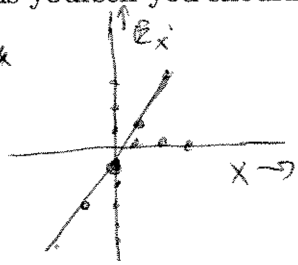
Exam 2, Physics 208 Name **Printed in Ink:** Key

Dr.Saslow, 7/01/05, Sects.301-305 **Section:**

Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.

1. (6 pts) Let $E_x = 2x - 1$, with E_x in volts/m and x in m. If $V = 0$ at $x = 0$, find $V(x)$. Evaluate E_x for $x = -2, -1, 0, 1, 2$ m and plot (no calculator plot; if you can't do this yourself you should ask yourself if you belong in math or science).

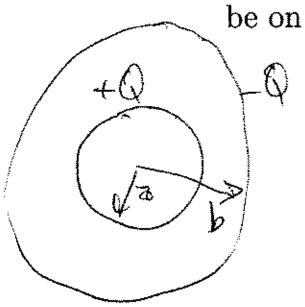
x	E_x
-2	-5
-1	-3
0	-1
1	1
2	3



$$V(x) - V(0) = -\int_0^x \vec{E}\cdot d\vec{s} = -\int_0^x E_x \underbrace{\hat{i}\cdot\hat{i}}_{=1}\,dx$$
$$= -\int_0^x E_x\,dx$$
$$= -\int_0^x (2x-1)\,dx$$
$$= -(x^2 - x)\Big|_0^x$$
$$= -x^2 + x$$

Since $V(0) = 0$, $V(x) = -x^2 + x$

2. Consider two concentric conducting shells of radii a and b with $a < b$. Let a charge Q be on the inner shell and $-Q$ be on the outer shell.



a (4 pts) Find the electric field in the region between the plates; explain your reasoning. By Gauss's law and spherical symmetry, between the plates (shells) the field is like that of a point charge, so $\vec{E} = E_r\hat{r} = \frac{kQ}{r^2}\hat{r}$

b (4 pts) Find $V(r) - V(a)$ for $a < r < b$.

$$V(r) - V(a) = -\int_a^r \vec{E}\cdot d\vec{s} = -\int_a^r \frac{kQ}{r^2}\underbrace{\hat{r}\cdot\hat{r}}_{=1}\,dr = \frac{kQ}{r}\Big|_a^r$$
$$= kQ\left(\frac{1}{r} - \frac{1}{a}\right)$$

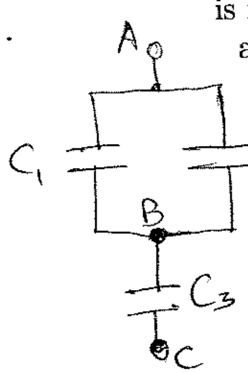
c (4 pts) Find the capacitance of this system.

$C = \frac{Q}{\Delta V}$, $\Delta V = V(a) - V(b) = -[V(b) - V(a)] = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$

where $+Q$ is

$$\Rightarrow C = \frac{Q}{kQ\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{ab}{k(b-a)}$$

3. Consider three capacitors. $C_1 = 40\ \mu$F and $C_2 = 20\ \mu$F are in parallel, and $C_3 = 30\ \mu$F is in series with them. $V_A = 12$ V and $V_C = -8$ V.



a. (12 pts) Find the charge and voltage difference for each capacitor. Find V_B.

$V_A - V_C = 12 - (-8) = 20\ V$

$C' = C_1 + C_2 = 60\ \mu F$

$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} = \frac{1}{60} + \frac{1}{30} = \frac{1+2}{60} = \frac{3}{60} = \frac{1}{20} \Rightarrow C = 20\ \mu F$

Thus $Q = C \Delta V = 20(20) = \boxed{400\ \mu C}$

But $Q_3 = Q = 400\ \mu C$. Then $\Delta V_3 = \frac{Q_3}{C_3} = \frac{400\ \mu C}{30\ \mu F} = \boxed{\frac{40}{3} V}$

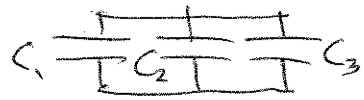
Hence $V_B = V_C + \Delta V_3 = -8 + \frac{40}{3} = +\frac{16}{3} = \boxed{5.33 V}$

Then $\Delta V_1 = \Delta V_2 = V_A - V_B = 12 - (+5.33) = \boxed{6.67 V}$, $Q_1 = C_1 \Delta V_1 = (40)(6.67) = \boxed{267\ \mu C}$

$Q_2 = C_2 \Delta V_1 = \boxed{133\ \mu C}$

b. (12 pts) If, using insulating gloves, C_3 is disconnected and then placed in parallel with C_1 and C_2, find the new charge and voltage difference for each capacitor.

