Chapter 8

Momentum, Impulse, and Collisions

PowerPoint® Lectures for
University Physics, Twelfth Edition
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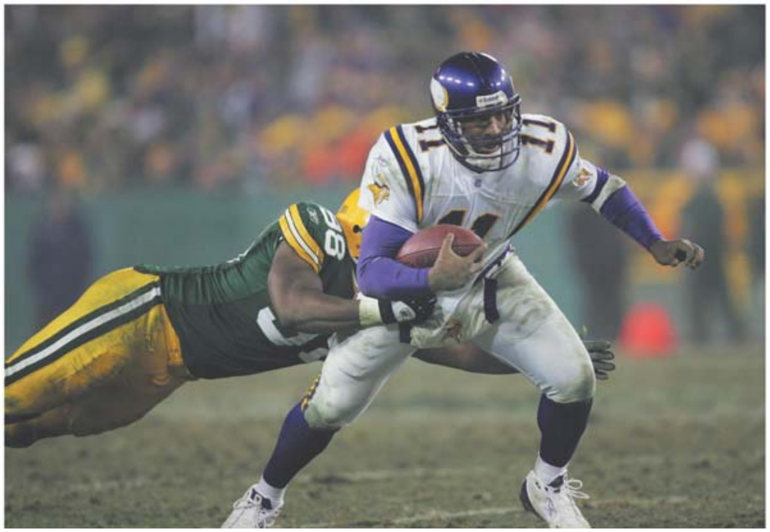
What could do more damage to the carrot? A .22 caliber bullet as shown or a twice light bullet with twice higher velocity?

Goals for Chapter 8

- To determine the momentum of a particle
- To add time and study the relationship of impulse and momentum
- To see when momentum is conserved and examine the implications of conservation
- To use momentum as a tool to explore a variety of collisions
- To understand the center of mass

What is momentum?



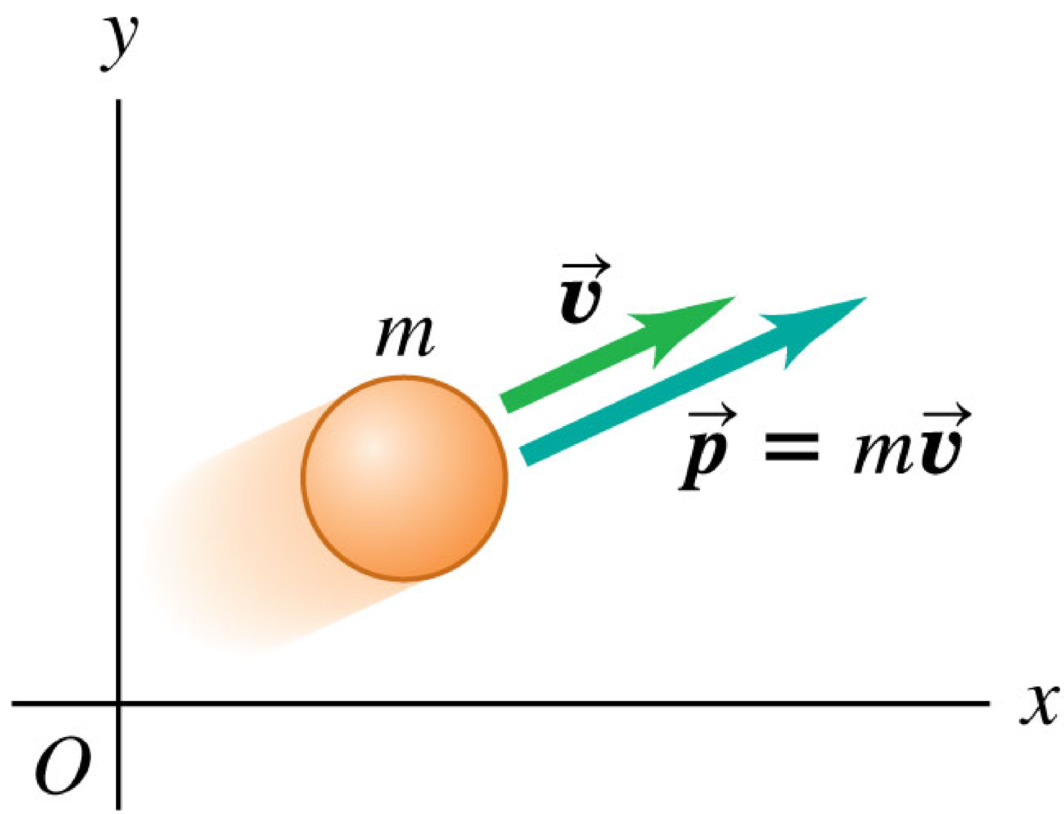
Chapter 8: Momentum

- Want to deal with more complicated systems
  - Collisions
  - Explosions
- Newton’s laws still work, but using them directly gets harder:
  - New tricks similar to energy conservation could help

Today's Lecture

- Begin with a definition of *Linear Momentum*
- Then show that *conservation of momentum* helps us solve certain types of problems
  - *Things colliding*
  - *Things exploding*

Figure 8.1



Momentum $\vec{p}$ is a vector quantity;
a particle's momentum has the same direction as its velocity $\vec{v}$.

Definition of Linear Momentum

<u>Vector equation!</u>

$$\vec{p} = m\vec{v}$$

$$\vec{p}_{system} = \sum m_i \vec{v}_i$$

Restating Newton's Second Law

"The rate of change of momentum of an object is equal to the net force applied to it"

$$\sum \vec{F} = \frac{d\vec{P}}{dt}$$

We can check for constant mass:

$$\frac{d\vec{P}}{dt} = \frac{d(m\vec{V})}{dt}$$

$$= m\frac{d(\vec{V})}{dt}$$

$$= m\vec{a}$$

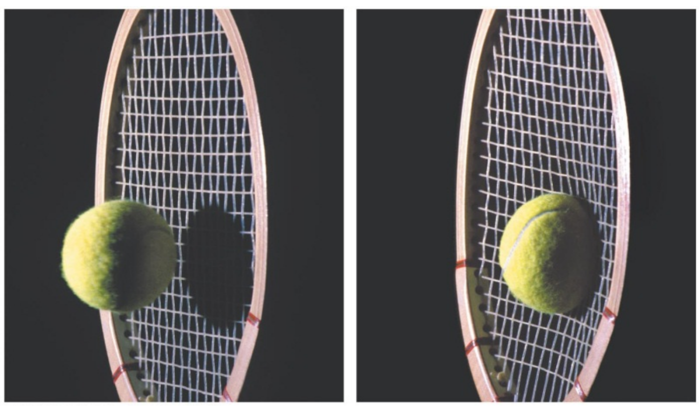
If we exert a net force on a body, the momentum of the body changes

Impulse: change in momentum

$$\sum \vec{F} = \frac{d\vec{P}}{dt} \Rightarrow \boxed{\Delta \vec{p}} = \int \vec{F}_{net}\, dt = \vec{F}_{net-ave} \Delta t$$

impulse

This is considered typically during a collision (short time) or explosion.



