Exam 1, Physics 208 6/17/05 **Name:** Key

Dr.Saslow, Summer 2005 **Section:** /

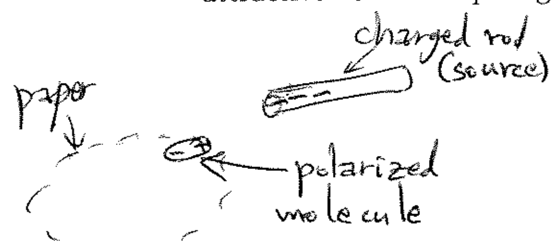
**Don't waste time on questions you aren't sure of. Be clear and concise. A cluttered response, some of which is correct and some of which is incorrect, will not get full credit.**

1. (10 pts) A non-conducting rod lies on the $y$-axis from $(0,0,0)$ to $(0,b,0)$, where $b$ is a constant. It has charge per unit length $\lambda = 8\beta \cancel{z}y^3$, where $\beta$ is a constant. What units must $b$ and $\beta$ have? In terms of $\beta$ and $b$, find the total charge $Q$ on the rod, and the average charge per unit length $\bar{\lambda}$. $(z \to y)$

"units of" $\to [b]$ is meters, $[\beta] = \dfrac{[\lambda]}{(\text{meter})^3} = \dfrac{C/m}{m^3} = \boxed{\dfrac{C}{m^4}}$

$$Q = \int dQ = \int \frac{dQ}{ds} ds = \int_0^b \lambda\, dy = \int_0^b (8\beta y^3) dy = 8\beta \tfrac{1}{4} y^4 \Big|_0^b$$
$$= \boxed{2\beta b^4}$$
$$\bar{\lambda} = \frac{Q}{\ell} = \frac{2\beta b^4}{b} = \boxed{2\beta b^3}$$

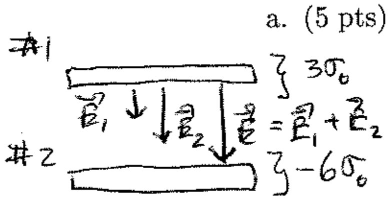
2. (10 pts) For the benefit of Bart Simpson's sixth grade teacher, concisely explain the physics of the "amber effect" (between a charged and a neutral object) and why it is attractive. Use a simple figure, with a negative source charge.




1. The charge on the charged object (source) polarizes the molecules of the neutral object
2. The nearer side of the polarized molecule has opposite charge to that of the source by opposites attract, likes repel.
3. Since the electrical force falls off with distance attraction of opposites dominates

3. Assume that the charged conducting sheets in the figure are infinite in extent. The one on the top has total charge per unit area $3\sigma_0$, and the one on the bottom has a total charge per unit area $-6\sigma_0$, where $\sigma_0 > 0$.

   a. (5 pts) Find the total electric field (magnitude and direction) between the plates.




$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2\pi k(3\sigma_0)\hat{\downarrow} + 2\pi k(-6\sigma_0)\hat{\uparrow}$$
$$= 6\pi k\sigma_0 \hat{\downarrow} + 12\pi k\sigma_0 \hat{\downarrow}$$
$$= 18\pi k\sigma_0 \hat{\downarrow}$$

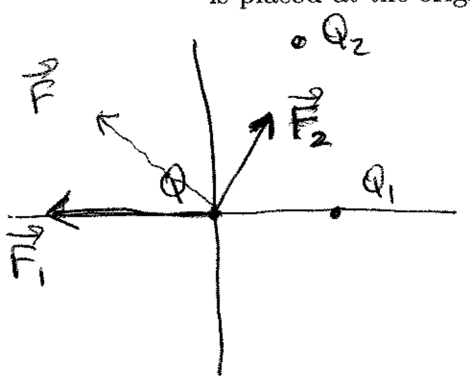
   b. (5 pts) Find the charge density on the top surface of the bottom sheet.

