

Don't waste time on questions you aren't sure of. Be clear and concise. A cluttered response, some of which is correct and some of which is incorrect, will not get full credit.

1. (10 pts) A non-conducting rod lies on the z-axis from (0,0,0) to (0,0,a), where a is a constant. It has charge per unit length $\lambda = 6\alpha z^5$, where α is a constant. What units must a and α have? In terms of α and a, find the total charge Q on the rod, and the average charge per unit length $\bar{\lambda}$.

$$[a] = \text{meter}$$

$$[\alpha] = \left[\frac{\lambda}{6z^5} \right] = \frac{C/m}{m^5} = C/m^6$$

$$Q = \int dQ = \int \frac{dQ}{dz} dz = \int_0^a \lambda dz = \int_0^a 6\alpha z^5 dz = 6\alpha z^6 \Big|_0^a = 6\alpha a^6$$

$$\bar{\lambda} = \frac{Q}{L} = \frac{6\alpha a^6}{a} = \boxed{6\alpha a^5}$$

2. (10 pts) For the benefit of Bart Simpson's sixth grade teacher, concisely explain the physics of the "amber effect" (between a charged and a neutral object) and why it is attractive. Use a simple figure, with a positive source charge.

1. The charge on the charged object polarizes the molecules of the neutral object

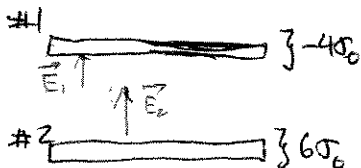
2. The nearer size of the polarized molecule has opposite charge to that of the source

3. Since electric force falls off with distances, attraction of opposites dominates



3. Assume that the charged conducting sheets in the figure are infinite in extent. The one on the top has total charge per unit area $-4\sigma_0$, and the one on the bottom has a total charge per unit area $6\sigma_0$, where $\sigma_0 > 0$.

a. (5 pts) Find the total electric field (magnitude and direction) between the plates.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2\pi k (-4\sigma_0) \downarrow + 2\pi k (6\sigma_0) \uparrow = 20\pi k \sigma_0 \uparrow$$

b. (5 pts) Find the charge density on the top surface of the bottom sheet.



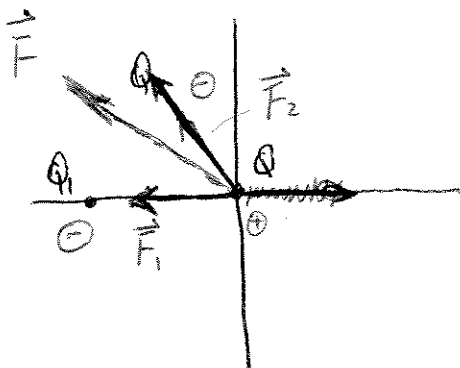
Choose imaginary surface like left

$$\oint \vec{E} \cdot d\vec{A} = 4\pi k \sigma_s \cdot A$$

$$20\pi k \sigma_0 = 4\pi k \sigma_s$$

$$1 \quad \sigma_s = \frac{20\pi k \sigma_0}{4\pi k} = 5\sigma_0$$

4. (35 pts) A point charge $Q_1 = -18.0 \times 10^{-9} \text{ C}$ is on the negative x -axis at $r_1 = 3 \text{ cm}$ from the origin. A point charge $Q_2 = -12.0 \times 10^{-9} \text{ C}$ makes a counterclockwise angle $\theta = 120^\circ$ to the positive x -axis, at $r_2 = 2 \text{ cm}$ from the origin. A charge $Q = 3.0 \times 10^{-9} \text{ C}$ is placed at the origin. Q_1 and Q_2 act on Q with forces \vec{F}_1 and \vec{F}_2 .



$$|\vec{F}_1| = \frac{k|Q_1Q|}{r_1^2} = \frac{(9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}) \cdot (3 \times 10^{-9} \text{ C}) \cdot (18 \times 10^{-9} \text{ C})}{(0.03 \text{ m})^2}$$

$$= \boxed{54 \times 10^{-5} \text{ N}}$$

a. Find $|\vec{F}_1|$ and $|\vec{F}_2|$.

$$|\vec{F}_2| = \frac{k|Q_2Q|}{r_2^2} = \frac{(9 \times 10^9)(3 \times 10^{-9})(12 \times 10^{-9})}{(0.02)^2}$$

$$= \boxed{81 \times 10^{-5} \text{ N}}$$

Note: $|\vec{F}_2| = 1.5 |\vec{F}_1|$

- b. On the figure, draw \vec{F}_1 and \vec{F}_2 with their tails on Q , and in relative proportion.

