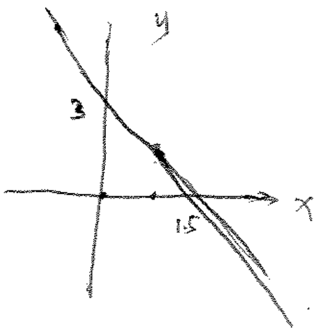
Exam 2, Physics 208 Name **Printed** in **Ink**: Key

Dr.Saslow, 7/01/05, Sects.301-305 **Section**:

**Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.**

1. (6 pts) Let $E_x = -2x + 3$, with $E_x$ in volts/m and $x$ in m. If $V = 0$ at $x = 0$, find $V(x)$. Evaluate $E_x$ for $x = -2, -1, 0, 1, 2$ m and plot (no calculator plot; if you can't do this yourself you should ask yourself if you belong in math or science).



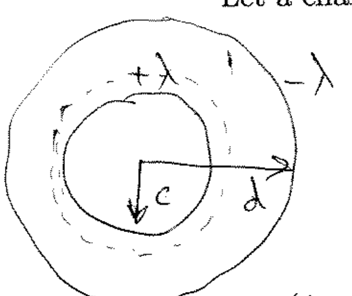

$E_x = -2x + 3$

| $x=$ | $E_x$ |
|---|---|
| $-2$ | $7$ |
| $-1$ | $5$ |
| $0$ | $3$ |
| $1$ | $1$ |
| $2$ | $-1$ |

$$V(x) - V(0) = -\int_0^x (-2x+3)\,dx = +x^2 - 3x \Big|_0^x$$

$$\boxed{V(x) = +x^2 - 3x}$$

2. Consider two long concentric conducting cylindrical shells of radii $c$ and $d$ with $c < d$. Let a charge per unit length $\lambda$ be on the inner shell and $-\lambda$ be on the outer shell.




a (4 pts) Find the electric field in the region between the plates; explain your reasoning.

$$\oint E \cdot dA = Q/\varepsilon_0$$

$$E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

b (4 pts) Find $V(r) - V(\cancel{a}\,c)$ for $\cancel{a}\,c < r < \cancel{b}\,d$.

$$V(c) - V(r) = \int_c^r \vec{E}\cdot d\vec{r} = \int_c^r \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\cdot d\vec{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{c}$$

c (4 pts) Find the capacitance of this system.

$$\Delta V = -(V(d) - V(c)) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{d}{c}$$

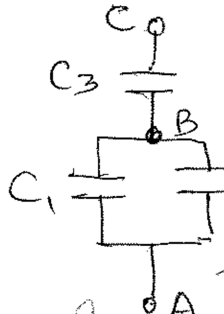
$$C = \frac{Q}{\Delta V} = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{d}{c}} = \frac{2\pi\varepsilon_0 L}{\ln\frac{d}{c}}$$

for capacitance per unit length

$$c = \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\frac{d}{c}}$$

3. Consider three capacitors. $C_1 = 10\ \mu$F and $C_2 = 20\ \mu$F are in parallel, and $C_3 = 15\ \mu$F is in series with them. $V_A = 6$ V and $V_C = -4$ V.

a. (2 pts) Find the charge and voltage difference for each capacitor. Find $V_B$.




$V_A - V_C = 6 - (-4) = 10\ V$

$C' = C_1 + C_2 = 30\ \mu F$

$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} = \frac{1}{30} + \frac{1}{15} = \frac{1}{10} \Rightarrow C = 10\ \mu F$

$\Rightarrow Q = C\Delta V = 10 \times 10 = 100\ \mu C$ $\quad Q_3 = Q = \boxed{100\ \mu C}$

$\Delta V_3 = \frac{Q_3}{C_3} = \frac{100\ \mu C}{15\ \mu F} = \boxed{\frac{20}{3} V}$; $V_B = V_C + \Delta V_3 = -4 + \frac{20}{3} = \frac{8}{3} = \boxed{2.67\ V}$; $\Delta V_1 = \Delta V_2 = V_A - V_B = \boxed{3.33\ V}$

$Q_1 = C_1 \Delta V_1 = 10\ \mu F \times 3.33 = 33.3\ \mu C$

$Q_2 = C_2 \Delta V_1 = 20\ \mu F \times 3.33 = 66.6\ \mu C$

b. (2 pts) If, using insulating gloves, $C_3$ is disconnected and then placed in parallel with $C_1$ and $C_2$, find the new charge and voltage difference for each capacitor.