20 m/s

15 m/s

m=0.1 Kg

$$p_{1x} = +(0.1\text{Kg})(20\text{m/s}) = 2\ \text{Kg}\cdot\text{m/s}$$

$$p_{2x} = -(0.1\text{Kg})(15\text{m/s}) = -1.5\ \text{Kg}\cdot\text{m/s}$$

$$\Rightarrow \Delta p_x = p_{2x} - p_{1x} = -3.5\ \text{Kg}\cdot\text{m/s}$$

What if $\Sigma F=0$?

If $\Sigma F=0$, then $dp/dt = 0$, p = constant

Momentum doesn't change

$$m\vec{v} = m'\vec{v}'$$

momentum before = momentum after

Conservation of Momentum

For a closed *system* (no external forces), by Newton's 3rd law, $\Sigma F=0$

Conservation of Momentum

$$\sum_i m_i \vec{v}_i = \sum_i m_i' \vec{v}_i'$$

Sum of all momentum before = Sum of all momentum after

True in *X* and *Y* directions separately!

Problem Solving

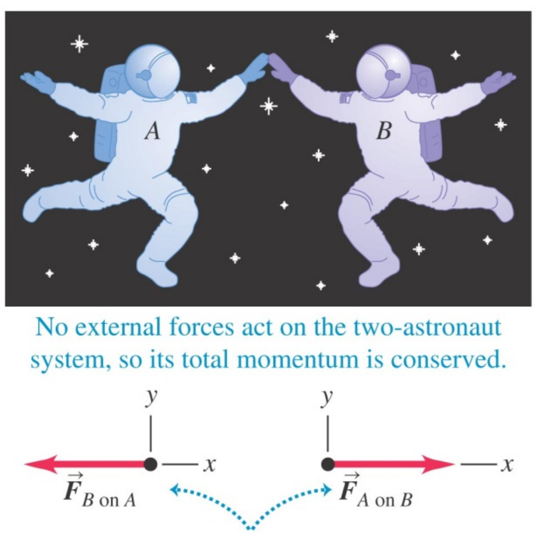
For Conservation of Momentum problems:

BEFORE and AFTER

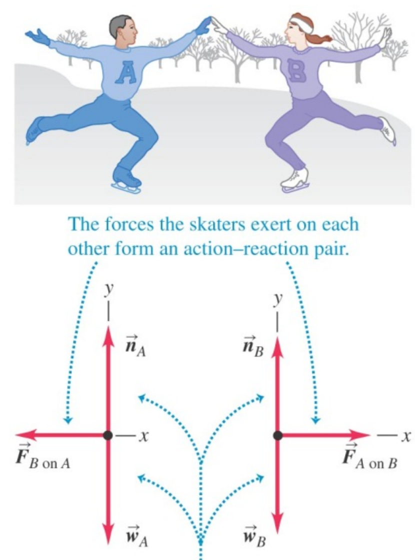
Everyday Experience?

Question: Why do you go backwards when you push someone on the ice?

Newton's Law's answer: When you exert a force on another person, then, by Newton's law, the person exerts an equal and opposite force on you.



The forces the astronauts exert on each other form an action–reaction pair.



Although the normal and gravitational forces are external, their vector sum is zero, so the total momentum is conserved.

Everyday Experience? Cont…

Question: Why do you go backwards when you push someone on the ice?

Momentum Conservation Answer:

- Before:
  - *The system starts with zero momentum (nobody is moving)*
- After:
  - *The system ends with zero momentum. You and your friend move in opposite directions (the one with least mass moves faster)*

Conceptual Example

- Ball of mass *m* is dropped from a height *h*:
  - What is the momentum before release?
  - What is the momentum before it hits the ground?
  - Is momentum conserved?

What if we add the Earth?

- What is the force on the ball?
- What is the force on the earth?
- Is there any net force in this <u>system</u>?
- Is momentum conserved?

$$\Sigma F=0\text{, then } dp/dt = 0, \rightarrow p = \text{constant}$$

Momentum for a System is Conserved

- Momentum is *ALWAYS* conserved for a *COMPLETE SYSTEM*, you just have to look at a big enough system to see it correctly.
  - Not conserved for a single ball in the field of gravity
  - A ball falling is not a big enough system. You need to consider what is *making* it fall.
- Momentum is conserved if the system is closed, i.e. either “large enough” or no external forces
  - Internal forces do not break momentum conservation

Conservation of Momentum

For a closed *system* (no external forces), by Newton's 3rd law, $\Sigma F=0$

Conservation of Momentum

$$\sum_i m_i \vec{v}_i = \sum_i m_i' \vec{v}_i'$$

Sum of all momentum before = momentum after Sum of all

True in *X* and *Y* directions separately!

