, if the density of the universe were more than some critical value, and there were no other forces to counteract gravity, the universe (which is presently expanding) might achieve some maximum size, then begin to contract. Many billions of years after the Big Bang, the universe could end in a Big Crunch.

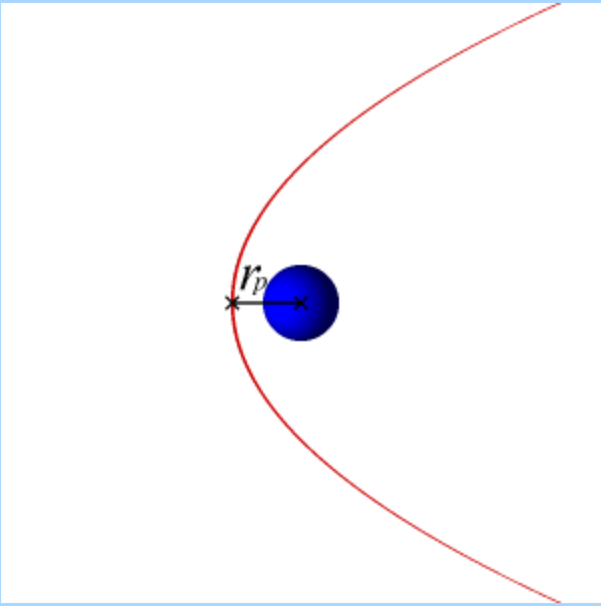
More on orbital velocity

How fast does an object of mass m have to move to orbit another object of mass M? If M is much, much greater than m, and if the distance of the orbiting object from the center of the other is r, then the orbital speed is:

$$V_c = \mathrm{sqrt}\ (GM/r)$$

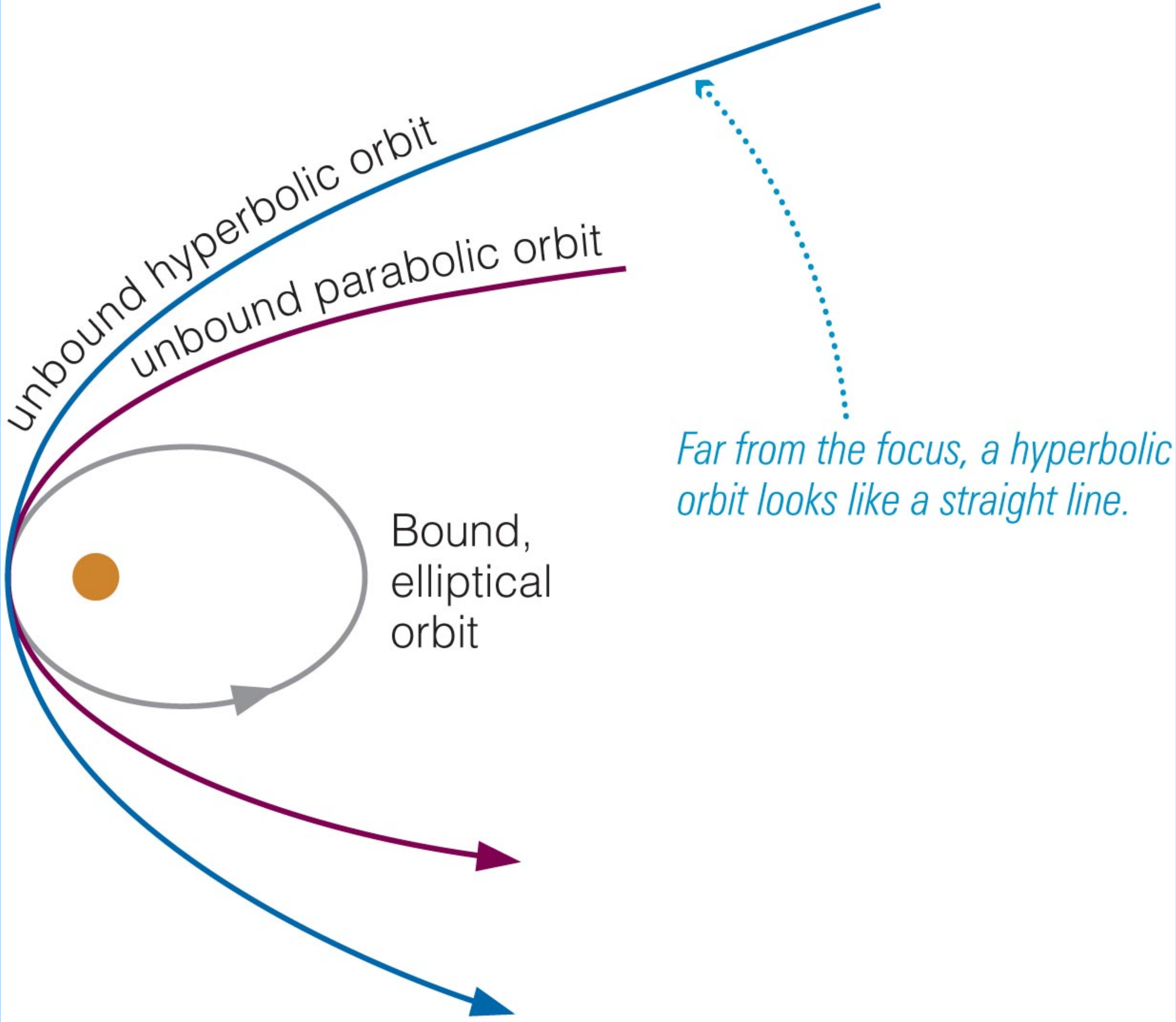
The Earth orbits the Sun at 30 km/sec. The space shuttle orbits the Earth at 7.9 km/sec. The Moon orbits the Earth at 1.02 km/sec.

If the space shuttle were to fire its rockets and achieve an orbital speed equal to sqrt(2) (= 1.414...) times the circular orbital speed, it would fly away from the Earth on a parabolic trajectory, no longer in orbit.