Now have



$Q^* = Q_1 + Q_2 + Q_3 = Q_3 + Q_3 = 2Q_3 = 2Q$
$= 800\ \mu C$

$C^* = C_1 + C_2 + C_3 = 40 + 20 + 30 = 90\ \mu F$

$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V = \frac{Q^*}{C^*} = \frac{800\ \mu C}{90\ \mu F} = 8.88\ V$

$Q_1 = C_1 \Delta V_1 = 40(8.88) = 355.55\ \mu C$, $Q_2 = C_2 \Delta V_2 = 177.77\ \mu C$, $Q_3 = C_3 \Delta V_3 = 266.77\ \mu C$

$Q_1 + Q_2 + Q_3 = 799.9 \approx 800\ \mu C$ ✓

4. A parallel plate capacitor has electrical energy 7.2×10^{-7} ergs when connected to a 6 V battery. It is now disconnected from the battery. A slab of dielectric constant $\kappa = 5$ and nearly the same thickness as the capacitor is slid into the capacitor.

a. (2 pts) What is the voltage difference now?

V decreases by K, to $\frac{6}{5} = 1.2\ V$

b. (2 pts) What is the electrical energy now?

$U_{initial} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} Q (\Delta V)$.

Since Q is unchanged but ΔV is down by 5, $U_{final} = \frac{1}{5} U_{initial} = 1.44 \times 10^{-7}$ ergs

c. (4 pts) Was the dielectric attracted, repelled, or did it feel no force when it was part way in the capacitor, and why? (No reason, no credit.)

Attracted, by the amber effect (dielectric is polarized and attracted to charge on plates)

5. You are given a voltaic cell with internal resistance of 2 Ω. When shorted, it briefly produces a current of 0.12 A.

   a. (4 pts) Find its emf and the rate at which energy is initially discharged.

$$I = \frac{\mathcal{E}}{r} \Rightarrow \mathcal{E} = Ir = 0.12 \times 2 = 0.24\ V$$

$$P = I^2 r = (0.12)^2 (2) = .0288\ W$$

   b. (4 pts) For what load resistance does this voltaic cell provide maximum power to load? Find that maximum power.

For impedance matching: $R = r = 2\ \Omega$

$$I = \frac{\mathcal{E}}{r+R} = \frac{0.24}{4} = 0.06\ A$$

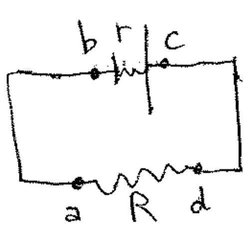
$$I^2 R = (36 \times 10^{-4})(2) = 72 \times 10^{-4}\ W = .0072\ W$$

   c. (4 pts) A 100% efficient flashlight bulb of resistance R produces 0.8 W when used with a AA cell. If R is much larger than the cell's internal resistance, find R and the efficiency at which the battery produces useful power.

$$P = 0.8\ W = \frac{\mathcal{E}^2}{R}, \quad R = \frac{\mathcal{E}^2}{P} = \frac{(1.5)^2}{0.8} = \frac{2.25}{0.8} = \cancel{2.875\ \Omega}\ 2.8125\ \Omega$$

6. A voltaic cell has internal resistance $r = 0.2\ \Omega$ and open circuit voltages across the left and right electrodes of 0.2 V and 1.4 V, for a net emf of $\mathcal{E} = 1.6$ V. It is in series with a resistor $R = 0.6\ \Omega$. Let $V_a = 0.4$ V. The connecting wires have zero resistance.

