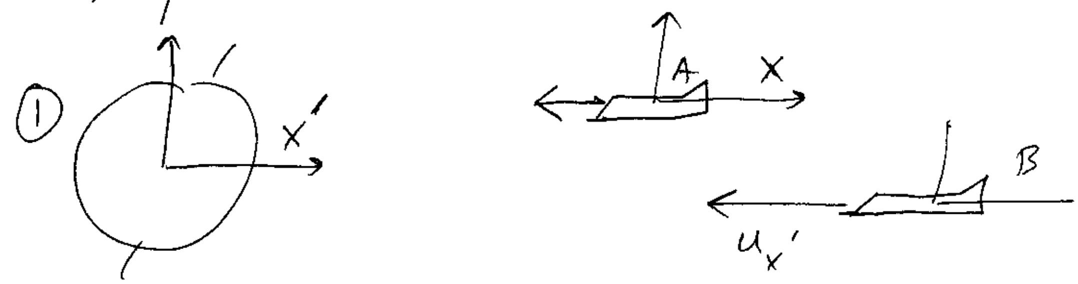
Phys 222 Homework 2 Solns

(1)



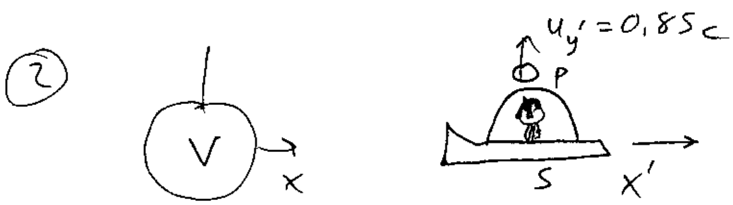
if Earth ≡ x' frame, A ≡ x frame, then
$v = +0.50c$ (boost of earth relative to A)
Also $u_{x'} = -0.80c$

hence $$u_x = \frac{(-0.80c) + (0.50c)}{1 + (0.50)(-0.80)} = \frac{-0.30c}{0.60}$$

$$u_x = -0.50c$$

Alternative: if A is x' frame, then v should be <u>negative</u>

(2)



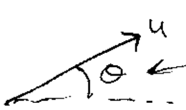
we make Spock = x' frame,
$u_x' = 0$ & $u_y' = 0.85c$

$$u_x = \frac{0 + 0.75c}{1 + 0} = +0.75c$$

with $v = +0.75c$,
$\gamma = (1 - (.75)^2)^{-1/2}$
$= 0.661^{-1} = 1.51$

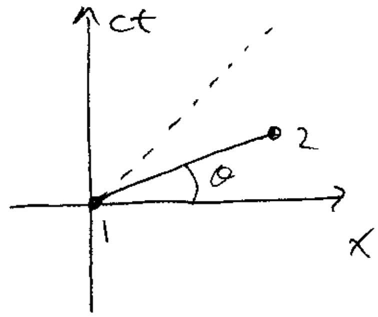
$$u_y = \frac{0.85c}{(1.51)(1 + 0)} = 0.56c$$

$|u| = (u_x^2 + u_y^2)^{1/2} = 0.94c$, at a forward angle,



$\tan\theta = \frac{0.56}{0.75} \Rightarrow 37° = \theta$

(3)



2 events are (x_1, ct_1) & (x_2, ct_2)

Now, we can always set our space time coordinates so that $x_1 = 0$ & $ct_1 = 0$, as shown;

Also we can rotate our space axes so that $y_2 = z_2 = 0$, so 2 is in the x-ct plane.

Spacelike means $\tan|\theta| < 1$, for the angle shown, so $\frac{|ct_2|}{|x_2|} < 1$, or $|ct_2| < |x_2|$

boosted frame, $x_1' = 0$ & $ct_1' = 0$ (still at origin);

$$x_2' = \gamma(x_2 - \beta ct_2)$$

$$ct_2' = \gamma(ct_2 - \beta x_2)$$

for same place we need $x_2' = 0$ [e.g. $x_2' = x_1'$];
that means $x_2 = \beta ct_2$

Now, β may be positive or negative but $|\beta| < 1$ always. In this case since $|ct_2| < |x_2|$, $|\beta ct_2|$ will always be less than $|x_2|$, so this will not work.