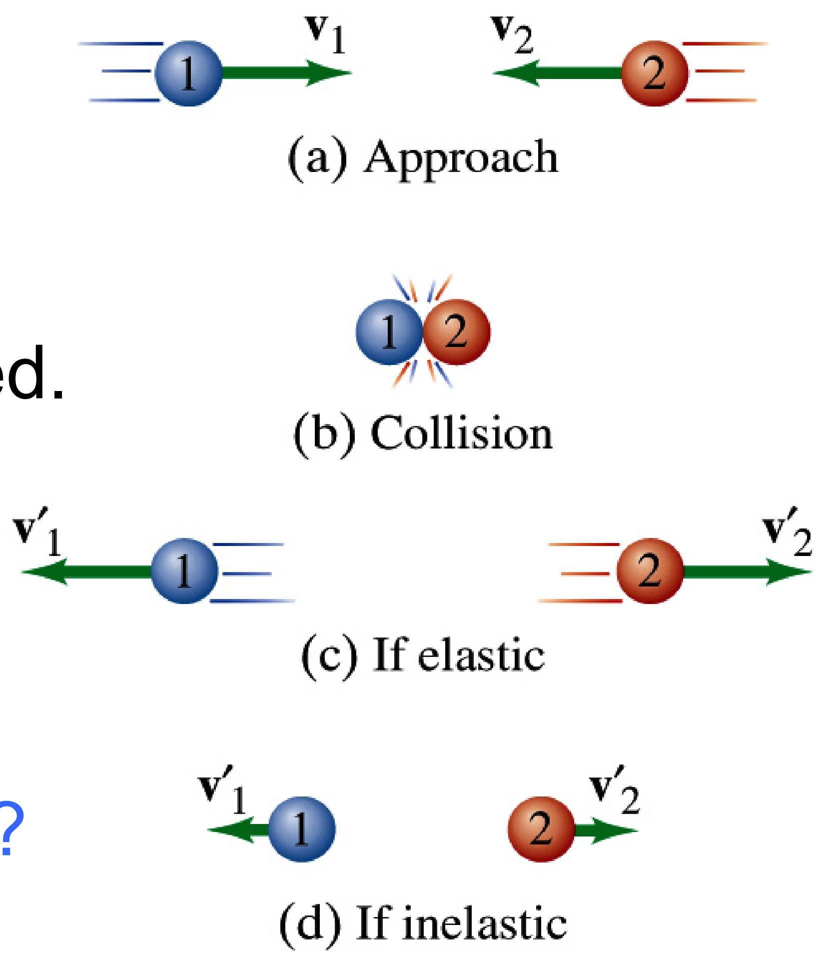
Types of collisions according to energy before and after the collision

Definitions:

- Elastic collision = TOTAL kinetic energy is conserved
- Inelastic collision = TOTAL kinetic energy is not conserved.

Keep in mind

- Momentum is ALWAYS conserved in a collision
- Total Energy may or may not. Q: where does the energy go?



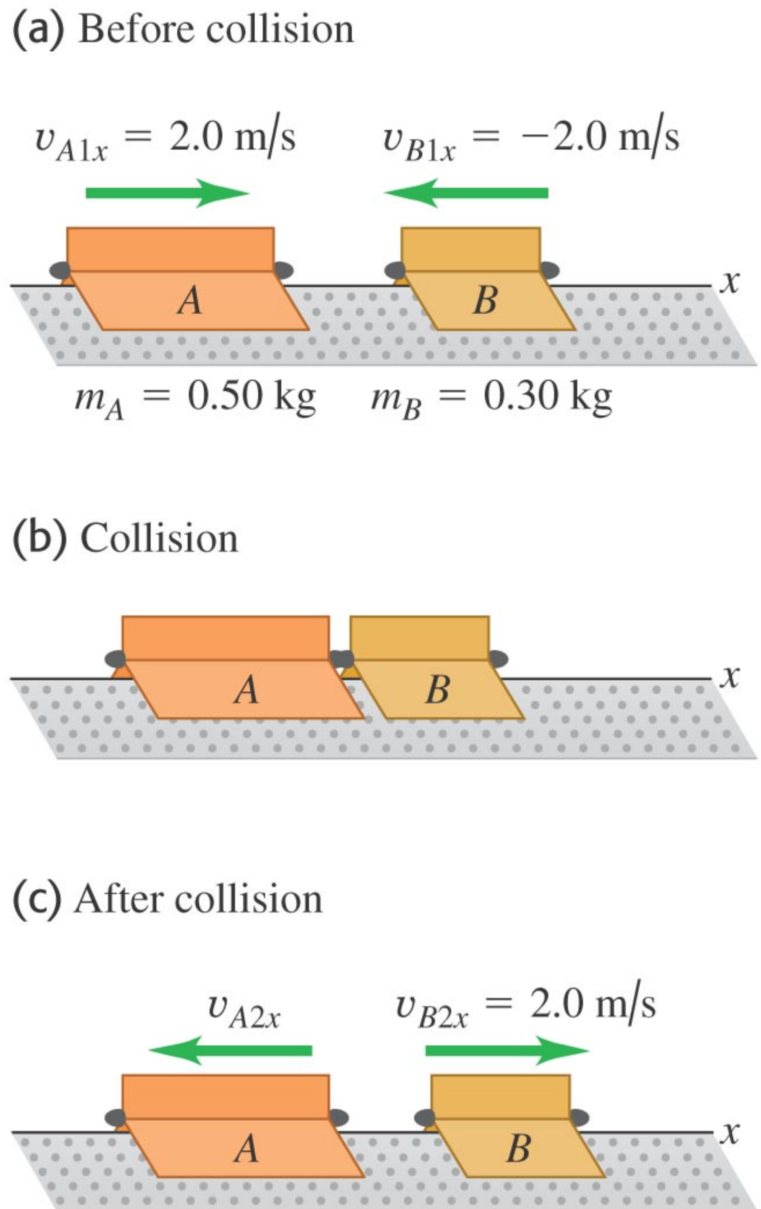
Objects colliding along a straight line

(use component notation)

