Key Pink

10. Answer the following questions about voltage.

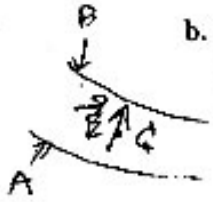
a. (5 pts) For two equipotentials A' and B', 0.4 cm apart, the electric field between them is of magnitude 90 N/C, and points to B'. If $V_{A'} = -1.2$ V, estimate $V_{B'}$.

$\vec{E}$ points from positions of higher to lower voltage, so $V_{A'} > V_{B'}$.

$|\Delta V| \approx |\vec{E}||\Delta \vec{s}| = 90\frac{N}{C} \times 0.004m = 0.36V$

Thus $V_{B'} = V_{A'} - 0.36V = -1.56V$

b. (5 pts) Equipotentials A and B, with $V_A = -8.1$ V and $V_B = -8.7$ V, are separated by 0.3 mm. For point C midway between them, estimate the field (magnitude and direction).



$|\vec{E}| \approx \frac{|\Delta V|}{|\Delta s|} = \frac{0.6V}{0.3\times10^{-3}m} = 2000\frac{V}{m}$

c. (5 pts) Let $V(y) = -2y^3$, with V in volts and y in meters. From the voltages at $y = 0.9$ m and $y = 1.1$ m, estimate $\vec{E}$ at $y = 1.0$ m (give magnitude and direction).

$V(1.1) = -2.904$, $V(0.9) = -1.458$

$\vec{E}$ points from high to low, or from 0.9 m to 1.1 m, so $\vec{E}$ is along $\uparrow$.

$|\vec{E}| \approx \frac{|\Delta V|}{|\Delta s|} = 7.23\frac{V}{m}$

d. (5 pts) For $V(y) = -2y^3$, with V in volts and y in m, find $\vec{E}$ at y=1.0 m exactly.

$E_y = \vec{E}\cdot\hat{y} = -\frac{dV}{dy} = -\left(\frac{d}{dy}(-2y^3)\right) = 2\frac{d}{dy}y^3 = 2(3y^2) = 6y^2$

At $y = 1$, $E_y = 6\frac{V}{m}$

e. (5 pts) On which equipotential in part c) will an electron have the lower energy, and by how much?

Higher V has lower $-eV$, so $y = 0.9$ m has lower energy.

The energy is lower by $e|\Delta V| = (1.6\times10^{-19}C)(1.446V) = 2.31\times10^{-19}J$

11. Two spheres, with radii $r_A = 2$ cm and $r_B = 1$ cm, sit upon two distant insulating platforms. Their initial voltages relative to infinity are $V_A^0 = -150$ V and $V_B^0 = +150$ V.

a. (6 pts) Find their initial charges Q_A^0 and Q_B^0

$V_A^0 = \frac{kQ_A^0}{r_A}$, so $Q_A^0 = \frac{V_A^0 r_A}{k} = -3.33\times10^{-10}C$

Similarly, $Q_B^0 = \frac{V_B^0 r_B}{k} = 1.667\times10^{-10}C$

b. (14 pts) Find their final charges Q_A and Q_B and their final potentials V_A and V_B, if the spheres are connected by a fine conducting wire.

$Q_A + Q_B = -1.667\times10^{-10}C$, by charge conservation.

Also, $V_A = \frac{kQ_A}{r_A} = \frac{kQ_B}{r_B} = V_B$ or $\frac{Q_A}{2} = \frac{Q_B}{1}$, so $Q_A = 2Q_B$.

Thus $3Q_B = -1.667\times10^{-10}C$, so that $Q_B = -0.555\times10^{-10}C$, and $Q_A = -1.111\times10^{-10}C$.

Then $V_A = V_B = 50.0V$