- c. Find F_x , the x -component of the total force \vec{F} on Q .

$$F_x = F_{1x} + F_{2x} = -|\vec{F}_1| + |\vec{F}_2| \cos 120^\circ = -54 \times 10^{-5} + 81 \times 10^{-5} \times (-0.5) = -94.5 \times 10^{-5} \text{ N}$$

- d. Find F_y , the y -component of \vec{F} .

$$F_y = F_{1y} + F_{2y} = 0 + |\vec{F}_2| \sin 120^\circ = 81 \times 10^{-5} \times \frac{\sqrt{3}}{2} = 70.1 \times 10^{-5} \text{ N}$$

- e. Find the angle \vec{F} makes with respect to the x -axis, and on the figure sketch the direction of \vec{F} . Angle is in 2nd quadrant

$$\tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \left(\frac{70.1}{-94.5} \right) = -36.6^\circ, \quad \theta = -36.6 + 180 = 143.4^\circ$$

- f. Find $|\vec{F}|$.

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(-94.5 \times 10^{-5} \text{ N})^2 + (70.1 \times 10^{-5} \text{ N})^2} = 117.7 \times 10^{-5} \text{ N}$$

- g. Q_1 and Q_2 are rotated clockwise by 35 degrees about the origin. Find the new F_x .

$$(F_x)_{\text{new}} = |\vec{F}|_{\text{new}} (\cos \theta)_{\text{new}} = |\vec{F}|_{\text{old}} \cos(143.4 - 35)$$

$$= 117.7 \times 10^{-5} \cos(108.4) = -37.2 \times 10^{-5} \text{ N}$$

5. A charge $Q_1 = -5.0 \times 10^{-9}$ C is on the y -axis, 8 cm above an infinitely long line charge $\lambda < 0$ along the x axis. The force on Q_1 has magnitude 3.5×10^{-6} N.

a. (10 pts) Find the electric field, in magnitude and direction.

$\lambda < 0$ \vec{E} is \downarrow since $\lambda < 0$

$$|\vec{E}| = \frac{|F|}{|Q_1|} = \frac{3.5 \times 10^{-6}}{5.0 \times 10^{-9}} = 7.0 \times 10^2 \frac{N}{C}$$

b. (5 pts) If Q_1 is moved to a distance of only 2 cm from λ , find the magnitude of the force on Q_1 .

$|\vec{E}| = 2k\lambda$ since r goes down by a factor of 4 (8 cm to 2 cm)
 $|\vec{E}|$ goes up a factor of 4 and so, force by 4; $|F| = 4 \times 3.5 \times 10^{-6}$
 $= 1.4 \times 10^{-5}$ N

c. (5 pts) Find λ .

$$|E| = \frac{2k\lambda}{r} \Rightarrow \lambda = \frac{|E|r}{2k} = \frac{7.0 \times 10^2 \frac{N}{C} \times (0.08)}{2 \times 9 \times 10^9} = -3.1 \times 10^{-9} \frac{C}{m}$$

6. Two line charges are normal to the page. A, with charge density 6λ , passes through the origin. B, with charge density 2λ , passes through $(6a, 0, 0)$.

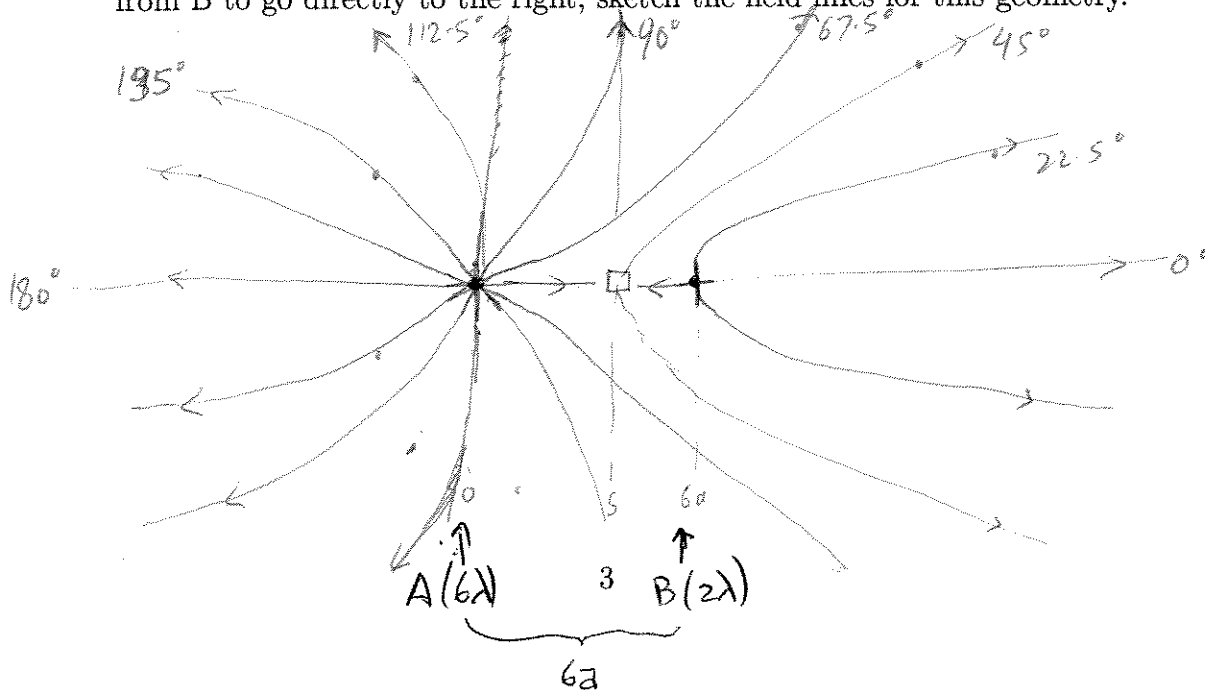
a. (8 pts) Find the position $(s, 0, 0)$ where the electric field is zero.