(4) a) $f = \sqrt{\dfrac{c+v}{c-v}}\, f_0$ for ~~light~~ object moving towards observer,

so here must have $v < 0$ (moving away)

here $\dfrac{450}{550} = \sqrt{\dfrac{c - |v|}{c + |v|}} = 0.818$

$$\frac{c-|v|}{c+|v|} = 0.669 \rightarrow |v| = c \times \frac{(1-0.669)}{1+0.669}$$
$$= 0.20\,c$$

(b) here $\dfrac{c+v}{c-v} = \left(\dfrac{700}{550}\right)^2 = 1.62$

$$v = \frac{1.62 - 1}{1.62 + 1} = 0.24\,c$$ moving ~~away~~ towards

(5) 

As shown in class, momenta are equal & opposite,

$$|p_0| = |p_+| \equiv p$$

also energy conservation, $765\,\text{MeV} = \underbrace{E^0}_{(p^2c^2 + (135\,\text{MeV})^2)^{1/2}} + E^+$

or, $(756\,\text{MeV} - E^+)^2 = (E^0)^2$

$$(756\,\text{MeV})^2 - 2E^+(756\,\text{MeV}) + (\cancel{p^2c^2} + (140\,\text{MeV})^2) =$$
$$\cancel{p^2c^2} + (135\,\text{MeV})^2$$

$$E^+ = \frac{1}{2 \times 756\,\text{MeV}}\left[(756)^2 + (140)^2 - (135)^2\right]\text{MeV}^2 \cong 383.4\,\text{MeV}$$

$$p^2c^2 + (140\,\text{MeV})^2 = (383.4\,\text{MeV})^2$$

$$\rightarrow pc = 357\,\text{MeV}$$

(6) Total energy is $\gamma m_0 c^2$, so :

$$15\text{ GeV} = \gamma \times 2.4\text{ GeV} \rightarrow \gamma = 6.25$$

so $1 - \frac{v^2}{c^2} = 1/6.25^2 = 0.0256$,

and $\frac{v}{c} = 0.987$

so to travel 4 ly the time will be very close to 4 years:

$$t = \frac{(4\text{ ly})}{0.987c} = 4.05\text{ y}$$

(7) Mass transform is $m = \gamma m_0$

m_0: 142 g rest mass

$v = 0.5c \rightarrow \gamma = (1 - 1/4)^{-1/2} = 1.15$

so $m = 164$ g

(8) p + d rest energies: $m_p c^2 + m_d c^2 = (m_p + m_d)c^2$

after, $\gamma = 5.5\text{ MeV} = pc$ (as we saw for photons)

but this is same as $|p|$ for ^3He (conservation of $\vec{p}$)

$$\text{so } \underbrace{(m_p + m_d)c^2}_{4.5_{08}\times 10^{-10}\,J = 2813.5\text{ MeV}} = \underbrace{5.5\text{ MeV}}_{\gamma} + (\underbrace{p^2c^2 + m_0^2 c^4}_{^3He,\text{ where } p = 5.5\text{ MeV}/c \text{ as above}})^{1/2}$$

4.5078

so $p^2c^2 + m_0^2c^4 = (2813.5 - 5.5\text{ MeV})^2 = 7.8851 \times 10^6\text{ MeV}^2$

p^2c^2: $(5.5\text{ MeV})^2 \leftarrow$ negligible

so $m_0 c^2 = (7.885 \times 10^6\text{ MeV}^2)^{1/2} = 2808\text{ MeV}$, $m_0 = 5.006 \times 10^{-27}$ kg