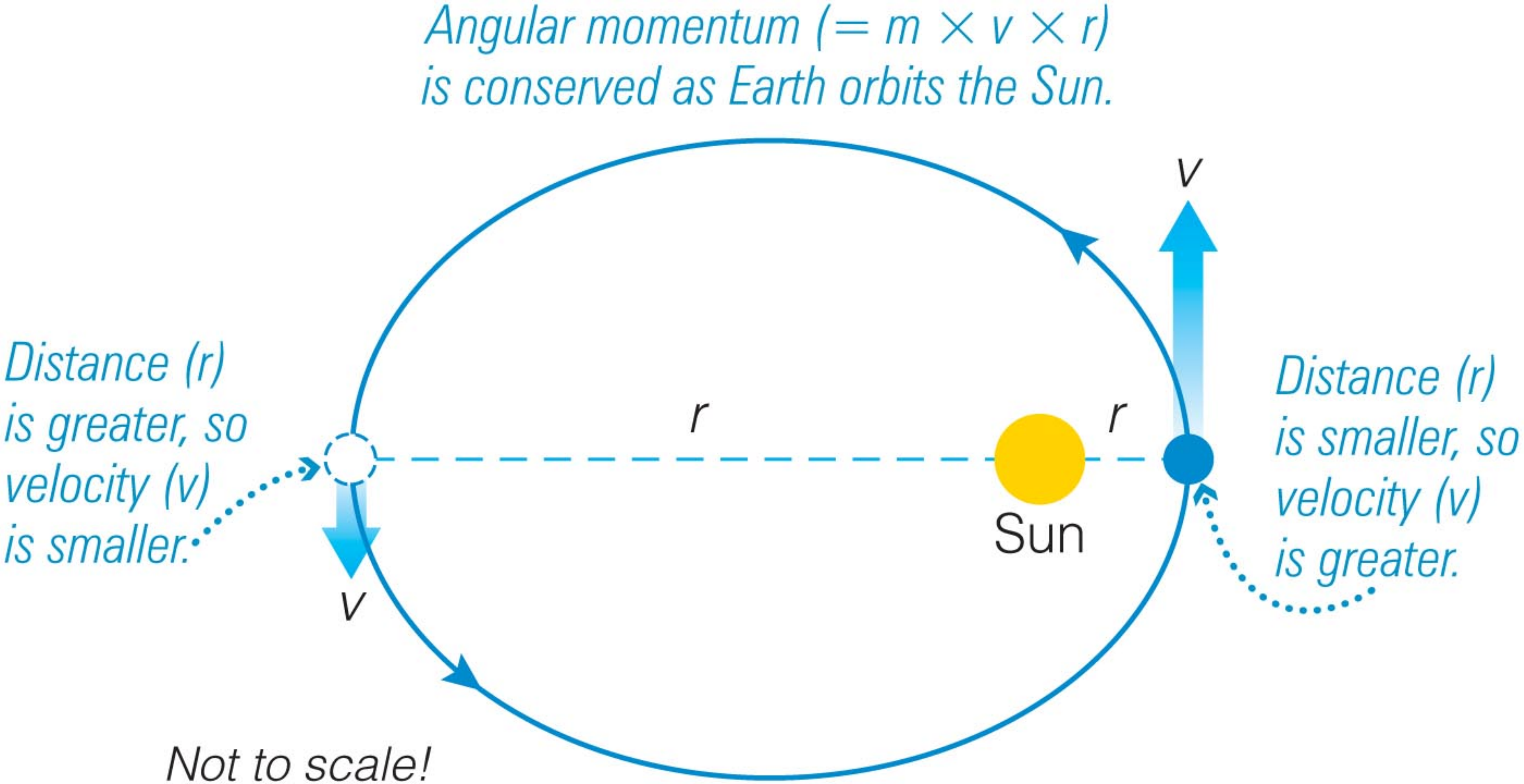
In the *Principia*, Newton proved that if gravity acts as an inverse-square law force, the trajectory of a planet or comet is a **conic section** (i.e. circle, ellipse, parabola, or hyperbola).

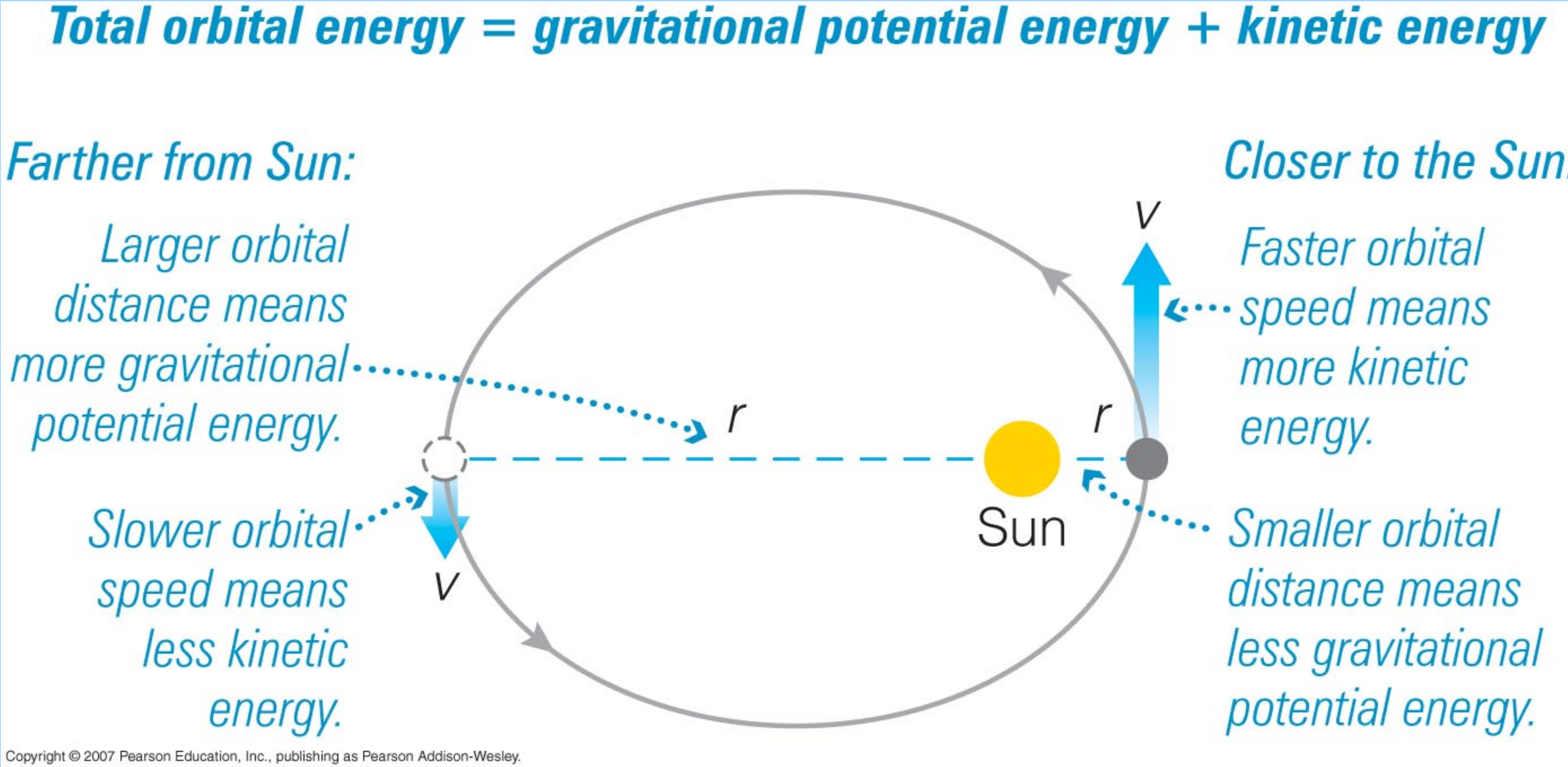
It turns out that if the force were proportional to distance (**F** = constant X **r**), one could also produce a circular or elliptical orbit. This is the force law for a spring. The further you stretch the spring, the harder it pulls back at you. However, if this were the force law in the solar system, the outer planets would move along on their orbits faster than the inner planets. This is definitely not the case. Gravity is an inverse-square law force.


unbound hyperbolic orbit
unbound parabolic orbit
Far from the focus, a hyperbolic orbit looks like a straight line.
Bound, elliptical orbit

Once Newton proved that circular, elliptical, parabolic, and hyperbolic trajectories were consequences of the inverse-square Law of Gravity, he could explain Kepler's laws of planetary motion.

Kepler's 2nd law (the radius vector of an orbiting body sweeps out equal areas in equal times) is a consequence of the conservation of angular momentum ($\mathbf{L} = m\mathbf{v}\mathbf{r}$). The mass of the planet is a constant. If the radius is smaller, the speed $|v|$ must be larger.


Angular momentum (= m × v × r)
is conserved as Earth orbits the Sun.
v
Distance (r)
is greater, so
velocity (v)
is smaller.
r
r
Sun
Distance (r)
is smaller, so
velocity (v)
is greater.
v
Not to scale!



Conservation of energy means conservation of *total* energy.

When a pole vaulter runs down the runway, plants the pole and springs into the air, the vaulter is converting

a. speed to velocity
b. momentum to angular momentum
c. mass into energy
d. kinetic energy to gravitational potential energy

The circular velocity of an orbiting object, $V_c = \text{sqrt}(GM/r)$. The orbital velocity is just distance (= circumference of the circle) divided by the orbital period, $V_c = 2\pi r/P$, it follows that

$$V_c^2 = GM/r = 4\pi^2 r^2/P^2 \quad .$$

Rearranging terms, we get

$$P^2 = 4\pi^2 r^3/GM \quad .$$

This is a more general form of Kepler's Third Law.

