

Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.

1. (10 pts) A parallel plate capacitor has plates that are collinear disks of radius a , separated by an air gap d , where $d \ll a$. Derive the capacitance.



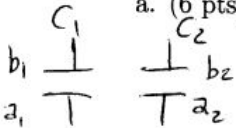
$$C = \frac{Q}{\Delta V} \quad A = \pi a^2 \text{ is plate area}$$

$$\Delta V = E d = 4\pi k \left(\frac{Q}{A}\right) d$$

$$C = \frac{Q}{4\pi k Q d / A} = \frac{A}{4\pi k d} = \frac{\pi a^2}{4\pi k d} = \frac{a^2}{4 k d}$$

2. Two capacitors $C_1 = 24 \mu\text{F}$ and $C_2 = 12 \mu\text{F}$ are separately connected to a 12 V battery, plates a_1 and a_2 being +, b_1 and b_2 being -. After charging up they are disconnected from the battery.

- a. (6 pts) Give the charges on each plate.



$$Q_1 = C_1 \Delta V_1 = (24 \mu\text{F})(12 \text{V}) = 288 \mu\text{C} = Q_{a_1} = -Q_{b_1}$$

$$Q_2 = C_2 \Delta V_2 = (12 \mu\text{F})(12 \text{V}) = 144 \mu\text{C} = Q_{a_2} = -Q_{b_2}$$

- b. (9 pts) Now they are connected, a_1 to b_2 and a_2 to b_1 . Hint: This is not a series connection. Find $V_{a_1} - V_{b_1}$ and $V_{a_2} - V_{b_2}$, and the charge on each plate.

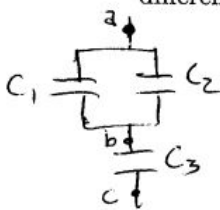


$$Q = Q_{a_1} - Q_{b_1} = 144 \mu\text{C}$$

$$\Delta V = V_{a_1} - V_{b_1} = -(V_{a_2} - V_{b_2}) = \frac{Q}{C} = \frac{144 \mu\text{C}}{36 \mu\text{F}} = 4 \text{V}$$

$$Q'_{a_1} = 96 \mu\text{C} = -Q'_{b_1}, \quad Q'_{a_2} = -48 \mu\text{C} = -Q'_{b_2}$$

3. (15 pts) Consider three capacitors. $C_1 = 24 \mu\text{F}$ and $C_2 = 12 \mu\text{F}$ are in parallel, and $C_3 = 18 \mu\text{F}$ is in series with them. 8V is across C_3 . Find the charge and voltage differences for each capacitor. Find Q_3 .



$$Q = Q_3 = C_3 \Delta V_3 = (18 \mu\text{F})(2 \text{V}) = 36 \mu\text{C}$$

$$C' = C_1 + C_2 = 36 \mu\text{F}$$

$$\Delta V' = \frac{Q'}{C'} = \frac{Q}{C} = \frac{36 \mu\text{C}}{36 \mu\text{F}} = 1 \text{V} = \Delta V_1 = \Delta V_2$$

$$Q_1 = C_1 \Delta V_1 = 24 \mu\text{F}, \quad Q_2 = C_2 \Delta V_2 = 12 \mu\text{F}$$

$$Q_1 + Q_2 = Q_3$$

or $C = (C_3^{-1} + C')^{-1}$
 $= \left(\frac{1}{36} + \frac{1}{18}\right)^{-1} = 12$

$$\Delta V = \frac{Q}{C} = \frac{36}{12} = 3 \text{V, etc.}$$