at that point $|E_A| = |E_B|$ or $\frac{2k(6\lambda)}{s} = \frac{2k(2\lambda)}{6a-s}$
 $s = \frac{9}{2} a$

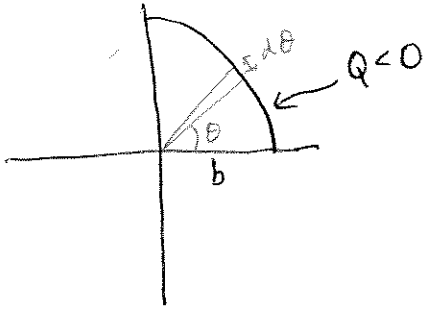
b. (7 pts) If λ is represented by two field lines, find the angle between the field lines as they originate from A. Repeat for B. Repeat for the angle between the field lines as viewed from far away from both.

A has 6λ or $6 \times 2 = 12$ lines $\text{at } \frac{360}{12} = 30^\circ$
 B has 2λ or $2 \times 2 = 4$ lines $\text{at } \frac{360}{4} = 90^\circ$
 total $12 + 4 = 16$ lines $\therefore \frac{360}{16} = 22.5^\circ$

c. (10 pts) Taking one field line from A to go directly to the left, and one field line from B to go directly to the right, sketch the field lines for this geometry.



7. (15 pts) A negative charge ($Q < 0$) is uniformly distributed over a quarter-ring of radius b , in the first quadrant. Indicate the direction of the field at the origin. Compute E_x at the origin.



Net \vec{E} is along \nearrow
 By symmetry $|E_x| = |E_y|$, lets compute E_x
 $\lambda = \frac{2Q}{\pi b}$; $dQ = \lambda b d\theta$
 $|d\vec{E}| = \frac{k dQ}{r^2} = \frac{k \lambda b d\theta}{b^2} = \frac{k \lambda}{b} d\theta$
 $|dE_x| = |d\vec{E}| \cos \theta = \frac{k \lambda}{b} \cos \theta d\theta$
 $E_x = \int_0^{\pi/2} \frac{k \lambda}{b} \cos \theta d\theta = \frac{k \lambda}{b} \sin \theta \Big|_0^{\pi/2} = \frac{k \lambda}{b} = \frac{2kQ}{\pi b^2}$

8. For a negatively-charged conductor, a surface element of area $dA = 4.5 \times 10^{-6} \text{ m}^2$ has its outward normal \hat{n} along $(3, 8, -4.5)$. For this element, $|\vec{E}| = 270 \text{ V/m}$.

- a. (5 pts) Find \hat{n} .

$$\sqrt{3^2 + 8^2 + (-4.5)^2} = 9.656$$

$$\hat{n} = \frac{(3, 8, -4.5)}{9.656} = (0.310, 0.828, -0.466)$$

- b. (5 pts) Find the direction of \vec{E} , called \hat{E} .

$$\hat{E} = -\hat{n} \quad (\text{field goes away from (+ve charge conductor)})$$

- c. (5 pts) Find the flux $d\Phi_E$ through dA .

$$d\Phi_E = \hat{E} \cdot \hat{n} dA = -|\vec{E}| dA = -270 \times 4.5 \times 10^{-6}$$

$$= -1.215 \times 10^{-3} \text{ Vm}$$

- d. (5 pts) Find the surface charge dQ_s .

$$d\Phi_E = 4\pi k dQ_s, \quad dQ_s = \frac{d\Phi_E}{4\pi k} = \frac{-1.215 \times 10^{-3}}{9 \times 10^9 \times 4\pi} = -1.35 \times 10^{-13} \text{ C}$$

$$= -1.07 \times 10^{-14} \text{ C}$$

9. Consider a conductor with a cavity. The total charge on the conductor itself is $+2$ units. If we associate 4 field lines with each unit of charge, then when viewed from afar there is a net of 12 field lines pointing toward the conductor, which is in equilibrium.