If you used the mass of the Sun, the distance of one Astronomical Unit and the number of seconds in a year, you could easily show that the Earth's period is 1.000 year.

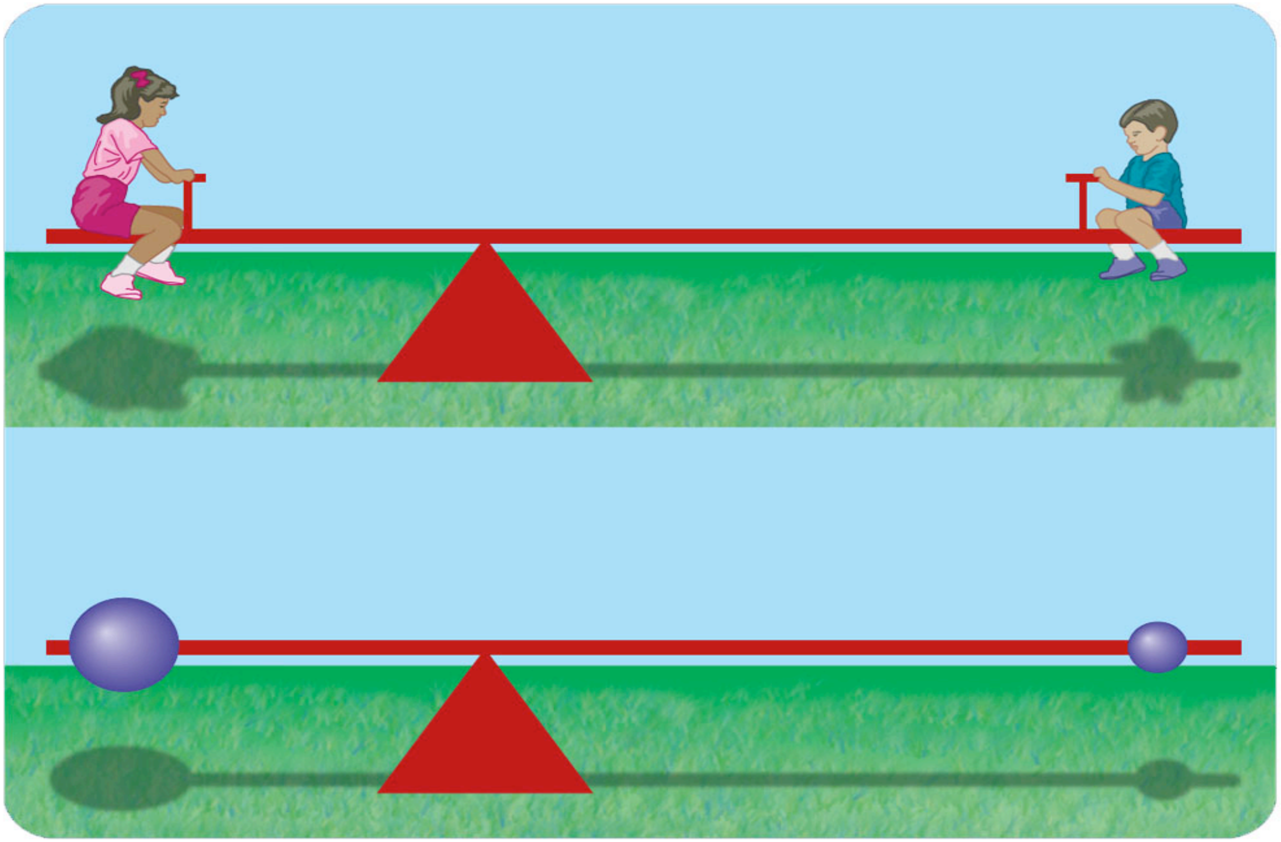
The more general form of Kepler's Third Law can also be used to determine the mass of Jupiter from the periods of revolution of its moons and their separations from Jupiter.

For elliptical orbits one uses the semi-major axis (a) in place of the radius of a circular orbit.

An even more general form of Kepler's Third Law is as follows:

$$P^2 = 4\pi^2 a^3/G(M+m) \quad ,$$

where m is the mass of the orbiting body. Since the Sun's mass is more than 300,000 times that of the Earth, the calculations are not affected much by ignoring the mass of the Earth. In reality, one body does not orbit the center of the other. They both orbit the **center of mass** of the system. This is important when considering double stars.



$$m_1 d_1 = m_2 d_2$$

In the previous graphic, if the girl is twice as massive as her younger brother, her distance from the balance point will be half as far as her brother's.

If you have two stars of one solar mass orbiting their center of mass at a distance of 1 AU, the period will not be equal to one year. It will be equal to one year divided by the sqrt of 2.

Simple question (if you’ve spent time near the ocean).
How many high tides are there each day?