$Q^* = Q_1 + Q_2 + Q_3 = 2Q_3 = 200\ \mu C$

$C^* = C_1 + C_2 + C_3 = 10 + 20 + 15 = 45\ \mu F$

$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V = \frac{Q^*}{C^*} = \frac{200\ \mu C}{45\ \mu F} = \frac{40}{9} V = 4.44\ V$

$Q_1' = C_1 \Delta V_1 = 10\ \mu F \times 4.44\ V = 44.4\ \mu C$

$Q_2' = C_2 \Delta V_2 = 20\ \mu F \times 4.44\ V = 88.8\ \mu C$

$Q_3' = C_3 \Delta V_3 = 15\ \mu F \times 4.44\ V = 66.6\ \mu C$

4. A parallel plate capacitor has electrical energy $3.6 \times 10^{-7}$ ergs when connected to a 12 V battery. It is now disconnected from the battery. A slab of dielectric constant $\kappa = 5$ and nearly the same thickness as the capacitor is slid into the capacitor.

a. (2 pts) What is the voltage difference now?

$\frac{12\ V}{5} = 2.4\ V$

b. (2 pts) What is the electrical energy now?

$\frac{1}{2} Q \cancel{V} = \frac{3.6 \times 10^{-7}}{5} = 0.72 \times 10^{-7}$

c. (4 pts) Was the dielectric attracted, repelled, or did it feel no force when it was part way in the capacitor, and why? (No reason, no credit.)

5. You are given a voltaic cell with internal resistance of 3 Ω. When shorted, it briefly produces a current of 0.15 A.

   a. (4 pts) Find its emf and the rate at which energy is initially discharged.

$$I = \frac{\mathcal{E}}{r} \Rightarrow \mathcal{E} = Ir = 0.15 \times 3 = 0.45\,V$$

$$P = I^2 r = (0.15)^2(3) = 0.0675\,W$$

   b. (4 pts) For what load resistance does this voltaic cell provide maximum power to load? Find that maximum power.

For impedance matching : $R = r = 3\,\Omega$

$$I = \frac{\mathcal{E}}{r+R} = \frac{0.45}{6} = 0.075\,A$$

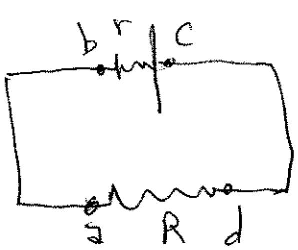
$$P_m = I^2 R = 0.075^2 \times 3 = 0.016875\,W.$$

   c. (4 pts) A 100% efficient flashlight bulb of resistance $R$ produces 1.2 W when used with a AA cell. If $R$ is much larger than the cell's internal resistance, find $R$ and the efficiency at which the battery produces useful power.

$$P = 1.2\,W = \frac{\mathcal{E}^2}{R}, \quad R = \frac{\mathcal{E}^2}{P} = \frac{(1.5)^2}{1.2} = 1.875\,\Omega$$

6. A voltaic cell has internal resistance $r = 0.2\ \Omega$ and open circuit voltages across the left and right electrodes of 0.4 V and 1.6 V, for a net emf of $\mathcal{E} = 2.0$ V. It is in series with a resistor $R = 0.8\ \Omega$. Let $V_a = 0.6$ V. The connecting wires have zero resistance.

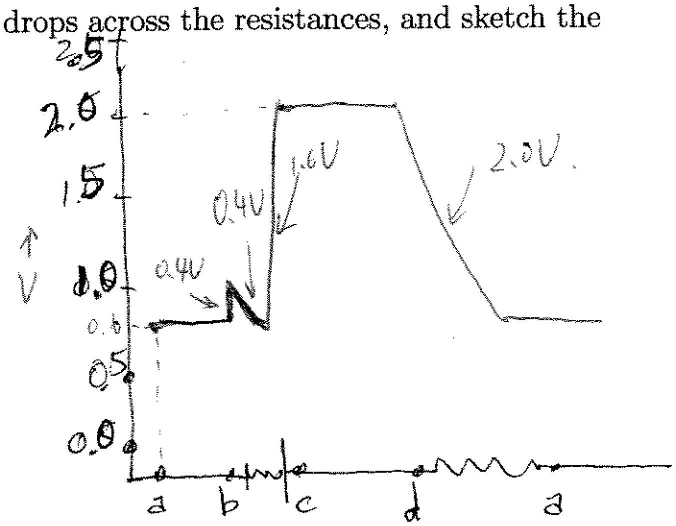
   a. (8 pts) Find the current, the voltage drops across the resistances, and sketch the voltage around the circuit.