$$x: \quad m_A \, v_{A1x} + m_B \, v_{B1x} = m_A \, v_{A2x} + m_B \, v_{B2x}$$

$$x: \quad (0.5)(2.0) - (0.3)(2) = (0.5) \, v_{A2x} + (0.3)(2)$$

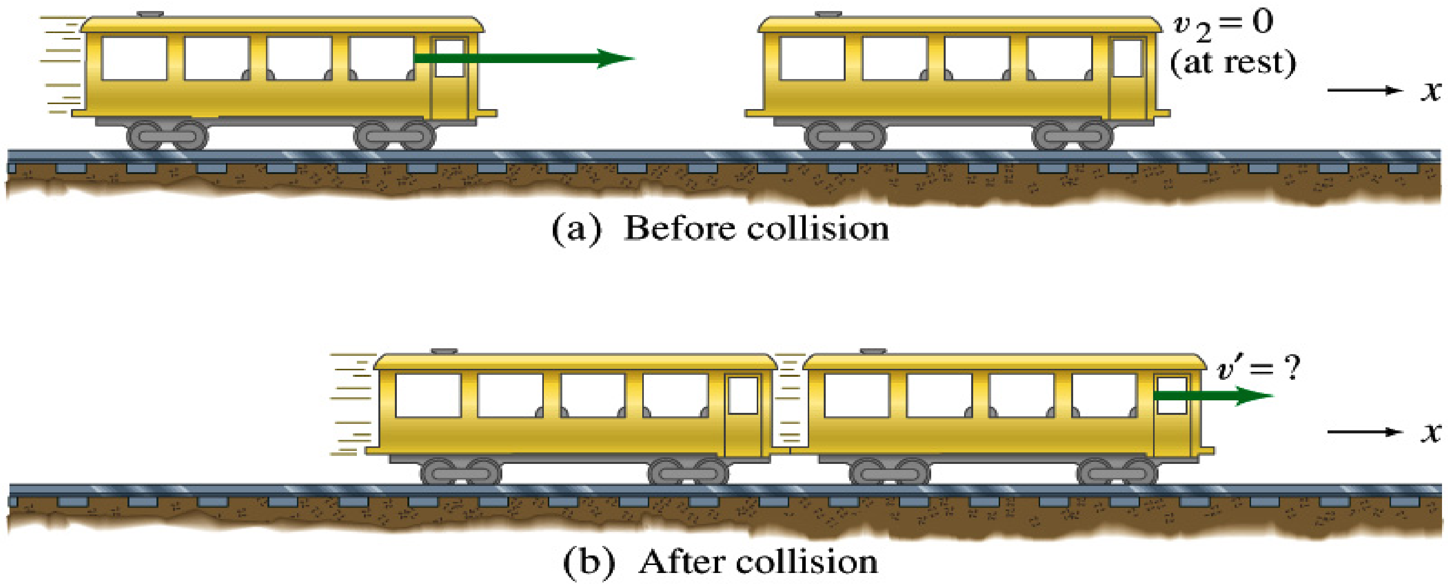
$$\Rightarrow v_{A2x} = -0.40 \text{ m/s}$$



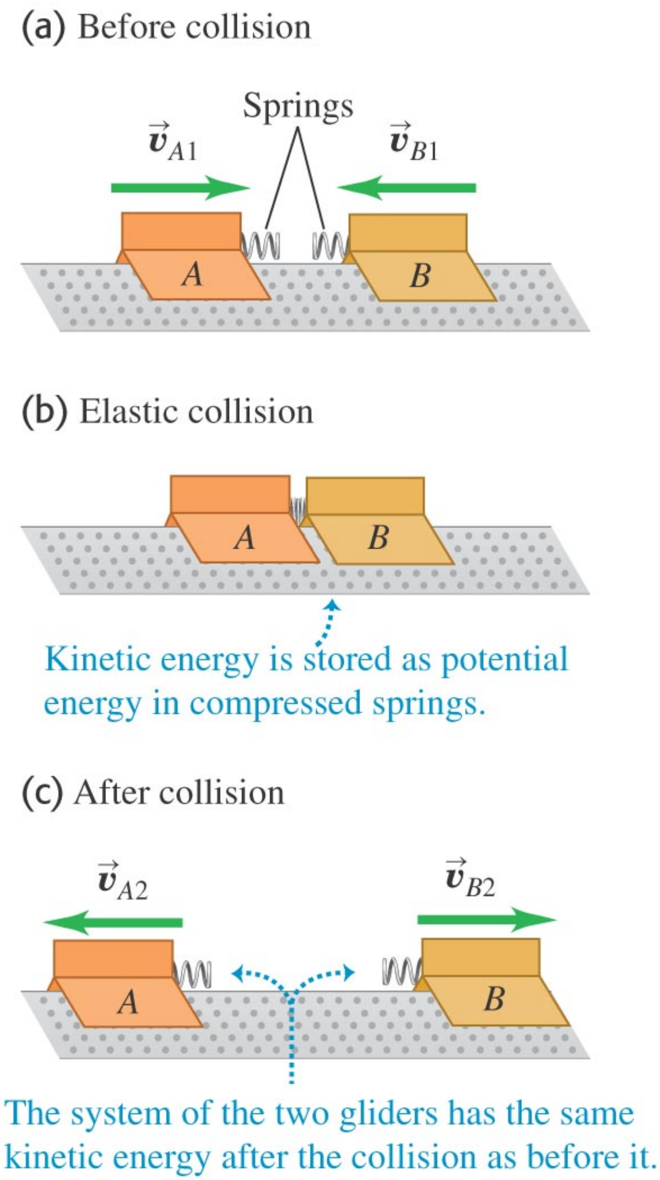
Colliding Trains: 1 Dimension

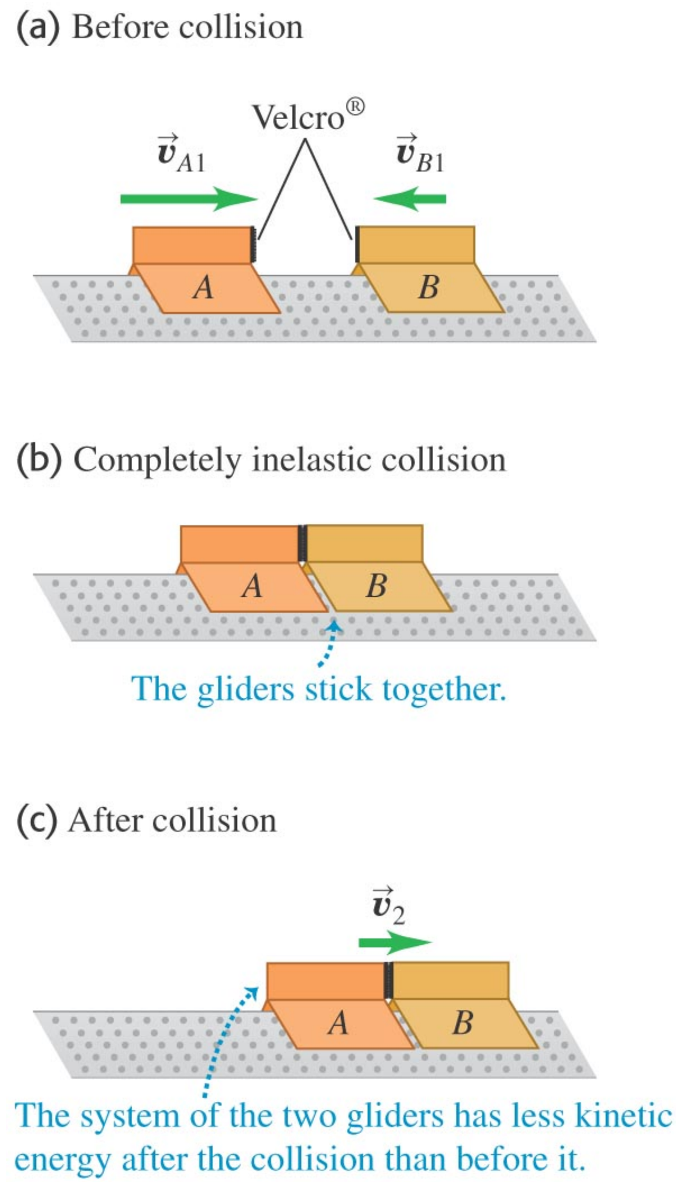
The train car on the left, mass m_1, is moving with speed V_o when it collides with a stationary car of mass m_2. The two stick together.