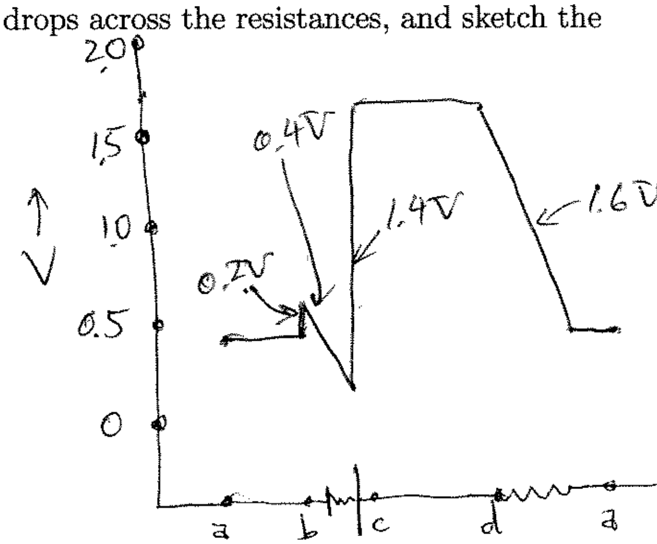
   a. (8 pts) Find the current, the voltage drops across the resistances, and sketch the voltage around the circuit.



$$I = \frac{\mathcal{E}}{r+R} = \frac{1.6}{0.8} = 2A$$

$$Ir = 0.4\ V$$

$$IR = 1.2\ V$$



   b. (2 pts) If the voltaic cell discharges in 40 minutes, find its initial "charge" and its initial energy.

$$\text{"}Q\text{"} = It = 2(40 \times 60) = 4800\ C \quad (\text{or } \tfrac{4}{3}\ \text{A-hr})$$

$$U_{batt} = \mathcal{E}\,\text{"}Q\text{"} = (1.6)(4800\ C) = 7680\ J$$

race-cars!

7. (6 pts) A strip of roadway has length 80 m and width 15 m. Automobiles move through it with average velocity 50 m/s, with each taking up, on average, an area of 40 m^2. Find the rate at which the autos pass through the roadway. If they each carry a charge of 5×10^{-7} C, find the electric current passing through the roadway.

time to cross $= \frac{80\text{m}}{50\text{m/s}} = 1.6\text{s}$

average # in roadway $= \frac{80\text{m} \times 15\text{m}}{40\text{m}^2} = 30$ cars

$\frac{dN}{dt}$ = rate of crossing $= \frac{30 \text{ cars}}{1.6\text{s}} = 18.75$ cars/sec

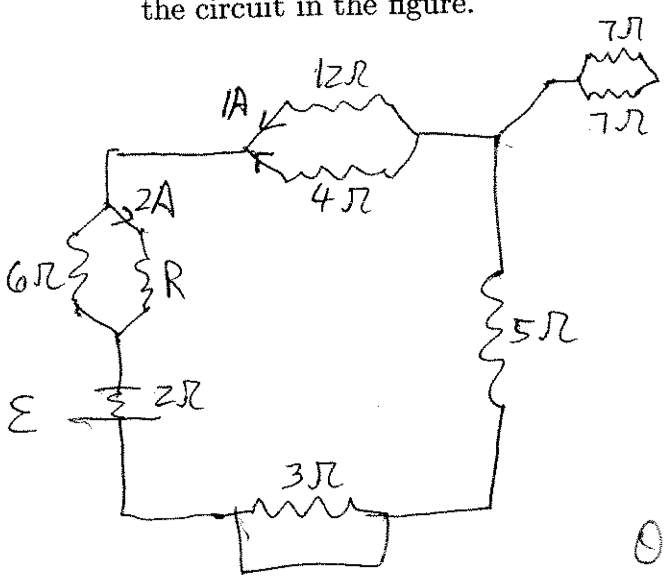
$I = \frac{18.75}{s} \times 5 \times 10^{-7}\,C = 9.375 \times 10^{-6}\,A$

8. (6 pts) Two cars, A and B, have dead batteries that are jump-started by a third car and then driven home, A going only a mile, but B going thirty miles. At home the cars are turned off and then successfully restarted. The next morning only one car will start. Which one? Why?

Car B will restart. It was given more of an average charge while being driven home ~~[illegible]~~

Note: Both cars initially re-started because they were given good "surface charges", but B had more of an average charge, which leveled out overnight to a large enough charge to start the car.

9. (8 pts) Find the unknown currents, the unknown resistance, and the unknown emf for the circuit in the figure.