A. 1

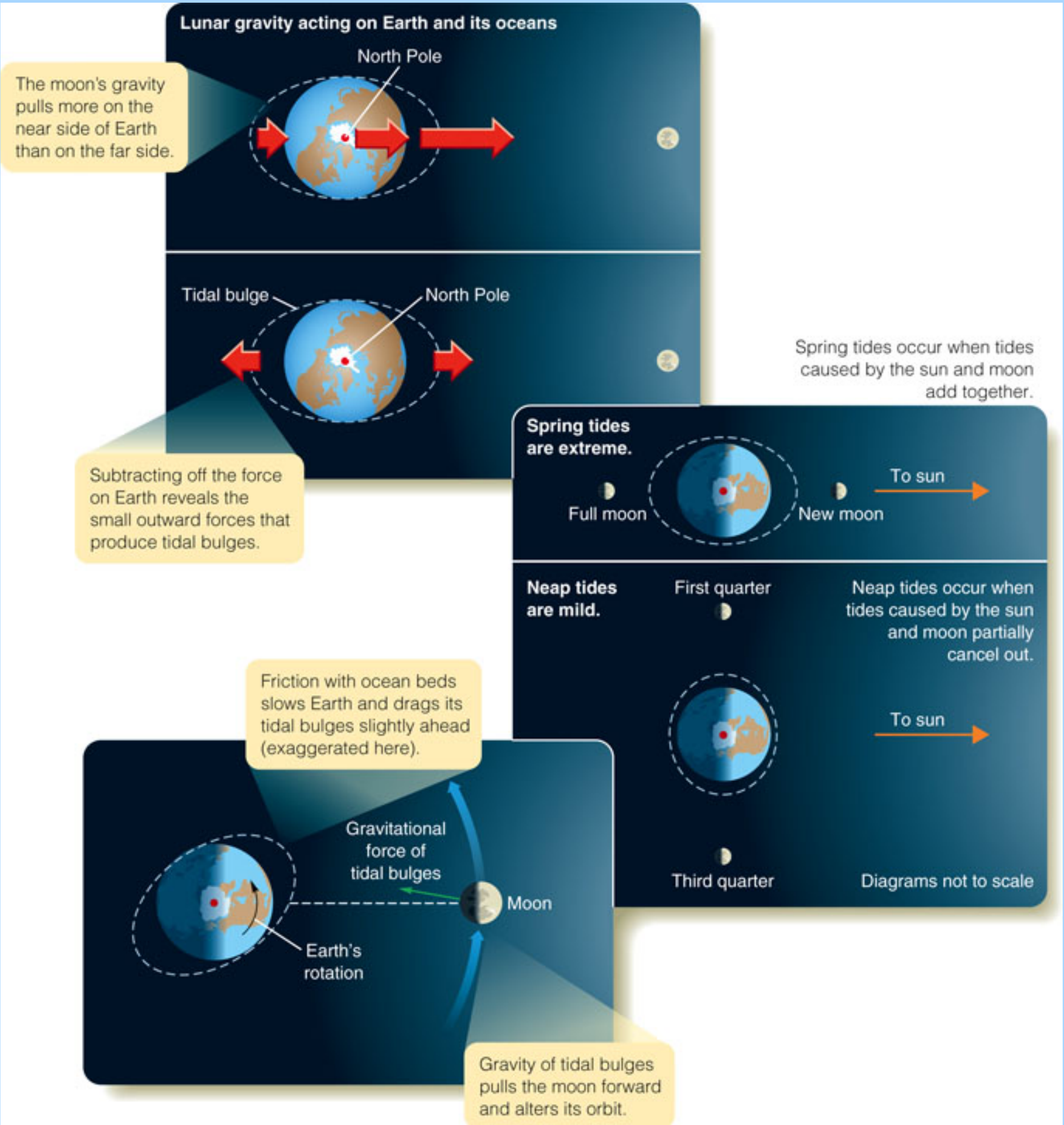
B. 2

C. 3

D. 4

The **tides** are caused by the gravitational pull of the Sun and Moon on the Earth. The oceans shows the larger movement, but the land distorts a bit too.

Recall that gravitational force is a directed force. It is a vector. The gravitational force exerted on the edge of the Earth closest to the Moon is greater than the force in effect at the center of the Earth, which in turn is greater than the force in effect at the edge of the Earth opposite the Moon. The net result is a bulge on *both* sides of the Earth.



Lunar gravity acting on Earth and its oceans
North Pole
The moon's gravity pulls more on the near side of Earth than on the far side.
Tidal bulge
North Pole
Subtracting off the force on Earth reveals the small outward forces that produce tidal bulges.
Spring tides occur when tides caused by the sun and moon add together.
Spring tides are extreme.
Full moon
New moon
To sun
Neap tides are mild.
First quarter
Neap tides occur when tides caused by the sun and moon partially cancel out.
To sun
Third quarter
Diagrams not to scale
Friction with ocean beds slows Earth and drags its tidal bulges slightly ahead (exaggerated here).
Gravitational force of tidal bulges
Moon
Earth's rotation
Gravity of tidal bulges pulls the moon forward and alters its orbit.

When would we experience the highest high tides and the lowest high tides?

a. At first quarter Moon
b. At 3rd quarter Moon
c. At new Moon
d. At full Moon
e. At both new Moon and full Moon

Let $\mathbf{F}_1$ be the gravitational force at the center of the Earth due to the presence of the Moon, and let $\mathbf{F}_2$ be the gravitational force at the edge of the Earth on the side of the Moon. The *tidal force* at the edge of the Earth on the Moon's side will be the difference of these two forces. It can be shown that

$$\mathbf{T} = \mathbf{F}_2 - \mathbf{F}_1 \sim G\, M_{water}\, m_{Moon}\, (2\, R_{earth} \,/\, d^3{}_{Moon}) \,,$$

where M_{earth} and m_{Moon} are the masses of the Earth and Moon, respectively, R_{earth} is the radius of the Earth, and d_{Moon} is the distance from the Earth to the Moon. Thus, the tidal force decreases as the *cube* of the distance, not the square of the distance. The tidal force is a *differential* gravitational force.

Those of you who know calculus will recognize that the derivative of $f(r) = A\,r^{-2}$ is $df/dr = -2A\,r^{-3}$. The gravitational force between the Earth and Moon is an attractive force, so is directed toward the center of the Earth. The tidal bulge of the oceans toward the Moon on the Moon's side of the Earth is opposite the direction of the center of the Earth. That is one interpretation of the change of arithmetic sign from f(r) above to df/dr.

If we measured the gravitational force of the side of the Earth *opposite* to the moon (say, $\mathbf{F}_3$) and calculated the tidal force $\mathbf{F}_1 - \mathbf{F}_3$, we would end up with the same size tidal force directed away from the center of the Earth. This gives a tidal bulge on the side of the Earth opposite to the Moon.

Consider the tidal force on the Earth due to the Moon compared to the tidal force on the Earth due to the Sun:

$$\mathbf{T}_{moon} / \mathbf{T}_{sun} \sim (m_{Moon} / m_{Sun}) (d_{Sun} / d_{Moon})^3 \; .$$

The mass of the Sun is about $2.71 * 10^7$ times the mass of the Moon. The distance to the Sun is on average about 389 times the mean distance to the Moon. It turns out that the tidal force on the Earth due to the Moon is about 2.2 times stronger than the tidal force on the Earth due to the Sun.

With similar reasoning we can calculate the tidal force on the Earth due to any of the other planets. For example, the Sun's mass is just over 1000 times that of Jupiter. Jupiter's distance from the Earth ranges roughly from 4.2 AU to 6.2 AU.

$4.2^3 \sim 74$, and $6.2^3 \sim 238$. The tidal force on the Earth due to Jupiter ranges from 74,000 times to 238,000 times *weaker* than the tidal force on the Earth due to the Sun.

The tidal forces on the Earth due to the planets are typically hundreds of thousands of times weaker than the tidal force due to the Sun, which is 2.2 times weaker than the tidal force on the Earth due to the Moon. For all intents and purposes the tidal force felt on the Earth is only that due to the combined action of the Moon and Sun.

If we moved the Moon 1/3 the distance that it presently is, how would that affect the gravitational force between the two?

A. It would remain the same.
B. It would be 3 times stronger.
C. It would be 9 times stronger.
D. It would be 1/3 as strong.
E. It would be 1/9 as strong.

Say the average high tide near your beach house is 2 ft high. if the Moon's orbit size were suddenly half as big as it is now, how high would the tides be now?

a. 2 feet
b. 4 feet
c. 8 feet
d. 16 feet

If all the planets, including the Earth, were lined up toward the Sun, the tidal forces of all the planets would just add together. What would be the result on the Earth?

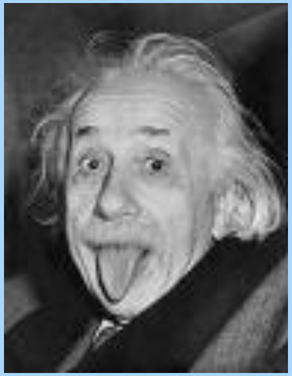
a. major tidal waves
b. tidal waves and earthquakes
c. outbreaks of plague, cholera, and zombies
d. There would be no significant destructive effect, as the tidal force of any planet on the Earth is very, very small compared to the tidal forces due to the Moon and Sun.

The number of degrees that a star is north or south of the celestial equator is called

a) right ascension
b) declination
c) elevation angle
d) azimuth

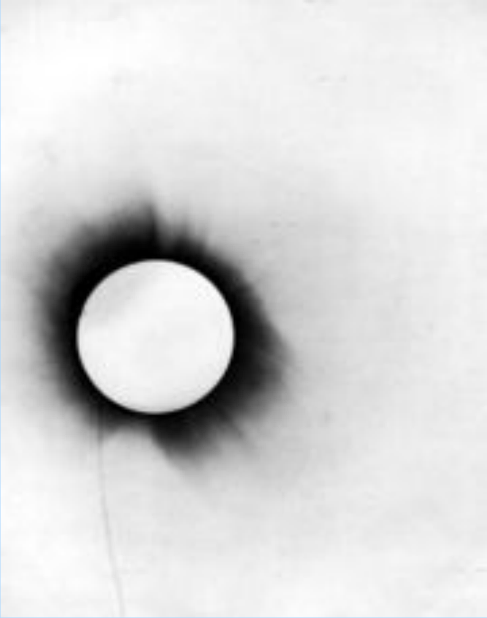
What is the declination of the Sun on the first day of autumn (aka the day of the autumnal equinox)?

a) -23.5 deg
b) 0 deg
c) +23.5 deg
d) It depends on your latitude.