For a conductor in equilibrium $\Big\}$ $\vec{E}_{out}\cdot\hat{n} = 4\pi k\sigma_s$

Here, $\hat{n} = \hat{\uparrow}$, so $\vec{E}_{out} = 18\pi k\sigma_0\hat{\downarrow}$

$$\sigma_s = \frac{1}{4\pi k}\left(18\pi k\sigma_0 \overbrace{\hat{\downarrow})\cdot\hat{\uparrow}}^{=-1} = -\frac{9}{2}\sigma_0$$

4. (35 pts) A point charge $Q_1 = 12.0 \times 10^{-9}$ C is on the positive ~~$y$~~ $x$-axis at $r_1$=2 cm from the origin. A point charge $Q_2 = -18.0 \times 10^{-9}$ C makes a counterclockwise angle $\theta = 60°$ to the positive $x$-axis, at $r_2 = 3$ cm from the origin. A charge $Q = 5.0 \times 10^{-9}$ C is placed at the origin. $Q_1$ and $Q_2$ act on $Q$ with forces $\vec{F}_1$ and $\vec{F}_2$. $y \to x$




a. Find $|\vec{F}_1|$ and $|\vec{F}_2|$.

$$|\vec{F}_1| = \frac{k|QQ_1|}{r_1^2} = \frac{\left(9\times10^9 \frac{N\text{-}m^2}{C^2}\right)\cdot(5\times10^{-9}C)(12\times10^{-9}C)}{(0.02m)^2}$$
$$= 135\times10^{-5}\ N$$
$$|\vec{F}_2| = \frac{k|QQ_2|}{r_2^2} = \frac{(9\times10^9)(5\times10^{-9})(18\times10^{-9})}{(0.03)^2}$$
$$= 90\times10^{-5}\ N \qquad \text{Note: } |\vec{F}_1| = 1.5\,|\vec{F}_2|$$

b. On the figure, draw $\vec{F}_1$ and $\vec{F}_2$ with their tails on $Q$, and in relative proportion.

c. Find $F_x$, the $x$-component of the total force $\vec{F}$ on $Q$.

$$F_x = F_{1x} + F_{2x} = -|\vec{F}_1| + |\vec{F}_2|\cos 60° = -135\times10^{-5} + 90\times10^{-5}\times0.5 = -90\times10^{-5}\ N$$

$$F_y = F_{1y} + F_{2y} = 0 + |\vec{F}_2|\sin 60° = 90\times10^{-5}\times\frac{\sqrt{3}}{2} = 77.9\times10^{-5}\ N$$

d. Find $F_y$, the $y$-component of $\vec{F}$.

e. Find the angle $\vec{F}$ makes with respect to the $x$-axis, and on the figure sketch the direction of $\vec{F}$. Angle is in 2nd quadrant

$$\tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{77.9}{-90}\right) = -40.9°, \quad \theta = -40.9 + 180 = 139.1°$$

f. Find $|\vec{F}|$.

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(-90\times10^{-5}N)^2 + (77.9\times10^{-5}N)^2} = 119.0\times10^{-5}\ N$$

g. $Q_1$ and $Q_2$ are rotated clockwise by 15 degrees about the origin. Find the new $F_x$.

$$(F_x)^{new} = |\vec{F}|^{new}(\cos\theta)^{new} = |\vec{F}|^{old}\cos(139.1-15)$$
$$= 119.\times10^{-5}\cos(124.1) = -66.7\times10^{-5}\ N$$

5. A charge $Q_1 = -8.0 \times 10^{-9}$ C is on the $y$-axis, 6 cm above an infinitely long line charge $\lambda > 0$ along the $x$ axis. The force on $Q_1$ has magnitude $9.6 \times 10^{-6}$ N.

$Q_1$ •

a. (10 pts) Find the electric field, in magnitude and direction.

$\lambda > 0$ $\quad \vec{E}$ is $\boxed{\uparrow}$, since $\lambda > 0$.

$$|\vec{E}| = \frac{|\vec{F}|}{|Q_1|} = \frac{9.6 \times 10^{-6} N}{8 \times 10^{-9} C} = \boxed{1.2 \times 10^{3} \frac{N}{C}}$$

b. (5 pts) If $Q_1$ is moved to a distance of only 2 cm from $\lambda$, find the magnitude of the force on $Q_1$.