1. **What is their speed after the collision?**
2. **Show that this is inelastic**

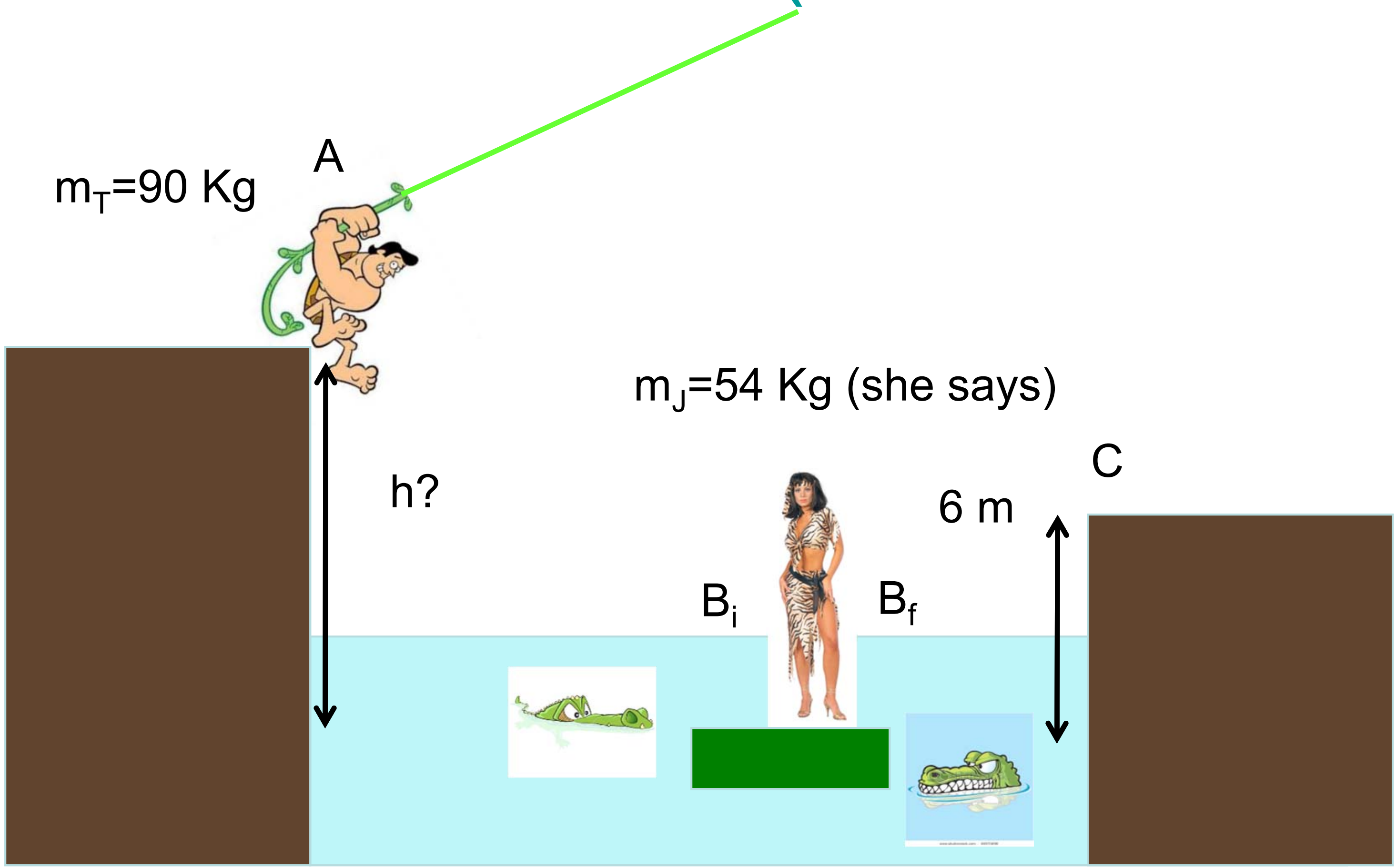


Elastic compared to inelastic





Tarzan and Jane (and the crocodile)



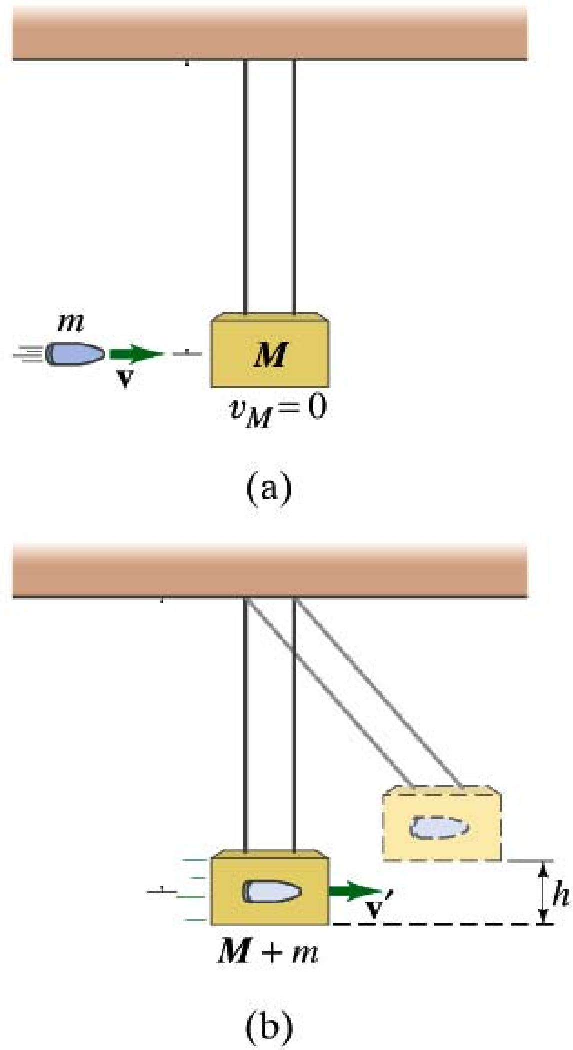
From A to B_i use conservation of energy

From B_i to B_f use conservation of momentum

From B_f to C use conservation of energy

Ballistic Pendulum

- A bullet of mass *m* and velocity V_o plows into a block of wood with mass *M* which is part of a pendulum.
  - How high, *h*, does the block of wood go?
  - Is the collision elastic or inelastic?