Albert Einstein (1879-1955) became well known to physicists in 1905 after publishing three key papers: on Special Relativity, the photoelectric effect, and Brownian motion.

Einstein's General Theory of Relativity (1916), which was a theory of how gravity curves space, predicted that starlight passing by the Sun during a total solar eclipse whould be bent by the gravity of the Sun. In 1919 this was measured. As a result, Einstein became famous to the rest of the world.




LIGHTS ALL ASKEW IN THE HEAVENS

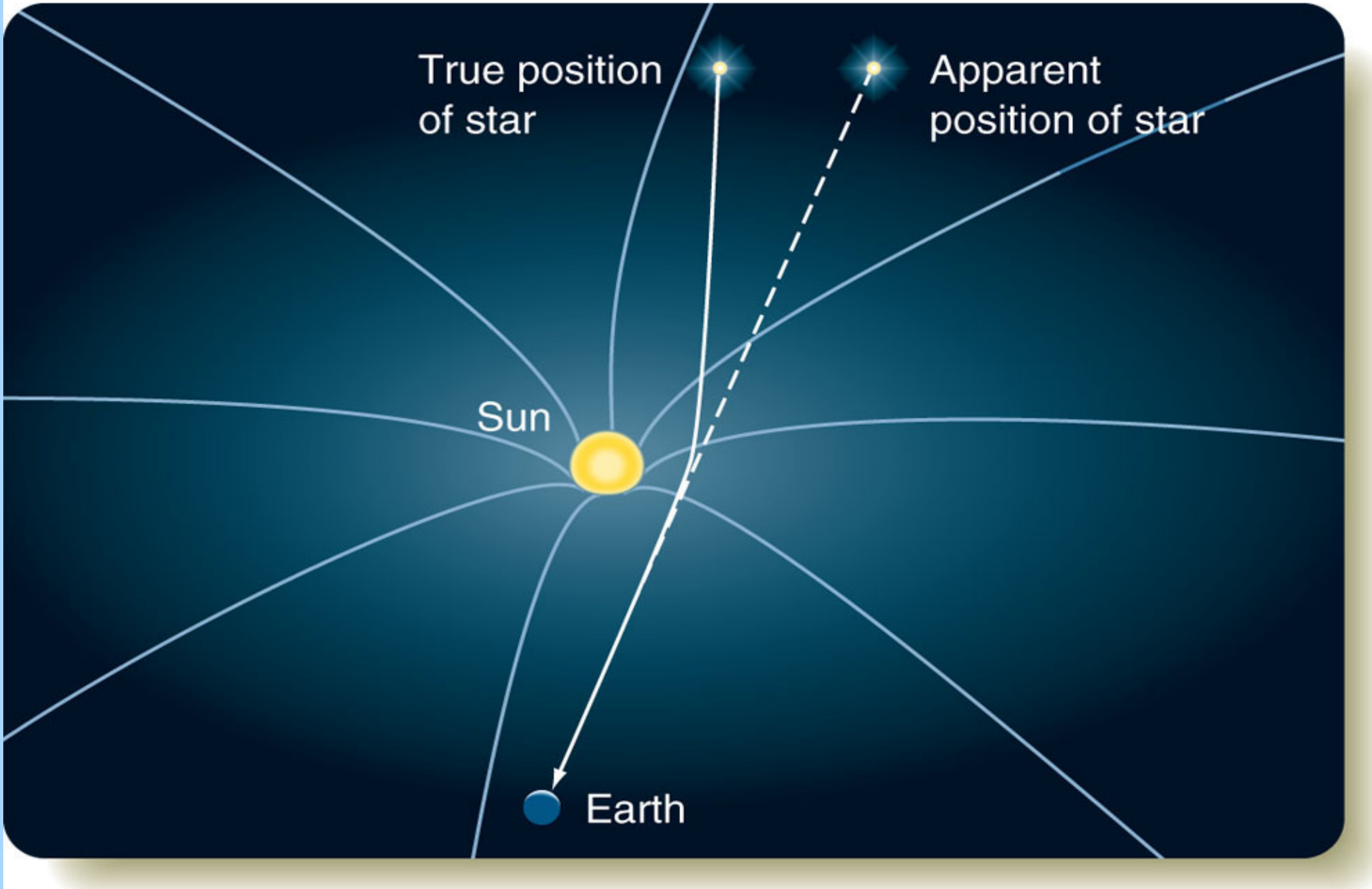
Men of Science More or Less Agog Over Results of Eclipse Observations.

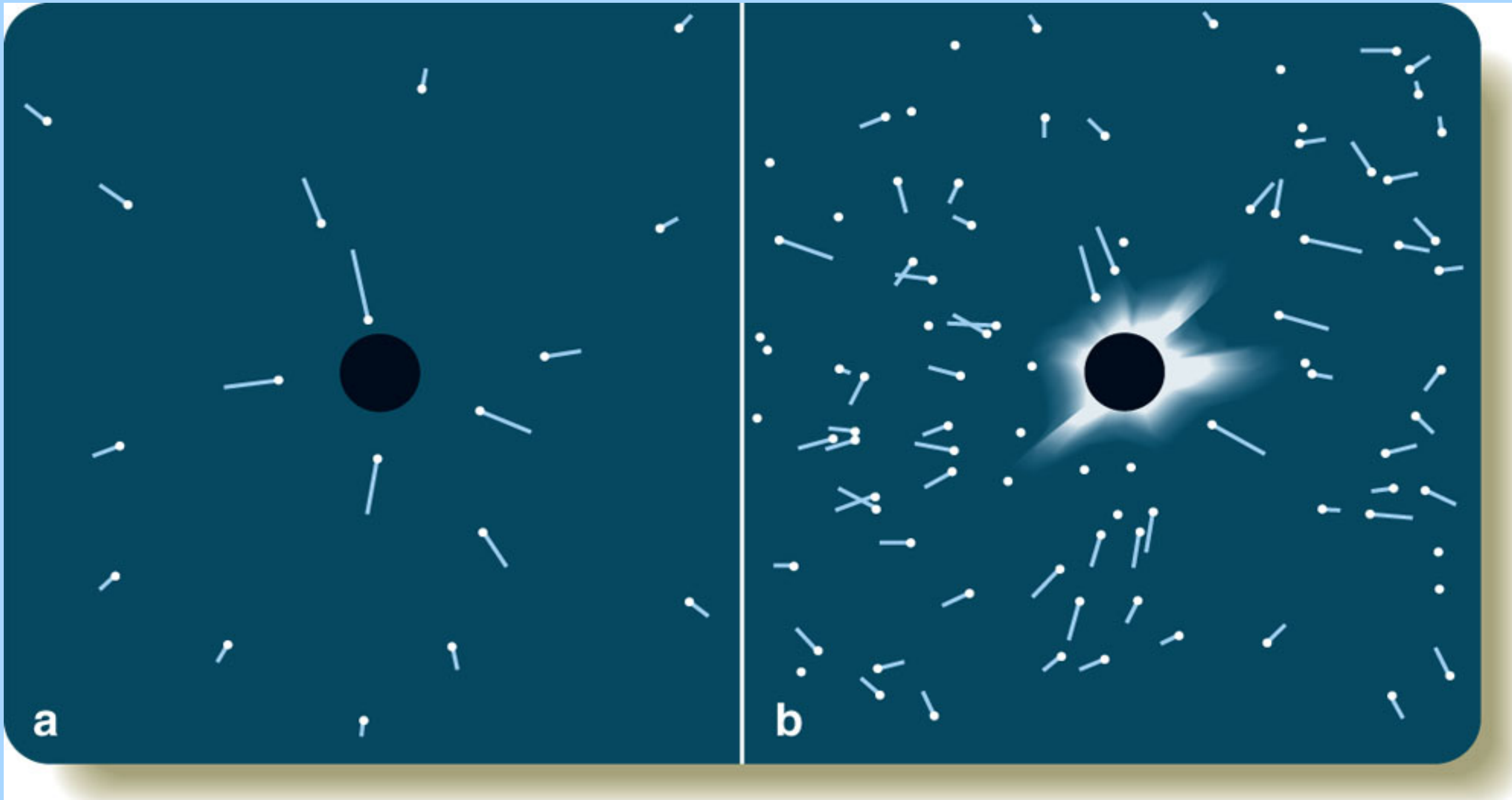
EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.