$I_{4\Omega} = \frac{(1A)(12\Omega)}{4\Omega} = 3A$

$I_{5\Omega} = 1 + 3 = 4A = I_{2\Omega}$

$I_{3\Omega} = 0$ (shorted)

$I_{7\Omega} = 0$ (off to the side)

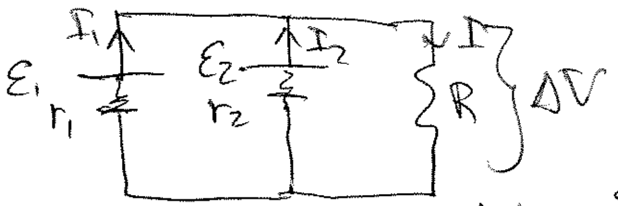
$I_{6\Omega} = 4 - 2 = 2A$

$\Rightarrow R = \frac{(2A)(6\Omega)}{2A} = 6\Omega$

On going around the circuit, the IR drops sum to: $5(4) + 1(12) + 2(6) + 4(2)$
$= 20 + 12 + 12 + 8 = 52\,V$

Hence $\varepsilon = 52\,V$

10. (12 pts) For the circuit below, take $\mathcal{E}_1 = 8$ V, $\mathcal{E}_2 = 12$ V, $r_1 = 0.01\ \Omega$, $r_2 = 0.04\ \Omega$, $R = 0.03\ \Omega$. (1) Indicate and label the directions of positive currents and indicate the positive side of the voltage ΔV across R. (2) Analyze the circuit using Kirchoff's rules. (3) Solve for the voltage across R. (4) Find the current through R and the currents provided by each of the batteries.



$$I = I_1 + I_2$$

$$\frac{\Delta V}{R} = \frac{\mathcal{E}_1 - \Delta V}{r_1} + \frac{\mathcal{E}_2 - \Delta V}{r_2}$$

$$\Delta V\left(\frac{1}{R} + \frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}$$

$$\Delta V(33.3 + 100 + 25) = \frac{8}{.01} + \frac{12}{.04} = 800 + 300 = 1100$$

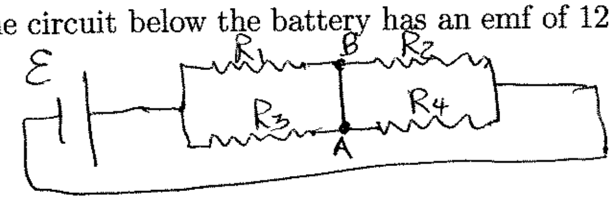
$$\Delta V = \frac{1100}{158.3} = 6.95\ V$$

$$I = \frac{\Delta V}{R} = 231.6A \qquad I_2 = \frac{\mathcal{E}_2 - \Delta V}{r_2} = \frac{12 - 6.95}{.04} = 126.3$$

$$I_1 = \frac{\mathcal{E}_1 - \Delta V}{r_1} = \frac{8 - 6.95}{.01} = 105.2A$$

Note: $I_1 + I_2 = 231.5A \approx I$ ✓

11. In the circuit below the battery has an emf of 12 V.



$R_1 = 3\Omega$, $R_3 = 6\Omega$
$R_2 = 12\Omega$, $R_4 = 6\Omega$

a. (4 pts) What is the equivalent resistance?

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}, \quad R' = 2\Omega$$

$$\frac{1}{R''} = \frac{1}{R_2} + \frac{1}{R_4} = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}, \quad R'' = 4\Omega$$

$$R = R' + R'' = 6\Omega$$

b. (4 pts) Find the currents I_1 and I_2, and the current from A to B, where upward current is taken to be positive.

$I = \frac{\mathcal{E}}{R} = 2A$. ~~Both~~ A & B are ~~at~~ the same voltage,

$$\Delta V_{1\&3} = IR' = 2(2) = 4V, \text{ so } I_1 = \frac{\Delta V_1}{R_1} = \boxed{\frac{4}{3}A}$$

$$\Delta V_{2\&4} = IR'' = 2(4) = 8V, \text{ so } I_2 = \frac{\Delta V_2}{R_2} = \frac{8}{12} = \boxed{\frac{2}{3}A}$$

A current of $\boxed{\frac{2}{3}A}$ flows from B to A (or $-\frac{2}{3}A$ from A to B