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## Goals for Chapter 6

- To understand and calculate work done by a force
- To study and apply kinetic energy
- To learn and use the work-energy theorem
- To calculate work done by a varying force along a curved path
- To add time to the calculation and determine the power in a physical situation


## Work, a force applied through a distance

As in the illustration, pushing in the same direction that the object move


If a body moves through a displacement $\overrightarrow{\boldsymbol{s}}$ while a constant force $\overrightarrow{\boldsymbol{F}}$ acts on it in the same direction ...

... the work done by the force on the body is $W=F s$.

## Work done by a constant force: direction matters

Work done by a constant force on an object is

-The maximum work (efficiency) done by a force is achieved when the force is in the direction of the displacement (it can be positive or negative)
-If the force is perpendicular to the displacement then it does not work (e.g. Normal forces do no work)

## Sign of work: positive, negative or zero

(a)


The force has a component in the direction of displacement:

- The work on the object is positive.
- $W=F_{\|} s=(F \cos \phi) s$
(b)


The force has a component opposite to the direction of displacement:

- The work on the object is negative.
- $W=F_{\|} s=(F \cos \phi) s$
- Mathematically, $W<0$ because $F \cos \phi$ is negative for $90^{\circ}<\phi<270^{\circ}$.
(c)


The force is perpendicular to the direction of displacement:
- The force does $n o$ work on the object.
- More generally, if a force acting on an object has a component $F_{\perp}$ perpendicular to the object's displacement, that component does no work on the object.


## What is the work done by the force of gravity?



$$
\begin{aligned}
& W_{m g}=\dot{'}_{m g} \cdot \stackrel{\stackrel{r}{S}}{s}=(-m g \hat{j}) \cdot(R \hat{i}-h \hat{j}) \\
& \quad=0+(-m g)(-h)=m g h
\end{aligned}
$$

It will be POSITIVE if the object goes downward and NEGATIVE if it goes up. The work DOES
NOT depend on the $x$-displacement!!

## Stepwise solution of work done by several forces Example 6.2

(a)

(b) Free-body diagram for sled


$$
W_{T}=F_{T} s \cos \theta=(5000 \mathrm{~N})(20 \mathrm{~m}) \cos \left(36.9^{\circ}\right)=80 \mathrm{~kJ}
$$

$$
W_{f}=F_{f} s \cos \theta=(3500 \mathrm{~N})(20 \mathrm{~m}) \cos \left(180^{\circ}\right)=-70 \mathrm{~kJ}
$$

$$
W_{m g}=F_{m g} s \cos \theta=m g s \cos \left(90^{\circ}\right)=0
$$

$$
W_{N}=F_{N} s \cos \theta=F_{N} s \cos \left(90^{\circ}\right)=0
$$

$$
W_{\text {tot }}=W_{F_{T}}+W_{f}+W_{m g}+W_{N}=10 \mathrm{~kJ}
$$

## Relation between Kinetic energy and the TOTAL work done on an object: the work-energy theorem

The next idea couples kinematics (changes in velocity of an object) and Netwon's second law of motion (total force on an object leading to an acceleration) to the total work done on an object.

The work done by the net (total) force on an object is equal to the change in the objects kinetic energy

$$
W_{\text {total }}=K E_{\text {final }}-K E_{\text {initial }} \quad \text { where } \quad K E=\frac{1}{2} m \mathrm{v}^{2}
$$

We first show this for constant forces but we will see later that this is true for any type of forces (that is why we call it a theorem)

According to Newton's $2^{\text {nd }}$ law $F_{\text {net }}=m a$ (here the forces are constant and we take the x-direction to point along the direction of the net force (sum of all forces) on the object. Then we can use the 1-D kinematic equation

$$
\begin{aligned}
& \mathrm{v}_{f}^{2}=\mathrm{v}_{i}^{2}+2 a\left(x_{f}-x_{i}\right)=\quad \square \\
& \mathrm{v}_{f}^{2}=\mathrm{v}_{i}^{2}+2 a s \\
& \mathrm{v}_{f}^{2}=\mathrm{v}_{i}^{2}+2 \frac{F_{\text {net }}}{m} \mathrm{~s}
\end{aligned} \quad \begin{aligned}
& \frac{1}{2} m \mathrm{v}_{f}^{2}=\frac{1}{2} m \mathrm{v}_{i}^{2}+F_{\text {net }} s \\
& \\
& F_{\text {net }} s=\frac{1}{2} m \mathrm{v}_{f}^{2}-\frac{1}{2} m \mathrm{v}_{i}^{2} \\
& \\
& W_{\text {net }}=K E_{f}-K E_{i}
\end{aligned}
$$

## Work and energy with varying forces

What is the work done when the force changes with distance (e.g. spring)?

$$
W=F_{a x} \Delta x_{a}+F_{b x} \Delta x_{b}+\mathrm{L}=\int_{x_{1}}^{x_{2}} F(x) d x
$$

Integral is geometrically the area under the curve (in this case the area of a $F(x)$ vs. $x$ graph has dimensions of work and it IS the work done by that force).