$|\vec{E}| = \frac{2k\lambda}{r}$. Since $r$ goes down by a factor of 3 (from 6cm to 2cm), $|\vec{E}|$ goes up by a factor of 3, to $3(1.2 \times 10^3 \frac{N}{C}) = 3.6 \times 10^3 \frac{N}{C}$. $\therefore F = 3 \times 9.6 \times 10^{-6} N = 2.8 \times 10^{-5} N$

c. (5 pts) Find $\lambda$.

$$1.2 \times 10^3 \frac{N}{C} = \frac{2(9 \times 10^9 N\text{-}m^2/C^2)\lambda}{(.06 m)} \Rightarrow \lambda = \frac{(1.2 \times 10^3)(0.06)}{2(9 \times 10^9)} \frac{C}{m} = \boxed{4 \times 10^{-9} \frac{C}{m}}$$

6. Two line charges are normal to the page. A, with charge density $6\lambda$, passes through the origin. B, with charge density $3\lambda$, passes through $(3b, 0, 0)$.

a. (8 pts) Find the position $(s, 0, 0)$ where the electric field is zero.

At that point, $|\vec{E}_A| = |\vec{E}_B|$, or $\frac{2k(6\lambda)}{s} = \frac{2k(3\lambda)}{(3b-s)}$,

so $2(3b-s) = s$, or $6b - 2s = s$, $6b = 3s$, $\boxed{s = 2b}$

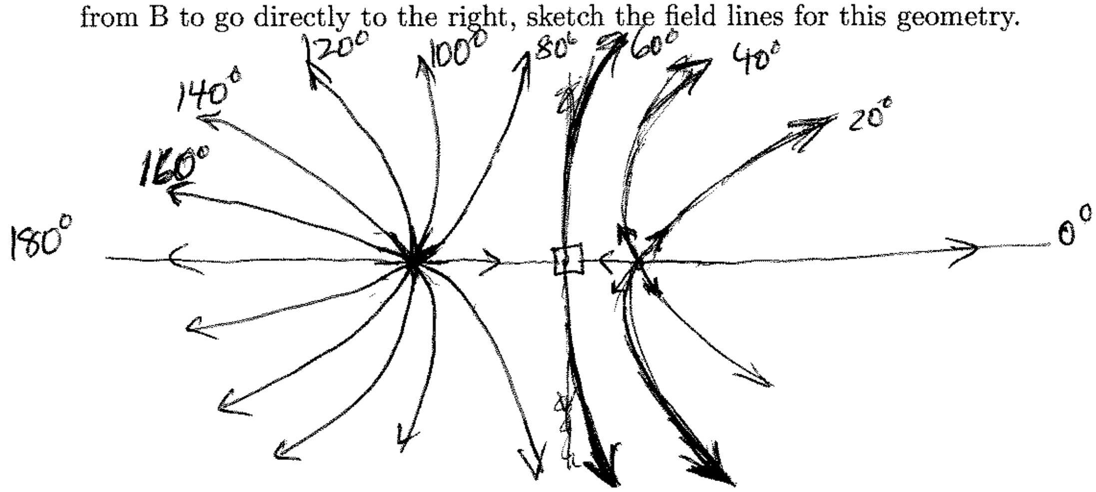
b. (7 pts) If $\lambda$ is represented by two field lines, find the angle between the field lines as they originate from A. Repeat for B. Repeat for the angle between the field lines as viewed from far away from both.

A has $6\lambda$, or 12 lines, at $\frac{360}{12} = 30^\circ$

B has $3\lambda$, or 6 lines, at $\frac{360}{6} = 60^\circ$

Together they have 18, at $\frac{360}{18} = 20^\circ$

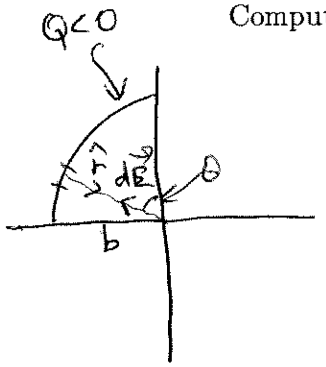
c. (10 pts) Taking one field line from A to go directly to the left, and one field line from B to go directly to the right, sketch the field lines for this geometry.