Two parts: 1-collision (momentum is conserved)
2-from low point (after collision) to high point: conservation of energy

1st part:

$$x:\quad m\text{v} + 0 = (M+m)\text{v}' \qquad \Rightarrow \qquad \text{v}' = \frac{m\text{v}}{(M+m)}$$
$$y:\quad 0+0 = 0+0$$

2nd part:

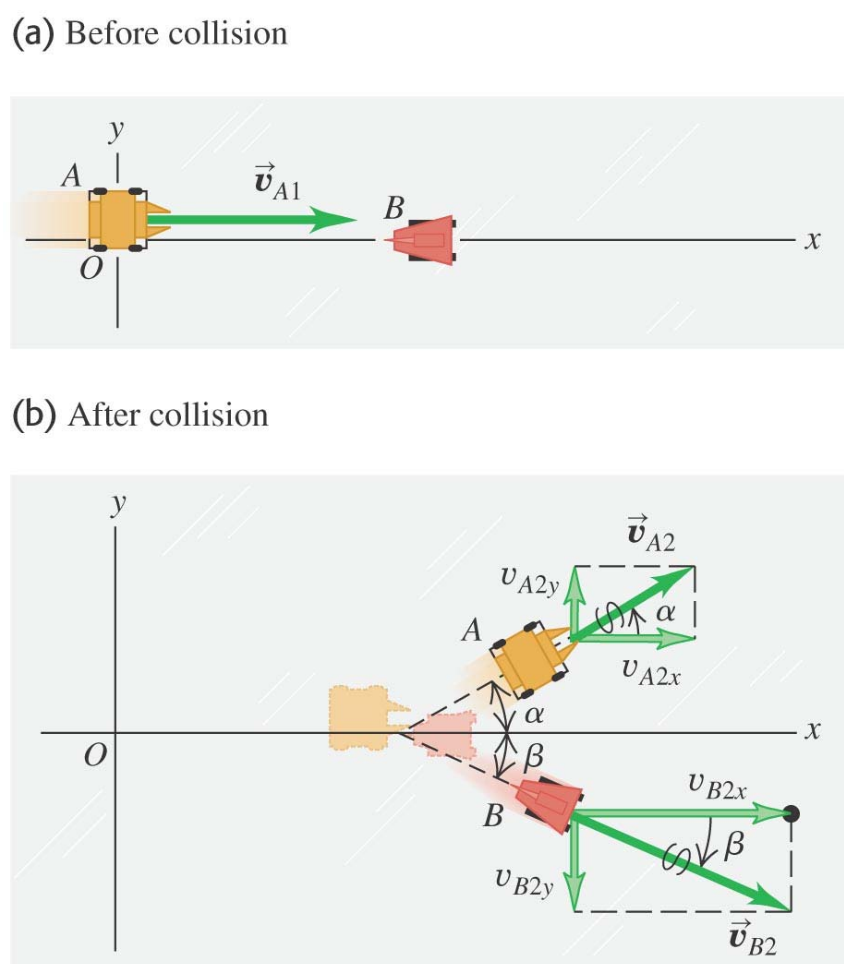
$$E_{bottom} = E_{top}$$

$$\frac{1}{2}(M+m)(\text{v}')^2 + 0 = 0 + (M+m)gh$$

$$\Rightarrow h = \frac{1}{2g}(\text{v}')^2 = \frac{m^2\text{v}^2}{2g(m+M)^2}$$

A two-dimensional collision

Robot A has a mass of 20 Kg, initially moves at 2.0 m/s parallel to the x-axis. After the collision with B, which has a mass of 12 Kg, robot A is moving at 1.0 m/s in a direction that makes and angle of 30 degrees. What is the final velocity of B?



Before After

$$x: \quad m_A v_{A1} + 0 \quad = \quad m_A v_{A2} \cos\alpha + m_B v_{B2-x}$$

$$y: \quad 0 + 0 \quad = \quad m_A v_{A2} \sin\alpha + m_B v_{B2-y}$$

$$x: \quad v_{B2-x} = \frac{m_A v_{A1} - m_A v_{A2} \cos\alpha}{m_B}$$

$$y: \quad v_{B2-y} = -\frac{m_A v_{A2} \sin\alpha}{m_B}$$

$$\Rightarrow \tan\beta = \frac{v_{B2-y}}{v_{B2-x}} = \frac{m_A v_{A2} \sin\alpha}{m_A v_{A1} - m_A v_{A2} \cos\alpha}$$

Center of Mass (CM)

