1 Solution to Homework 4

1.The energy of the photon is

$$E = hf = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{500 \times 10^{-9} m} = 3.9756 \times 10^{-19} J = 2.48ev \quad (1)$$

The momentum is given by

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} J.s}{500 \times 10^{-9} m} = 1.32 \times 10^{-27} kg.m.s^{-1} \quad (2)$$

2. (a) The energy of the electrons is

$$E = 300Kev = 300 \times 10^3 \times 1.6 \times 10^{-19} J = 4.8 \times 10^{-14} J \quad (3)$$

The wavelength of the X ray is

$$\lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{4.8 \times 10^{-14} J} = 4.141 \times 10^{-12} m \quad (4)$$

Accoridng to the Compton scattering formula, the shift can be calculated as

$$\begin{aligned} \Delta\lambda = \frac{h}{mc}(1 - \cos\theta) &= \frac{6.626 \times 10^{-34} J.s}{9.11 \times 10^{-31} kg \times 3 \times 10^8 ms^{-1}}(1 - \cos 30) \\ &= 2.426 \times 10^{-12} m \times 0.134 = 3.25 \times 10^{-13} m \end{aligned} \quad (5)$$

(b) After the collision, the wavelength becomes

$$\lambda' = \lambda + \Delta\lambda = 2 \times 10^{-11} m + 3.25 \times 10^{-13} m = 4.466 \times 10^{-12} m \quad (6)$$

and the energy is

$$\begin{aligned} E_f = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{4.466 \times 10^{-12} m} &= 4.451 \times 10^{-14} j \\ = \frac{4.451 \times 10^{-14} j}{1.6 \times 10^{-19}} ev &= 2.78 \times 10^5 ev \end{aligned} \quad (7)$$

(c) The kinetic energy is given by the difference of the X ray energy

$$E_k = E - E_f = 300kev - 278kev = 22kev \quad (8)$$

3.(a).The minimum wavelength occur when all the initial kinetic energy is used to produce a single photon:

$$eV = hf = \frac{hc}{\lambda} \quad (9)$$

Therefore,

$$\lambda = \frac{hc}{eV} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{1.6 \times 10^{-19} C \times 6 \times 10^4 V} = 2.07 \times 10^{-11} m \tag{10}$$

(b)According the conclusion of part (a), each photon carries the energy $E_0 = 6 \times 10^4 ev$, and the total energy of the electron beam per second is

$$E_t = 500 \times 10^3 J = \frac{5 \times 10^5}{1.6 \times 10^{-19}} ev = 3.125 \times 10^{24} ev \tag{11}$$

And the number of the photons are

$$n = \frac{E_t}{E_0} = \frac{3.125 \times 10^{24} ev}{6 \times 10^4 ev} = 5.2 \times 10^{19} \tag{12}$$

4. For head on collisions between alpha particles and the nucleus, all the kinetic energy ($E = \frac{1}{2}mv^2$) of the alpha particle i s turned into potential energy and the particle is at rest. The distance from the center of the alpha particle to the centre of the nucleus (b) at this point is a maximum value for the radius, if it is evident from the experiment that the particles have not hit the nucleus. Applying the inverse-square law between the charges on the electron and nucleus, one can write:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{b} \tag{13}$$

Rearranging:

$$b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{E} \tag{14}$$

Put $E = 13.9 Mev$,$q_1 = 2 \times (1.6 \times 10^{-19}) C$, $q_2 = 29 \times (1.6 \times 10^{-19}) C$,$\frac{1}{4\pi\epsilon} = 9 \times 10^9 N.m^2 C^{-2}$,

$$b = 9 \times 10^9 N.m^2 C^{-2} \frac{2 \times (1.6 \times 10^{-19}) C \times 29 \times (1.6 \times 10^{-19}) C}{13.9 \times 10^6 \times 1.6 \times 10^{-19} J} = 6.0 \times 10^{-15} m \tag{15}$$

5. (a)The energy of the photon is

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{15 \times 10^{-9} m} = 1.325 \times 10^{-17} J = \frac{1.325 \times 10^{-17} J}{1.6 \times 10^{-19}} ev = 82.8 ev \tag{16}$$

Since the photon energy is bigger than the ground state energy of the hydrogen atom $-13.6 ev$, the atom can be ionized.

(b)The kinetic energy is given by

$$E_k = E - E_0 = 82.8 ev - 13.6 ev = 69.2 ev \tag{17}$$

6. First we can calculate the energy range between 400nm and 650nm:

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{400 \times 10^{-9} m} = 4.97 \times 10^{-19} J = 3.1 ev \tag{18}$$

$$E_2 = \frac{hc}{\lambda_2} = \frac{6.626 \times 10^{-34} J.s \times 3 \times 10^8 ms^{-1}}{650 \times 10^{-9} m} = 1.9ev \tag{19}$$

The energy level of the hydrogen atom is $-13.6ev, -3.4ev, -1.51ev, 0.85ev, ...$; To emit photon with wavelength between 400nm and 650nm, the energy should lie between 3.1ev and 1.9ev, so it is only possible for the transition from $n = 3$ to $n = 2$, the energy difference is 1.89ev, which is not in the energy range, so it is impossible to get the desired wave

7.(a)We can use the Moseley's empirical formula to calculate the K_α line

$$\frac{hc}{\lambda_\alpha} = \frac{3}{4} 13.6(Z-1)^2 ev \tag{20}$$

For the Cu atom, $Z = 29$, we can solve the wavelength, which is simply

$$\lambda_\alpha = 0.15497nm \tag{21}$$

The K_β line is also calculated by this empirical formula

$$\frac{hc}{\lambda_\beta} = 13.6(\frac{1}{1^2} - \frac{1}{3^2})(Z-1)^2 ev = \frac{8}{9} 13.6(Z-1)^2 \tag{22}$$

The wavelength is then

$$K_\beta = 0.1307nm \tag{23}$$

(b) For the N_b atom, $Z = 41$, The K_α line is also calculated by the Moseley's formula

$$\lambda_\alpha = 0.07593nm \tag{24}$$

and

$$\lambda_\beta = 0.06406nm \tag{25}$$