$\underbrace{A(6\lambda) \qquad B(3\lambda)}_{3b}$

7. (15 pts) A negative charge ($Q < 0$) is uniformly distributed over a quarter-ring of radius $b$, in the second quadrant. Indicate the direction of the field at the origin. Compute $E_x$ at the origin. Net $\vec{E}$ is along ↖




By symmetry, $|E_x| = |E_y|$, so let's compute $E_y$ ($>0$).

$$|d\vec{E}| = \frac{k\,dQ}{r^2} = \frac{k\lambda\,ds}{b^2} = \frac{k\lambda(b\,d\theta)}{b^2} = \frac{k\lambda}{b}d\theta$$

$$dE_y = |d\vec{E}|\cos\theta = \frac{k\lambda}{b}\cos\theta\,d\theta$$

$$E_y = \frac{k\lambda}{b}\int_0^{\pi/2}\cos\theta\,d\theta = \frac{k\lambda}{b}\sin\theta\Big|_0^{\pi/2} = \frac{k\lambda}{b} = \frac{k}{b}\left(\frac{Q}{\frac{1}{4}(2\pi b)}\right) = \frac{2kQ}{\pi b^2}$$

$$E_x = -E_y = -\frac{2kQ}{\pi b^2}, \quad |\vec{E}| = \frac{2\sqrt{2}\,kQ}{\pi b^2} \quad \left(\text{by } \sqrt{E_x^2 + E_y^2}\right)$$

8. For a negatively-charged conductor, a surface element of area $dA = 2.8 \times 10^{-7}$ m$^2$ has its outward normal $\hat{n}$ along (3,4.5,–8). For this element, $|\vec{E}| = 160$ V/m.

a. (5 pts) Find $\hat{n}$. $\sqrt{3^2 + (4.5)^2 + (-8)^2} = 9.656$

$$\hat{n} = \frac{(3, 4.5, -8)}{9.656} = (0.310,\ 0.466,\ -0.828)$$

b. (5 pts) Find the direction of $\vec{E}$, called $\hat{E}$.

$\hat{E} = -\hat{n}$ (field lines go to negatively charged conductor)

c. (5 pts) Find the flux $d\Phi_E$ through $dA$.

$$d\Phi_E = \vec{E}\cdot\hat{n}\,dA = |\vec{E}|((-\hat{n})\cdot\hat{n})\,dA = -|\vec{E}|\,dA = -160\,\frac{V}{m}\left(\frac{N}{C}\right) \times 2.8\times10^{-7}$$

$$= -4.48\times10^{-5}\ \text{V-m}$$

d. (5 pts) Find the surface charge $dQ_S$.

$$d\Phi_E = 4\pi k\,dQ_S, \quad dQ_S = \frac{d\Phi_E}{4\pi k} = \frac{-4.48\times10^{-5}\ \text{V-m}}{4\pi\times9\times10^{9}\ \text{N-m}^2/\text{C}^2} = -4.98\times10^{-15}\ \text{C}$$

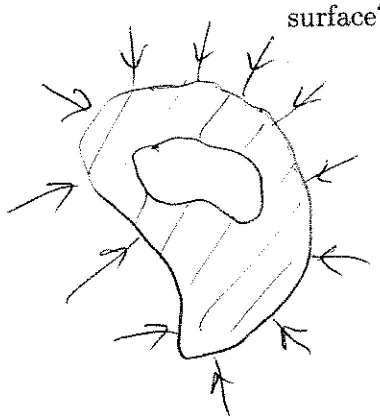
$= -3.9\times10^{-6}$ C

9. Consider a conductor with a cavity. The total charge on the conductor itself is $-2$ units. If we associate 4 field lines with each unit of charge, then when viewed from afar there is a net of 12 field lines pointing toward the conductor, which is in equilibrium.

a. (5 pts) How much charge is within the cavity?

