

PHYSICS 208 Exam 3/Final Exam: Spring 2008

Formula/Information Sheet

- Basic constants:

Gravitational acceleration	g	=	9.8 m/sec ²
Permittivity of free space	ϵ_0	=	$8.8542 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$
Coulomb constant	$k = 1/4\pi\epsilon_0$	=	$8.9875 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$ [$k_m = \mu_0/4\pi = 10^{-7} \text{ Wb}/\text{A}\cdot\text{m}$]
Elementary charge	e	=	$1.60 \times 10^{-19} \text{ C}$
Unit of energy: electron volt	1 eV	=	$1.60 \times 10^{-19} \text{ J}$
Unit of energy: kilowatt-hour	1 kWh	=	$3.6 \times 10^6 \text{ J}$
Planck's Constant	h	=	$6.626 \times 10^{-34} \text{ J sec}$

- Properties of some particles:

Proton	mass = $1.67 \times 10^{-27} \text{ kg}$	charge = $+1.60 \times 10^{-19} \text{ C}$
Electron	mass = $9.11 \times 10^{-31} \text{ kg}$	charge = $-1.60 \times 10^{-19} \text{ C}$
Neutron	mass = $1.67 \times 10^{-27} \text{ kg}$	charge = 0

- Some indefinite integrals:

$$\int \frac{dx}{x} = \ln x \quad \left| \quad \int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx) \right.$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2+a^2}} \quad \left| \quad \int \frac{x dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}} \right.$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) \quad \left| \quad \int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} \right.$$

- Basic equations for Electromagnetism:

Maxwell equations:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + \epsilon_0 \frac{d\Phi_E}{dt})$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{enclosed}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Basic Equations for Waves, Interference and Diffraction:

Wave Equation	$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f(x,t)}{\partial t^2}$
Plane EM wave traveling in the $+x$ direction	$E(x,t) = E_m \cos(kx - \omega t)$ $B(x,t) = B_m \cos(kx - \omega t)$
Speed of an EM wave [m/s]	$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_m}{B_m} = \frac{E(x,t)}{B(x,t)}$
Wave length of an EM wave [m]	$\lambda = \frac{c}{f}$
Wave number of an EM wave	$k = \frac{2\pi}{\lambda}$
Poynting vector [J/s·m ²]	$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$
Time-averaged S [J/s·m ²]	$S_{ave} = \frac{E_m B_m}{2\mu_0}$
Intensity of an EM wave [J/s·m ²]	$I = S_{ave}$
Total energy of an EM wave [J]	$U = I A t$
Total momentum of an EM wave	$ \vec{p} = \frac{U}{c}$
Law of Reflection	$\theta_{incident} = \theta_{reflected}$
Snell's Law	$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
Lens Equation	$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$
Lens Maker's Equation	$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$
Magnification	$m = \frac{y'}{y} = \frac{-s'}{s}$
Double Slit Constructive Int.	$d \sin(\theta) = m\lambda$
Double Slit Destructive Int.	$d \sin(\theta) = (m + \frac{1}{2})\lambda$
Intensity Maxima	$I = I_0 \cos^2(\phi/2)$ $\phi = \frac{2\pi}{\lambda} (r_2 - r_1)$
Energy of an EM Wave (photon)	$E = hf$
Single Slit Dest. Int.	$\sin(\theta) = \frac{m\lambda}{a}$

• Basic equations for Electric Fields:

Coulomb's law		$ \vec{F} = k \frac{ q_1 q_2 }{r^2}$
Electric field [N/C = V/m]	(point charge q)	$\vec{E}(r) = k \frac{q}{r^2} \hat{r}$
	(group of charges)	$(\hat{r} = \text{unit vector radially from } q)$ $\vec{E} = \sum \vec{E}_i = k \sum \frac{q_i}{r_i^2} \hat{r}_i$
	(continuous charge distribution)	$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$
Electric force [N]	(on q in \vec{E})	$(\hat{r} = \text{unit vector radially from } dq)$ $\vec{F} = q \vec{E}$

Electric flux	(through a small area ΔA_i)	$\Delta \Phi_i = \vec{E}_i \cdot \Delta \vec{A}_i = E_i \Delta A_i \cos \theta_i$
	(through an entire surface area)	$\Phi_{surface} = \lim_{\Delta A \rightarrow 0} \sum \Delta \Phi_i = \int \vec{E} \cdot d\vec{A}$
Gauss' law	(through a closed surface area)	$\Phi_{closed} \equiv \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$