What is the *"Center of Mass?"*

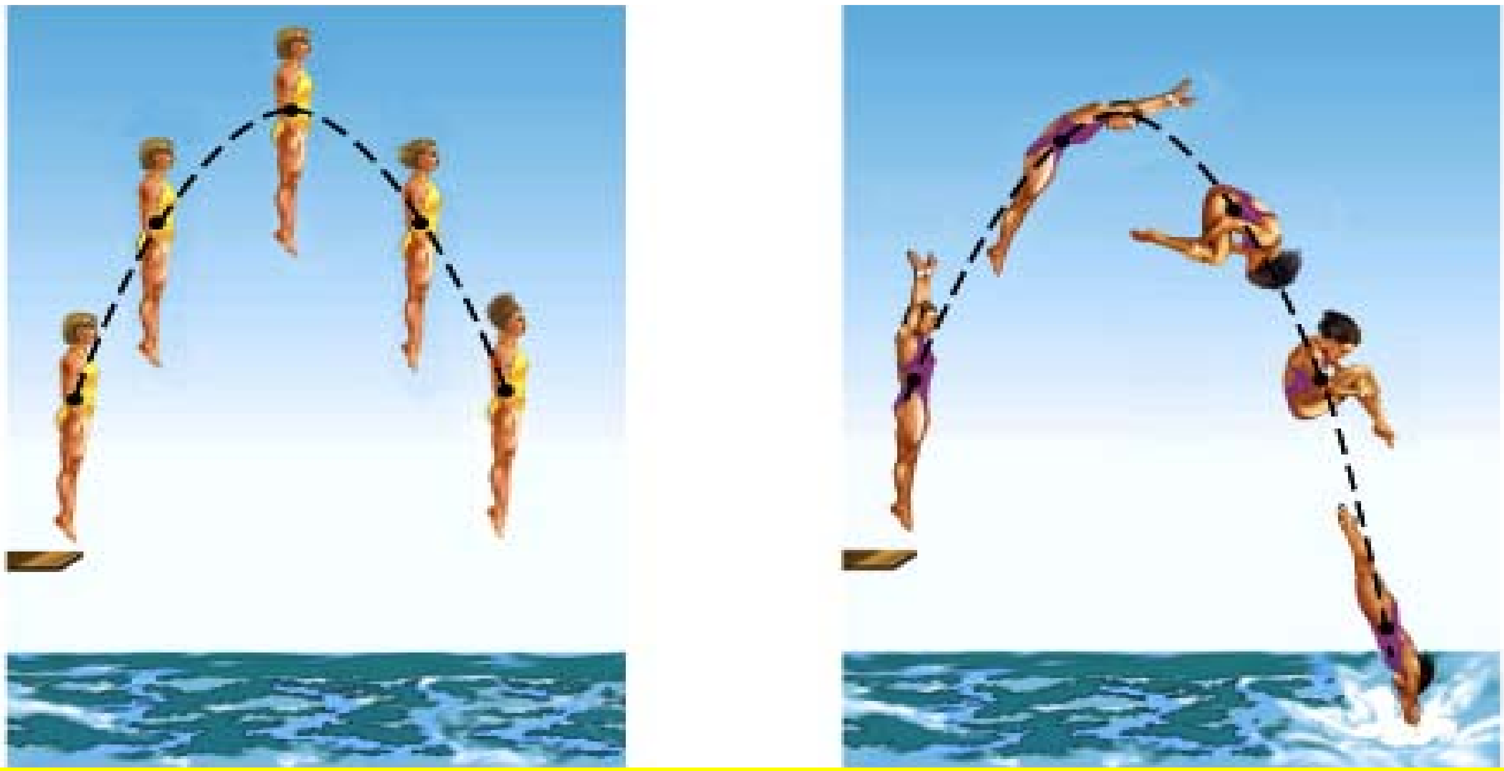
- More importantly *"Why do we care?"*
- This is a special point in space where *"it's as if the object could be replaced by all the mass at that one little point"*

Center of Mass (CM) Cont…

Examples where this is useful:

- We can model the earth moving around the sun as a single point at *“the center of the earth”*
- There is only one point on a stick that you can put your finger under and hold it up
- At some level we’ve been assuming these things for doing problems all semester

Visual Examples



The center of mass has the same trajectory in both cases since both have the same acceleration and initial velocity

How do you calculate CM?

1. Pick an origin
2. Look at each “piece of mass” and figure out how much mass it has and how far it is (vector displacement) from the origin. Take mass times position
3. Add them all up and divide out by the sum of the masses

The center of mass is a displacement vector *“relative to some origin”*

Spelling out the math:

$$\vec{X}_{\text{cm for 2 particles}} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

$$\vec{X}_{\text{cm for 3 particles}} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + m_3\vec{x}_3}{m_1 + m_2 + m_3} =$$

$$\frac{m_1\vec{x}_1 + m_2\vec{x}_2 + m_3\vec{x}_3}{M}$$

etc...

Note that $\vec{x}$ is the 3-D vector displacement

Simple Example

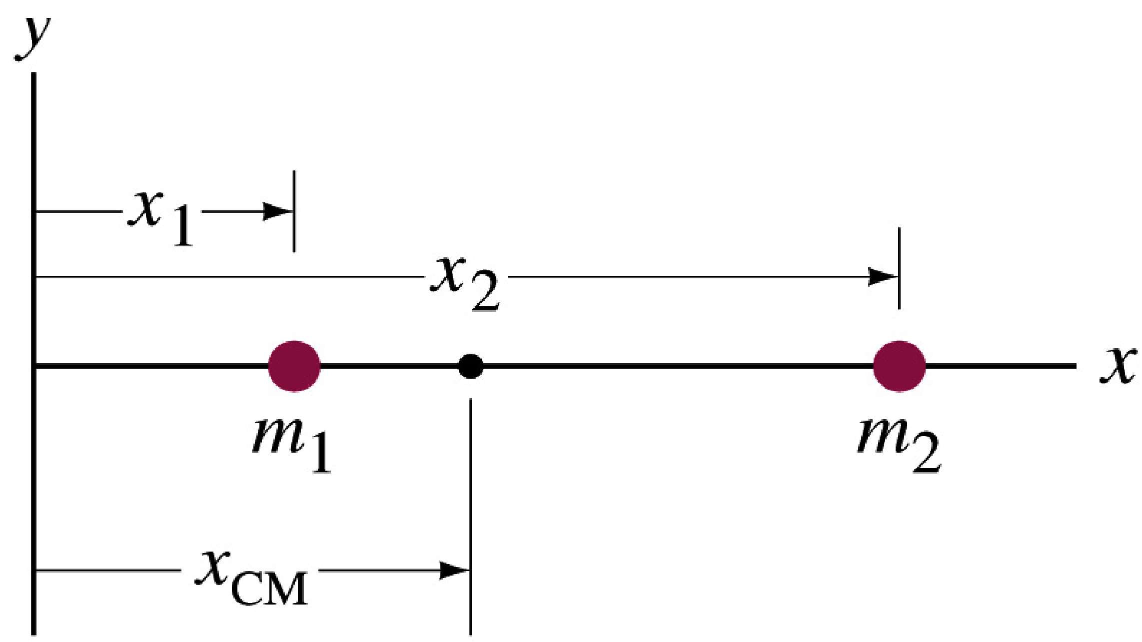
- **We are given two balls with masses m_1 and m_2 and an origin. The balls are placed at a distance x_1 and x_2 from the origin.**

Where is the center of mass if:

1. In general

2. $m_2 = m_1$?

3. $m_1 = 0$



How Does it Help?