12 lines toward ⇒ $-3$ net charge on conductor ($-2$) + cavity,
so $Q_{cavity} = -1$

b. (10 pts) How much charge is on the inner surface of the conductor? On the outer surface? In the bulk?



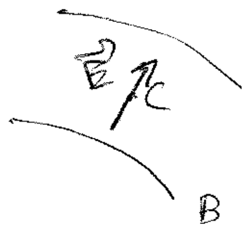
$Q_{inner} = -Q_{cavity} = +1$

$Q_{outer} + Q_{inner} = Q_{conductor}$

$Q_{outer} = Q_{conductor} - Q_{inner} = -2 - 1 = -3$

10. Answer the following questions about voltage.

a. (5 pts) Equipotentials A and B have $V_A = 8.4$ V and $V_B = 8.7$ V, and a separation of 4 mm. For a point C midway between them, estimate the electric field (magnitude and direction).




$\vec{E}$ is along ↗ (from region of high V to region of low V)

$|\vec{E}| \approx \left|\frac{\Delta V}{\Delta s}\right| = \left(\frac{8.7-8.4\,V}{4\,mm}\right) = 750\,\frac{V}{m}$ 75 V/m

b. (5 pts) Let $V(y) = 2y^4$, where $V$ is in volts and $y$ in in meters. From the voltages at $y$=1.9 m and $y$=2.1 m, estimate the electric field at $y$=2.0 m, including magnitude and direction. $V(1.9) = 26.06\,V$, $V(2.1) = 38.39\,V$

$\vec{E}$ points from 2.1 to 1.9, ~~at~~ ←. $|\vec{E}| \approx \left|\frac{\Delta V}{\Delta s}\right| = \left|\frac{38.39-26.06}{2.1-1.9}\right|\frac{V}{m}$

$= \left|\frac{12.33}{.2}\right|\frac{V}{m} = 61.65\,\frac{V}{m}$

c. (5 pts) Let $V(y) = 2y^4$, where $V$ is in volts and $y$ in in meters. Compute the electric field at $y$=2.0 m, exactly, including magnitude and direction.

$E_x = -\frac{dV}{dy} = -8y^3$. For $y=2$, $E_x = -8(2)^3 = -64\,\frac{V}{m}$.

11. An electron sits at rest at the origin. A uniform electric field of magnitude $4.0 \times 10^4$ N/C is suddenly applied along the $-\hat{\jmath}$ direction.

a. (5 pts) Find the force acting on the electron, in magnitude and direction.

$\vec{F} = q\vec{E} = (-1.6\times10^{-19}\,C)\left(4\times10^4\,\frac{N}{C}\,(-\hat{j})\right) = 6.4\times10^{-15}\,N\,\hat{j}$

b. (5 pts) Find how long it will take for the electron to move 2 cm.

$\vec{a} = \frac{\vec{F}}{m} = \frac{6.4\times10^{-15}\,N\,\hat{j}}{9.1\times10^{-31}\,kg} = 0.703\times10^{16}\,\frac{m}{s^2}\,\hat{j}$

$s = \frac{1}{2}at^2$, so $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2(0.02\,m)}{0.703\times10^{16}\,m/s^2}} = 2.385\times10^{-9}\,s$

12. A dime and a penny each is sitting on an insulator. The dime has charge $-8 \times 10^{-9}$C, and the penny has charge $3 \times 10^{-9}$C. They are now connected by a thin insulated wire, which is then removed.

a. (5 pts) The dime is found to have a charge of $-2.1 \times 10^{-9}$C. What is the charge on the penny? use charge conservation

$-8\times10^{-9} + 3\times10^{-9} = -2.1\times10^{-9} + Q'_{penny}$



$Q'_{penny} = -2.9\times10^{-9}\,C$

b. (5 pts) The voltage of the penny is found to be 2.4V relative to a copper doorknob. What is the voltage of the dime?

same as penny: 2.4 V relative to doorknob