True position of star
Apparent position of star
Sun
Earth



Schematic diagram of starlight deflections (left), and actual data from a 1922 eclipse.

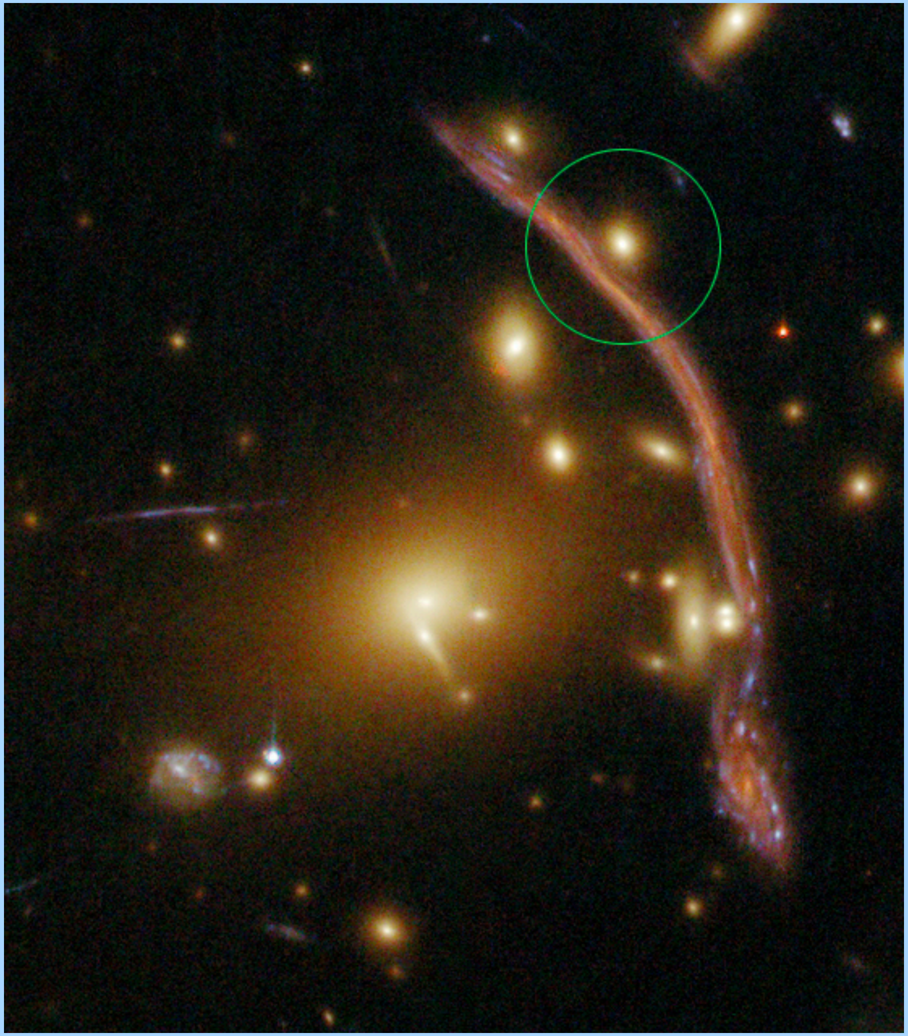
Optical Deflection of Starlight During Eclipses

Date	Location	arc secs
29 May 1919	Sobral	1.98 ± 0.16
	Principe	1.16 ± 0.40
21 Sep 1922	Australia	1.77 ± 0.40
		1.42 to 2.16
		1.72 ± 0.15
		1.82 ± 0.20
9 May 1929	Sumatra	2.24 ± 0.10
19 June 1936	USSR	2.73 ± 0.31
	Japan	1.28 to 2.13
20 May 1947	Brazil	2.01 ± 0.27
25 Feb 1952	Sudan	1.70 ± 0.10
30 Jun 1973	Mauritania	1.66 ± 0.19

Astronomers in the 18th and 19th centuries made very accurate measures of the positions of the planets. They noted that the direction of the elliptical orbit of Mercury was changing direction time. The shift amounts to 5601 arc seconds per century, less than 1.6 degrees. Newton's gravitational theory could account for 5558 arcseconds of this advance of the perihelion. Einstein's theory of gravity explained the extra 43 arc seconds per century of the shift.

Table 5-2 | Precession in Excess of Newtonian Physics

PLANET	Observed Excess Precession (Sec of arc per century)	Relativistic Prediction (Sec of arc per century)
Mercury	43.11 ± 0.45	43.03
Venus	8.4 ± 0.48	8.6
Earth	5.0 ± 1.2	3.8
Icarus	9.8 ± 0.8	10.3



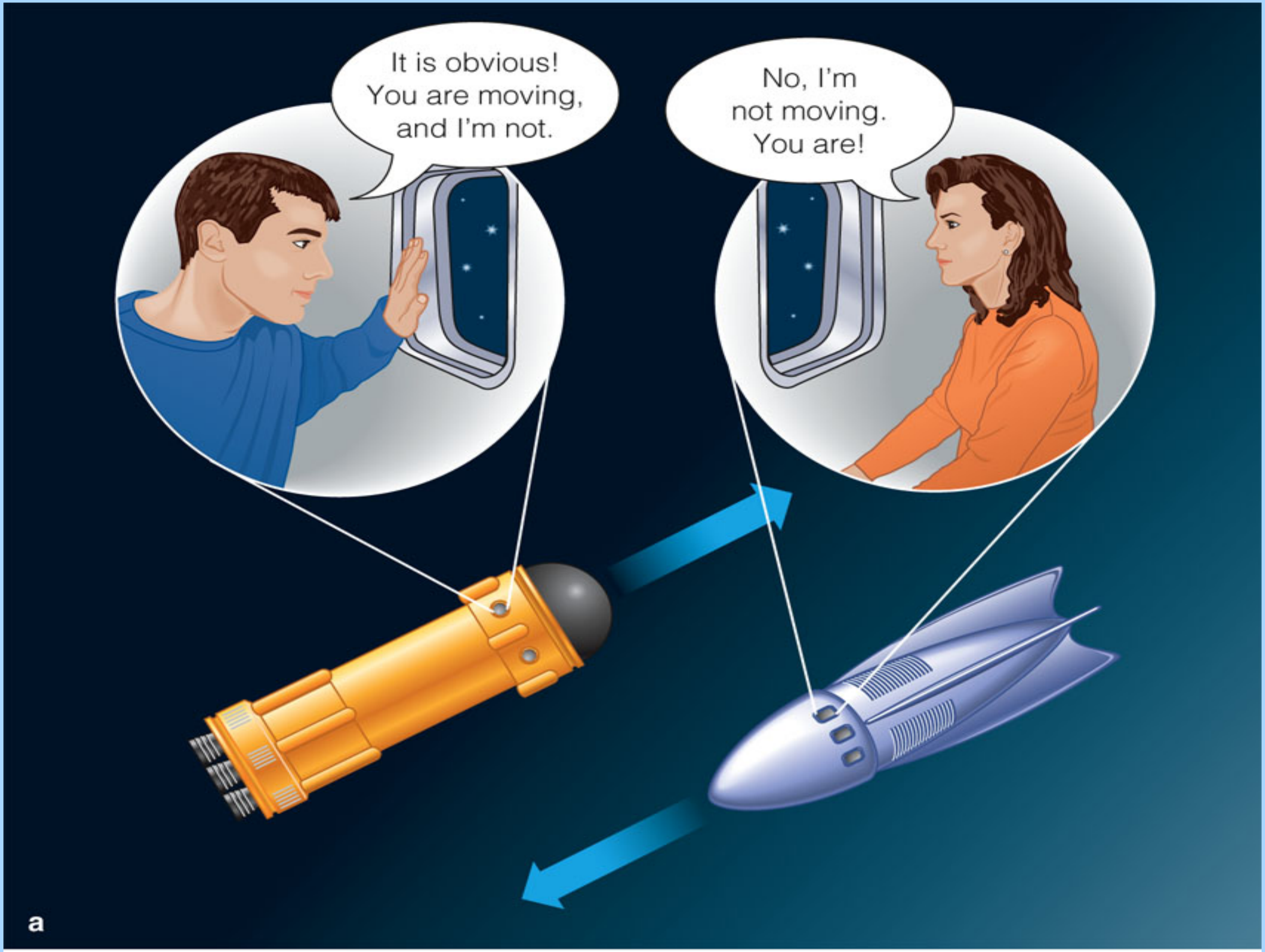
The foreground galaxy in the cluster Abell 370 is further distorting the light of a more distant galaxy that is being lensed by the cluster.