- Remember equation for momentum of a system of objects? $$P = \sum_{i} m_i \vec{v}_i = \sum_{i} m_i' \vec{v}_i'$$
- Now I can re-write it as:

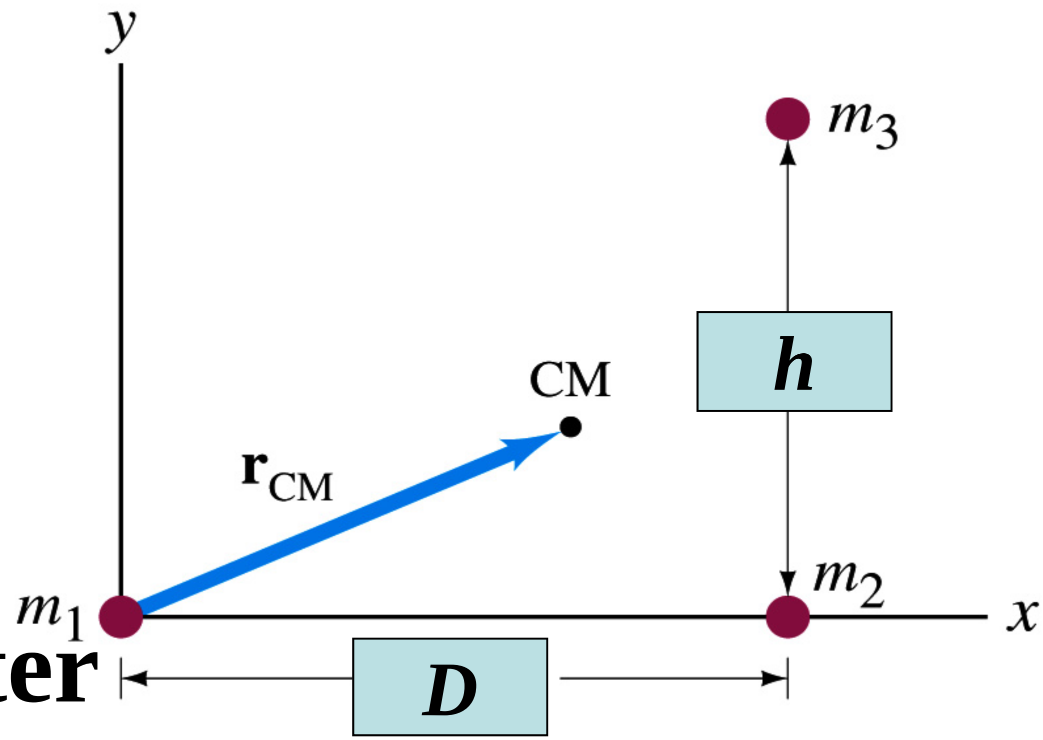
$$P = \sum_{i} m_i \vec{v}_i = \sum_{i} m_i \frac{d\vec{x}_i}{dt} = \sum_{i} \frac{d(m_i \vec{x}_i)}{dt} = \sum_{i} M \frac{d(m_i \vec{x}_i / M)}{dt} =$$

$$M \sum_{i} \frac{d(m_i \vec{x}_i / M)}{dt} = M \frac{d}{dt}(\sum_{i} m_i \vec{x}_i / M)) = M \frac{d}{dt} X_{CM} = M \vec{V}_{CM}$$

- I can write one equation for multi-component system and treat it as a single object, where e.g. momentum is perfectly conserved
  - Think of describing Solar system motion as a whole: do I want to calculate equations of motion for every planet in the system?

2-D Example

- **Three balls with masses m_1, m_2 and m_3 are are located at the points given to the right.**



Where is the center of mass?

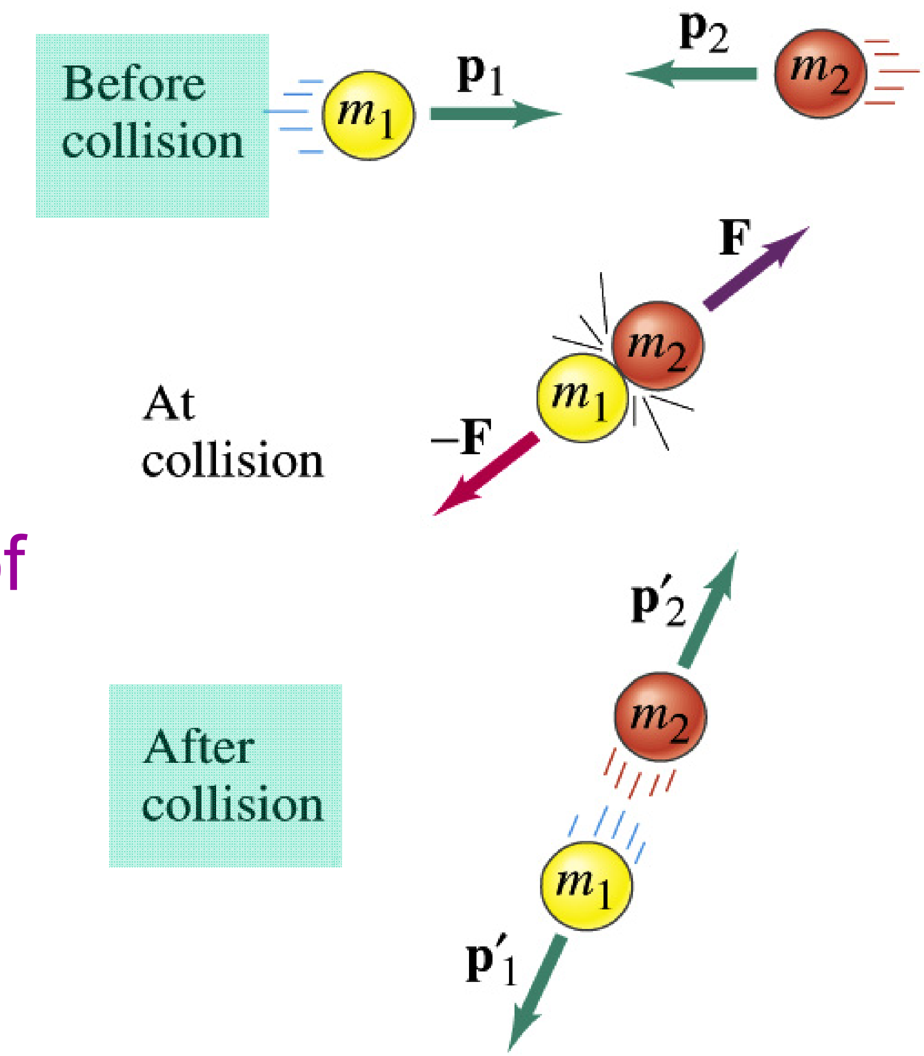
CM and Problems

Solving collision/explosion problems:

1. Conservation of Momentum in all directions
2. Watching the Center of Mass

Need to be able to do both

– Pick easier method



Toy Rocket Problem

Your friend fires a toy rocket into the air with an unknown velocity. You observe that at the peak of its trajectory it has traveled a distance *d* in the *x*-direction and that it breaks into two equal mass pieces. Part I falls straight down with no initial velocity.

Where does the 2nd half of the toy end up?