Some indefinite integrals (no limits):

$$
\int x^{n}=\frac{1}{n+1} x^{n+1}
$$

Some definite integrals (with limits):

$$
\int_{a}^{n} x^{n} d x=\frac{1}{n+1} b^{n+1}-\frac{1}{n+1} a^{n+1}
$$

(a) Particle moving from $x_{1}$ to $x_{2}$ in response
to a changing force in the $x$-direction


(c)


## The stretch of a spring and the force that caused it

- The force applied to an ideal spring will be proportional to its stretch.
- The graph of force on the $y$ axis versus stretch on the $x$ axis will yield a slope of $k$, the spring constant.

Force due to a spring is:
x is positive for stress and

$$
F_{\text {spring }}=-k x \longleftarrow \begin{aligned}
& \text { tor stress and } \\
& \text { negative for } \\
& \text { compression }
\end{aligned}
$$

$$
W_{\text {spring }}=\int_{a}^{b}(-k x) d x=-\frac{1}{2} k b^{2}+\frac{1}{2} k a^{2}
$$



The area under the graph represents the work done on the spring as the spring is stretched from $x=0$ to a maximum value $X$ :


## Stepping on a scale-Example 6.6

A woman weighting 600 N steps onto a scale and compresses it 1 cm . What is the k of the spring and the work done ON the spring during compression?


## Power: rate of making work

$$
\begin{aligned}
& \text { Average power } \\
& P_{a v e}=\frac{\Delta W}{\Delta t}
\end{aligned}
$$

Instant power

$$
P=\frac{d W}{d t}
$$

If the force creating such a power is constant then $d W=F$ ds so

$$
P=\frac{\dot{F} \cdot d s^{\mathrm{l}}}{d t}=\stackrel{\mathrm{r}}{F} \cdot \stackrel{\mathrm{r}}{\mathrm{v}}
$$

## Heart power

Each day the heart takes in and out 7500 L of blood. Assume that the work done by the heart is equal to the work required to lift this amount of blood a height equal to the average height of a person ( 1.63 m ). The density (mass per unit volume) of blood is $1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$.
(a)How much work does the heart do in a day?
(b)What is the power output of the heart in watts?

The mass that is lifted is $\mathrm{m}=\rho \mathrm{V}=7.5 \mathrm{~m}^{3} \times 1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}=7.88 \times 10^{3} \mathrm{~kg}$
The work done by against gravity to lift it is $\mathrm{mgh}=1.26 \times 10^{5} \mathrm{~J}$

The average power is this work divided by the time
$P=\left(1.26 \times 10^{5} \mathrm{~J}\right) /(24 \times 3600 \mathrm{~s})=1.46$ Watts

## Stopping a block with a spring

Find the maximum distance the spring will compress (using the work-energy theorem)


The key idea is that the kinetic energy changes to zero by the work done by the spring

$$
\begin{aligned}
& W_{\text {spring }}=\int_{0}^{a}(-k x) d x=-\frac{1}{2} k a^{2}+\frac{1}{2} k(0)^{2}=K E_{f}-K E_{i} \\
& -\frac{1}{2} k a^{2}=0-\frac{1}{2} m v_{0}^{2} \Rightarrow a=\sqrt{\frac{m v_{0}^{2}}{k}}
\end{aligned}
$$

## Connected blocks and final speeds using energy methods

If the table has a kinetic coefficient of friction of 0.250 , what is the final speed of the blocks after they have moved a distance $\mathrm{s}=1.50 \mathrm{~m}$ ?


$$
\begin{aligned}
& \mathrm{W}_{\text {net } 1}=\mathrm{KE}_{\mathrm{f} 1}-\mathrm{KE}_{\mathrm{i} 1} \\
& \mathrm{~W}_{\text {net } 2}=\mathrm{KE}_{\mathrm{f} 2}-\mathrm{KE}_{\mathrm{i} 2}
\end{aligned} \longrightarrow
$$

$$
\begin{gathered}
\mathrm{W}_{\text {net } 1}+\mathrm{W}_{\text {net } 2}=\mathrm{KE}_{\mathrm{f} 1} \text { and } 2-\mathrm{KE}_{\mathrm{i} 1 \text { and } 2} \\
-\mu_{k} F_{N} s+T s-T s+m_{2} g s=\frac{1}{2} m_{1} \mathrm{v}^{2}+\frac{1}{2} m_{2} \mathrm{v}^{2}-0
\end{gathered}
$$

$$
\mathrm{v}=\sqrt{\frac{2\left(-\mu_{k} m_{1}+m_{2}\right) g s}{\left(m_{1}+m_{2}\right)}}
$$