Electric potential [V = J/C]	(definition)	$\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$
	($\vec{E} = \text{constant}$)	$\Delta V = -\vec{E} \cdot (\vec{r}_B - \vec{r}_A)$
	(point charge q)	$V(r) = k \frac{q}{r}$ (with $V(\infty) = 0$)
	(group of charges)	$V(\vec{r}) = \sum V_i(\vec{r} - \vec{r}_i) = k \sum \frac{q_i}{ \vec{r} - \vec{r}_i }$ $(V_i(\infty) = 0)$
	(continuous charge distribution)	$V(\vec{r}) = k \int \frac{dq'}{ \vec{r} - \vec{r}' }$ $(V(\infty) = 0)$
Electric potential energy [J]	(definition)	$\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$
\vec{E} from V		$\vec{E} = -\vec{\nabla} V,$
	where $\vec{\nabla} = \text{gradient operator can be expressed}$	$\hat{i}(\partial/\partial x) + \hat{j}(\partial/\partial y) + \hat{k}(\partial/\partial z)$
Electric potential energy of two-charge system		$U_{12} = k \frac{q_1 q_2}{r_{12}}$

Capacitance [F]	(definition)	$C \equiv \frac{Q}{ \Delta V }$
	(parallel-plate capacitance)	$C = K \frac{\epsilon_0 A}{d}$
Dielectrics in capacitors		$C = K C_o$
Electrostatic potential energy [J] stored in capacitance		$U_E(t) = \frac{1}{2} \frac{Q(t)^2}{C}$
Energy density [J/m ³] in an Electric field		$u = \frac{1}{2} \epsilon E^2$
Electric dipole moment ($2a = \text{separation between two charges}$)		$ \vec{p} = 2aq$
Torque on electric dipole moment		$\vec{\tau} = \vec{p} \times \vec{E}$
Potential energy of an electric dipole moment		$U = -\vec{p} \cdot \vec{E}$

- Basic equations for current and resistance:

Current [A]	(definition)	$I \equiv \frac{dQ(t)}{dt}$
	with motion of charges	$I = nqv_d A$
Current density [A/m ²]		$J = \frac{I}{A}$ (where $I = \int \vec{J} \cdot \vec{n} dA$)
Resistivity [$\Omega \cdot m$]		$\rho = \frac{ \vec{E} }{ \vec{J} }$
Resistance [Ω]	(definition)	$R \equiv \frac{V}{I}$
	for uniform cross-sectional area A	$R = \rho \frac{\ell}{A}$
Energy loss rate on R [J/s]		$P = I^2 R = V^2/R = IV$
Time constant in RC circuit [s]		$\tau_{RC} = RC$
Charging an RC circuit [q(t)]		$q(t) = q_f(1 - e^{-t/RC})$
Discharging an RC circuit [q(t)]		$q(t) = q_0 e^{-t/RC}$

- Basic equations for Magnetism and Induction:

Magnetic force [N]	on charge q	$\vec{F} = q \vec{v} \times \vec{B}$
	on current-carrying conductor	$\vec{F} = \int Id\vec{l} \times \vec{B}$
	or	$F = IlB \sin \theta$
cyclotron motion	cyclotron radius	$r = \frac{mv}{qB}$
	cyclotron frequency	$\omega = \frac{qB}{m}$
Magnetic moment [A·m ² or J/T]		$\vec{\mu} = NI\vec{A}$
Torque [N·m]	on a current loop	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Energy [J]	of a current loop	$U = -\vec{\mu} \cdot \vec{B}$

Field of moving charge		$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$
Ampere's law		$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
Biot-Savart law		$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$
Magnetic field [T]	a long straight wire	$ \vec{B} = \mu_0 I / (2\pi a)$
	inside a toroid	$ \vec{B} = \mu_0 NI / (2\pi r)$
	inside a solenoid	$ \vec{B} = \mu_0 NI / \ell$
	a straight wire segment	$ \vec{B} = k_m I (\cos \theta_1 - \cos \theta_2) / a$
	a circular arc (radius R)	$ \vec{B} = k_m I \theta / R$
Displacement current [A]	(definition)	$i_d \equiv \epsilon_0 \frac{d\Phi_E}{dt}$
Ampere-Maxwell law		$\oint \vec{B} \cdot d\vec{s} = \mu_0 (i_c + i_d)$

Faraday's Law		$\mathcal{E} = -\frac{d\Phi_m}{dt}$
Self Inductance [H]	(definition)	$L = \frac{N\Phi_m}{I}$
Self Induced electromotive force [V]		$\mathcal{E} = -L \frac{dI}{dt}$
Mutual Inductance [H]	(definition)	$M_{21} = N_2 \frac{\Phi_{21}}{I_1}$
Electromotive force induced by mutual induction [V]		$\mathcal{E}_2 = -M_{21} \frac{dI_1}{dt}$
Magnetic field energy density		$u_{magnetic} = \frac{1}{2} \mu_0 B^2$
Magnetic energy stored in L [J]		$U_B(t) = \frac{1}{2} LI(t)^2$
Time constant in LR circuits [s]		$\tau_{LR} = \frac{L}{R}$
Energizing an LR circuit [I(t)]		$I(t) = I_f(1 - e^{-tR/L})$
De-energizing an LR circuit [I(t)]		$I(t) = I_0 e^{-tR/L}$
Angular frequency of LC circuit [rad/sec]		$\omega = \sqrt{\frac{1}{LC}}$

- Some indefinite integrals:

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$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2+a^2}} \quad \left| \int \frac{x dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}} \right.$$

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