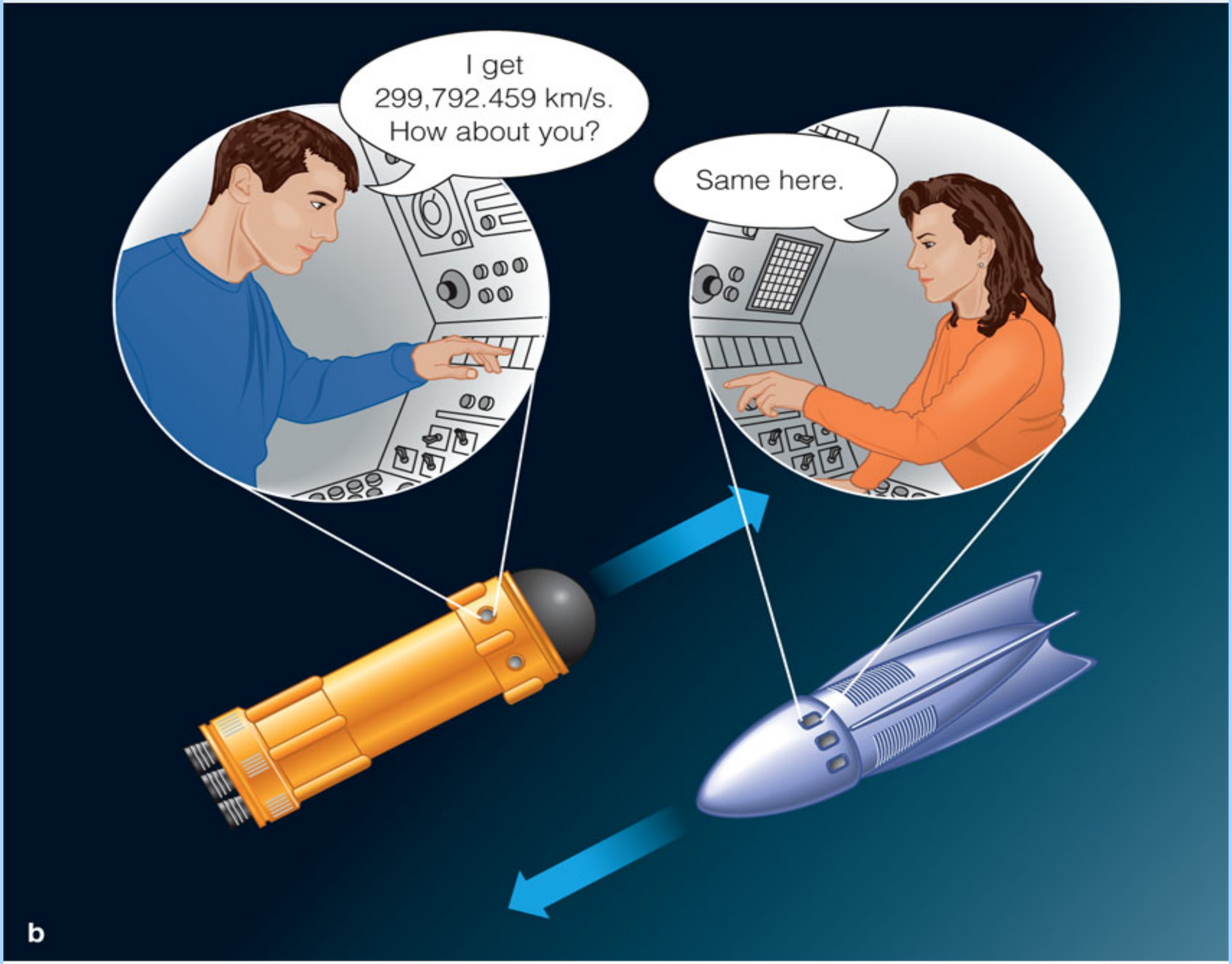
In 1939, under the urging of Leo Szilard, Einstein wrote to President Franklin D. Roosevelt to support research on nuclear fission for possible military use. He feared that the Nazis were carrying out just such research. Thus began the Manhattan Project, which led to the first nuclear test on 16 July 1945.

The Theory of Special Relativity relates to *inertial frames of reference*. This means that none of the observers are being subjected to any forces. They are not experiencing any acceleration. There are two consequences of this situation:

1) The laws of physics are the same for all observers, no matter what their motion, so long as they are not accelerated.

2) The velocity of light is constant and will be the same for all observers independent of their motion relative to the light source.