$$I = \frac{\mathcal{E}}{r+R} = \frac{2.0}{1.0} = 2A$$

$$Ir = 0.4\,V$$

$$IR = 1.6\,V$$




   b. (2 pts) If the voltaic cell discharges in 40 minutes, find its initial "charge" and its initial energy.

$$\text{"}Q\text{"} = It = 2(40 \times 60) = 4800\,C \quad (\text{or } \tfrac{4}{3}\,A\text{-}hr)$$

$$U = \mathcal{E}\,\text{"}Q\text{"} = (2.0) \times (4800\,C) = 9600\,J$$

7. (6 pts) A strip of roadway has length 120 m and width 20 m. Automobiles move through it with average velocity 60 m/s, with each taking up, on average, an area of 50 m$^2$. Find the rate at which the autos pass through the roadway. If they each carry a charge of $4 \times 10^{-7}$ C, find the electric current passing through the roadway.

$$\text{time to cross} = \frac{120\text{ m}}{60\text{ m/sec}} = 2\text{ sec}$$

$$\text{Average \# in roadway} = \frac{120 \times 20}{50} = \cancel{36}\ 48\text{ cars}$$

$$\frac{dN}{dt} = \text{rate of crossing} = \frac{48\text{ cars}}{2\text{ sec}} = 24\text{ cars/sec}$$

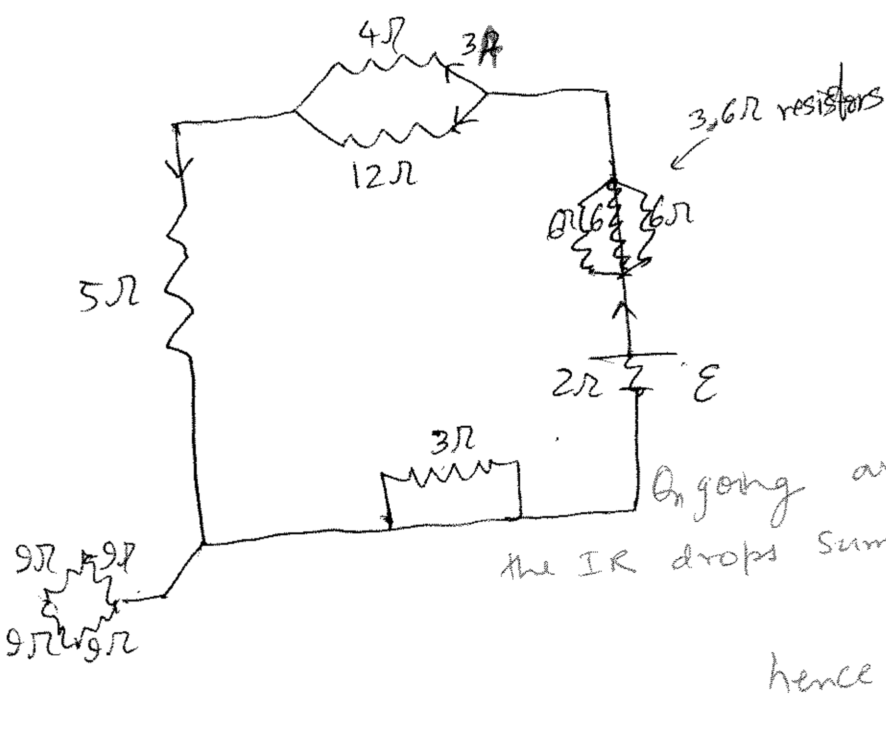
$$I = 24\ \frac{\text{cars}}{\text{sec}} \times 4 \times 10^{-7} = 9.6 \times 10^{-6}\text{ A}$$

8. (6 pts) Two cars, A and B, have dead batteries that are jump-started by a third car and then driven home, A going only a mile, but B going thirty miles. At home the cars are turned off and then successfully restarted. The next morning only one car will start. Which one? Why?

Car B will restart. It was given more of an average charge while being driven home.

Note: Both cars initially restarted because they were given good "surface charges", but B had more of an average charge, which leveled out over night to a large enough charge to start the car.

9. (8 pts) Find the unknown currents, ~~the unknown resistance~~, and the unknown emf for the circuit in the figure.




$$I_{12\Omega} = \frac{4\Omega \times 3A}{12\Omega} = 1A$$

$I_{5\Omega} = 1A + 3A = 4A$

$I_{3\Omega} = 0$ as shorted.