- a. (5 pts) How much charge is within the cavity?

12 lines toward \Rightarrow -3 net charge conductor + cavity $\downarrow +2$

$$-3 = +2 + Q_{\text{cavity}} \quad Q_{\text{cavity}} = -5$$

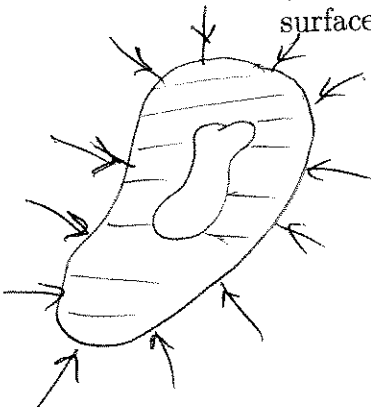
- b. (10 pts) How much charge is on the inner surface of the conductor? On the outer surface? In the bulk?

$$Q_{\text{inner}} = -Q_{\text{cavity}} = +5$$

$$Q_{\text{outer}} + Q_{\text{inner}} = Q_{\text{conductor}}$$

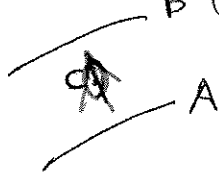
$$Q_{\text{outer}} = Q_{\text{conductor}} - Q_{\text{inner}}$$

$$= 2 - 5 = -3$$



10. Answer the following questions about voltage.

- a. (5 pts) Equipotentials A and B have $V_A = 8.4 \text{ V}$ and $V_B = 8.1 \text{ V}$, and a separation of 6 mm. For a point C midway between them, estimate the electric field B (magnitude and direction).



$$|\vec{E}| = \left| \frac{\Delta V}{\Delta s} \right| = \left(\frac{8.4 \text{ V} - 8.1 \text{ V}}{6 \text{ mm}} \right) = 5 \text{ (V/m)}$$

- b. (5 pts) Let $V(y) = 3y^3$, where V is in volts and y in meters. From the voltages at $y=1.4 \text{ m}$ and $y=1.6 \text{ m}$, estimate the electric field at $y=1.5 \text{ m}$, including magnitude and direction. $V(1.4) = 8.232 \text{ (V)}$ $V(1.6) = 12.288 \text{ (V)}$

\vec{E}
from 1.6 to 1.4

$$|\vec{E}| = \left| \frac{\Delta V}{\Delta s} \right| = \left| \frac{12.288 - 8.232}{1.6 - 1.4} \right| = 20.28 \text{ (V/m)}$$

- c. (5 pts) Let $V(y) = 3y^3$, where V is in volts and y in meters. Compute the electric field at $y=1.5 \text{ m}$, exactly, including magnitude and direction.

$$E_x = -\frac{dV}{dy} = -9y^2 \quad \text{for } y=1.5, \quad E = -20.25 \text{ V/m}$$

11. An electron sits at rest at the origin. A uniform electric field of magnitude $5.0 \times 10^4 \text{ N/C}$ is suddenly applied along the $+\hat{i}$ direction.

- a. (5 pts) Find the force acting on the electron, in magnitude and direction.

$$\vec{F} = q\vec{E} = (-1.6 \times 10^{-19} \text{ C}) (5.0 \times 10^4 \text{ N/C}) (\hat{i}) = 8 \times 10^{-15} \text{ N } (-\hat{i})$$

- b. (5 pts) Find how long it will take for the electron to move 1 cm.

$$\vec{a} = \frac{\vec{F}}{m} = \frac{8 \times 10^{-15} \text{ N } (-\hat{i})}{9.1 \times 10^{-31} \text{ kg}} = 0.88 \times 10^{16} \text{ m/s}^2 (-\hat{i})$$

$$s = \frac{1}{2} a t^2 \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0.01}{0.88 \times 10^{16}}} = 0.15 \times 10^{-8} \text{ s}$$

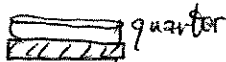
12. A quarter and a penny each is sitting on an insulator. The quarter has charge $-6 \times 10^{-9} \text{ C}$, and the penny has charge $3 \times 10^{-9} \text{ C}$. They are now connected by a thin insulated wire, which is then removed.

- a. (5 pts) The penny is found to have a charge of $-1.1 \times 10^{-9} \text{ C}$. What is the charge on the quarter?

use charge conservation

$$-6 \times 10^{-9} + 3 \times 10^{-9} = -1.1 \times 10^{-9} + Q_{\text{quarter}}$$

$$Q_{\text{quarter}} = -1.9 \times 10^{-9} \text{ C}$$



- b. (5 pts) The voltage of the penny is found to be 2.4V relative to a copper doorknob. What is the voltage of the quarter?

Same . 2.4V