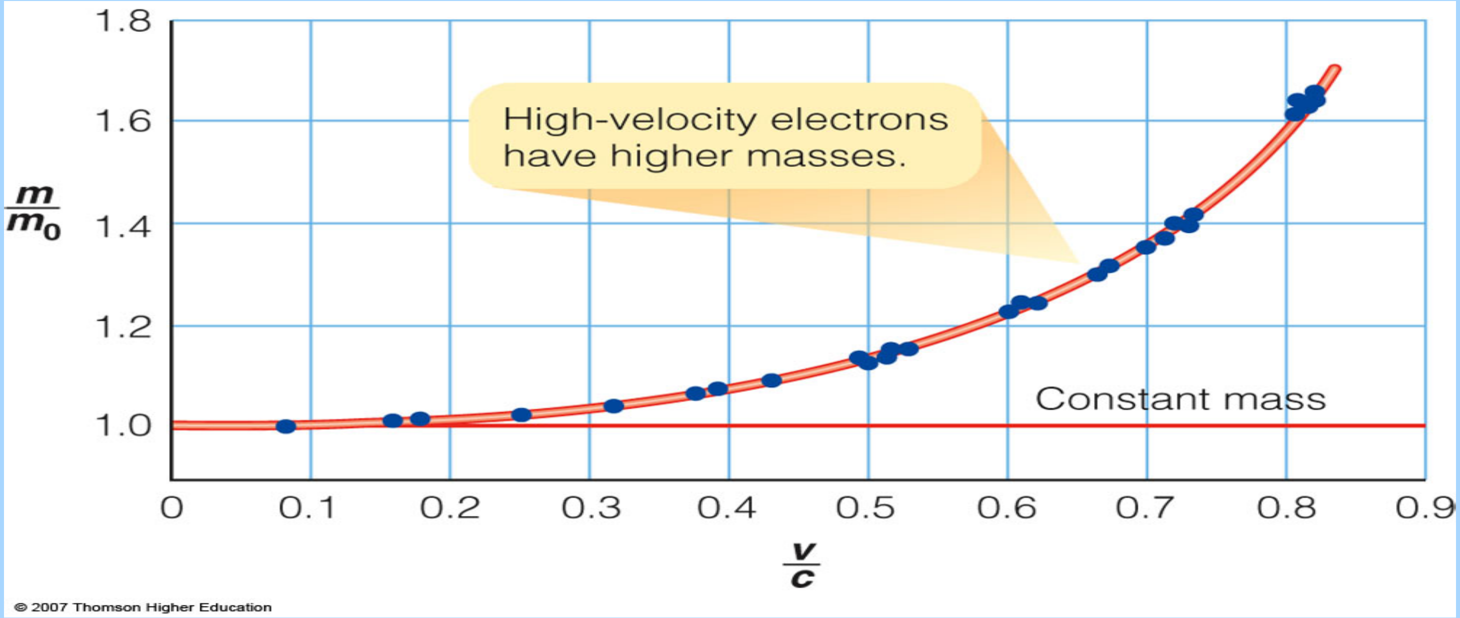

It is obvious! You are moving, and I'm not.
No, I'm not moving. You are!
a


I get 299,792.459 km/s. How about you?
Same here.
b

You may recall reading that the ancient Greeks spoke of the the four fundamental elements: Earth, water, air, and fire. They also postulated a substance which filled the region of the universe above the terrestrial sphere, called *quintessence* (literally the 5th essence). This was also known as the ether.

In the latter half of the 19th century physicists were able to measure the velocity of light to within a small fraction of one percent. They expected to obtain different values of the speed of light in different directions, hypothesizing that there was ether moving in some direction or other. But no such variations were ever proven.

One of the consequences of Special Relativity is that particles moving a sizable fraction of the speed of light would have larger masses. This has in fact been measured in the lab.

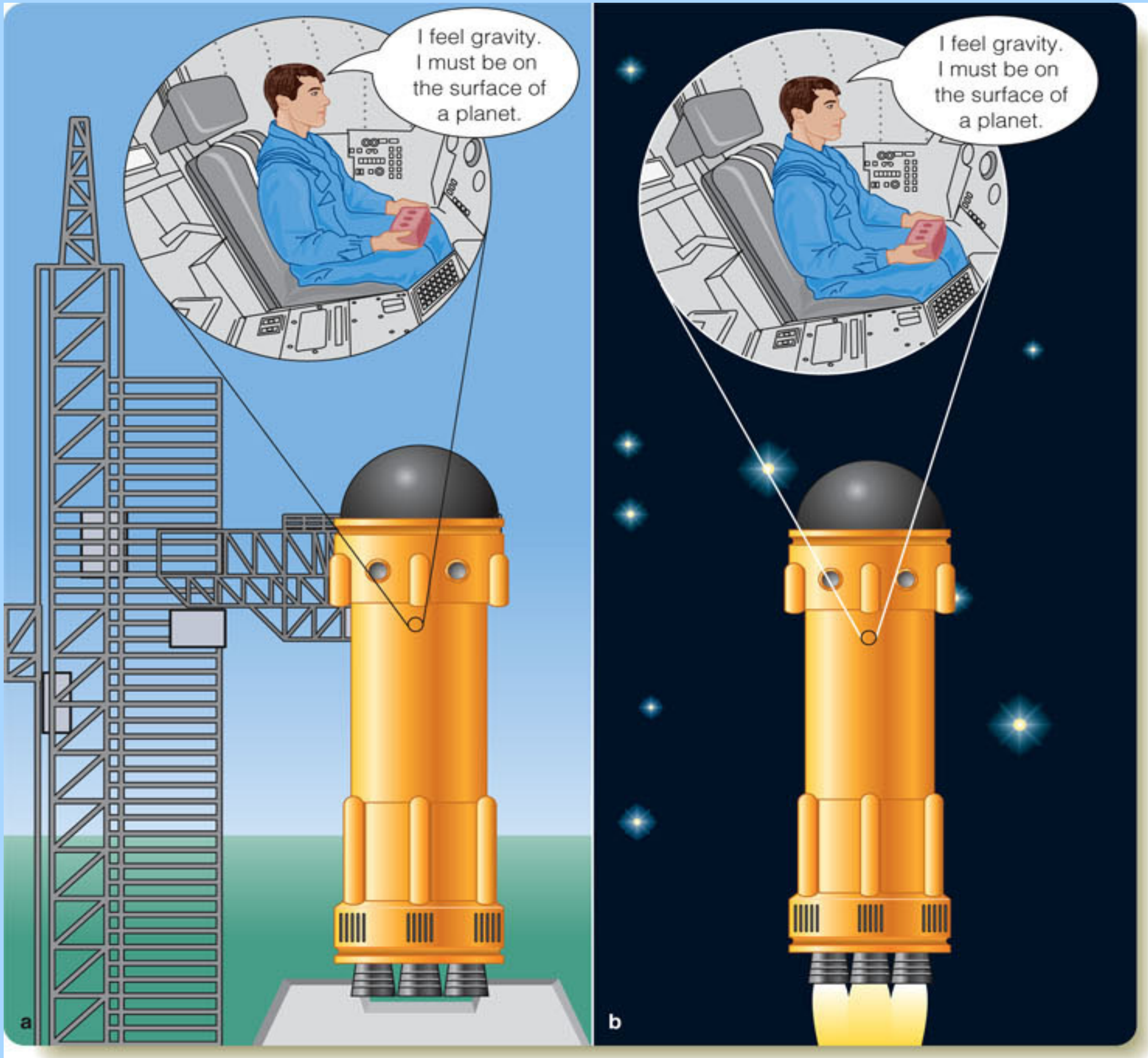


The most famous consequence of Special Relativity is the equivalence of mass and energy. For an object of mass m at rest, its equivalent energy is $E = mc^2$.

This is the basic principle behind nuclear fusion. In the Sun's core protons are converted into helium nuclei, and a certain percentage of the mass is converted into energy according to Einstein's formula. A *huge* amount of energy comes from a small amount of mass that is converted. This is how the Sun has been able to shine for billions of years and how it will continue to shine for several billion more years.

General Relativity (1916)

An observer in a windowless spaceship cannot distinguish between two situations: 1) he/she is accelerating through space; 2) he/she is sitting on a planet and subject to the planet's gravitational force. This relativity of perspective is known as the **Equivalence Principle**.


I feel gravity. I must be on the surface of a planet.
a
I feel gravity. I must be on the surface of a planet.
b

According to GR, mass tells space-time how to curve, and the curvature of space-time (gravity) tells mass how to accelerate.

The Sun, having a certain mass within a certain volume, will cause space-time to warp. This is why the positions of the stars near the edge of the Sun are different during a total solar eclipse. The maximum shift in this case is only 1.75 seconds of arc, but it is measurable.

Predictions and confirmations of Special Relativity

1) Matter can be transformed into energy (matter/antimatter annihilation); energy can be transformed into particles and anti-particles

2) Speed of light is a constant even if observer(s) are moving a substantial fraction of the speed of light (Michelson-Morley experiment of 1887)

3) An object of rest mass m_0 has a measurably larger mass when moving at a substantial fraction of the speed of light

4) time dilation (radioactive decay of unstable moving particles is slower than the same particles at rest; supernova light-curves)

The General Theory of Relativity was very successful because it explained a number of observational phenomena:

1) Bending of starlight during solar eclipses

2) Advance of the perihelion of Mercury (Newtonian theory could not explain all the rotation of the orbit)

3) Precession of orbiting gyroscopes