$I_{9\Omega} = 0$ as off to the side

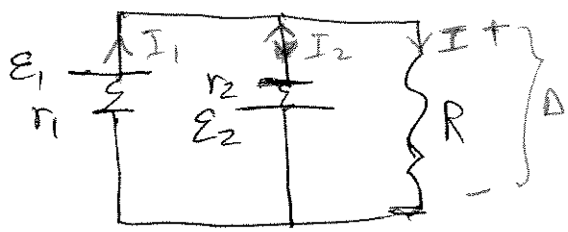
$I_{2\Omega} = I_{5\Omega} = 4A$

$I_{6\Omega} = \frac{4}{3}$ A [currents equally divided]

On going around the circuit the IR drops sum to $= 4(3) + 5(4) + 2(4) + 6(\frac{4}{3}) = 48$ V

hence $\mathcal{E} = 48$ V

10. (12 pts) For the circuit below, take $\mathcal{E}_1 = 12$ V, $\mathcal{E}_2 = 10$ V, $r_1 = 0.02\ \Omega$, $r_2 = 0.01\ \Omega$, $R = 0.03\ \Omega$. (1) Indicate and label the directions of positive currents and indicate the positive side of the voltage $\Delta V$ across $R$. (2) Analyze the circuit using Kirchoff's rules. (3) Solve for the voltage across $R$. (4) Find the current through $R$ and the currents provided by each of the batteries.




$$I = I_1 + I_2$$

$$\frac{\Delta V}{R} = \frac{\mathcal{E}_1 - \Delta V}{r_1} + \frac{(\mathcal{E}_2 - \Delta V)}{r_2}$$

$$\Delta V\left[\frac{1}{R} + \frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\mathcal{E}_1}{r_1} - \frac{\mathcal{E}_2}{r_2}$$

$$\Delta V\left(\frac{1}{0.03} + \frac{1}{0.02} + \frac{1}{0.01}\right) = \frac{12}{0.02} - \frac{10}{0.01} = 600 - 1000 = -400$$

$$\Delta V(183.3) = -400$$

$$\Rightarrow \Delta V = -2.18\text{ V}$$

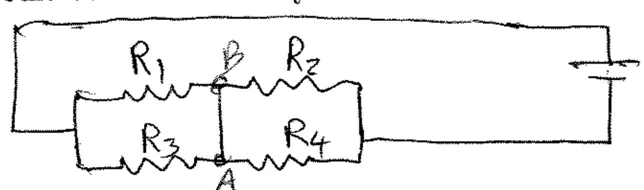
$$I = \frac{\Delta V}{R} = \frac{-2.18}{0.03} = -72.7\text{ A}$$

$$I_1 = \frac{12 + 2.18}{0.02} = 709\text{ A}$$

$$I_2 = -\frac{10 + 2.18}{0.01} = \frac{-7.82 \ \cancel{12.18}}{0.01} = -782\text{ A} \ \cancel{12.18\text{ A}}$$

$$I = I_1 + I_2$$

11. In the circuit below the battery has an emf of 6 V.




$R_1 = 6\,\Omega$, $R_3 = 12\,\Omega$

$R_2 = 24\,\Omega$, $R_4 = 12\,\Omega$

a. (4 pts) What is the equivalent resistance?

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12} \Rightarrow R' = 4\,\Omega$$

$$\frac{1}{R''} = \frac{1}{R_2} + \frac{1}{R_4} = \frac{1}{24} + \frac{1}{12} = \frac{3}{24} \Rightarrow R'' = 8\,\Omega$$

$$R = R' + R'' = 12\,\Omega$$

b. (4 pts) Find the currents $I_1$ and $I_2$, and the current from A to B, where upward current is taken to be positive.

$$I = \frac{\mathcal{E}}{R} = \frac{6}{12} = 0.5\text{ A}$$

$$\Delta V_{1\&3} = IR' = 0.5(4) = 2\text{ V}, \text{ So, } I_1 = \frac{\Delta V_1}{R_1} = \frac{2}{6} = 0.33\text{ A}$$

$$\Delta V_{2\&4} = IR'' = 0.5(8) = 4\text{ V}, \text{ So, } I_2 = \frac{\Delta V_2}{R_2} = \frac{4}{24} = \frac{1}{8}\text{ A} = 0.125$$

A current of $\left(\frac{1}{3} - \frac{1}{8}\right) = 0.208$ A flows from B to A

or $-0.208$ A from A to B