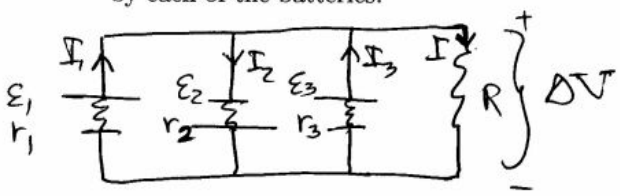
Key Prob

6. (20 pts) For the circuit below, take $\mathcal{E}_1 = 6$ V, $\mathcal{E}_2 = 10$ V, $\mathcal{E}_3 = 12$ V, $r_1 = 0.02\ \Omega$, $r_2 = 0.01\ \Omega$, $r_3 = 0.03\ \Omega$, $R = 0.04\ \Omega$. Analyze the circuit using Kirchoff's rules. Solve for the voltage across R. Find the current through R and the currents provided by each of the batteries.



(1) Draw sign conventions for currents and voltage across R.

(2) Current conservation gives $I_1 + I_3 = I_2 + I$

(3) $I_1 = \frac{\mathcal{E}_1 - \Delta V}{r_1}$, $I_2 = \frac{\mathcal{E}_2 + \Delta V}{r_2}$, $I_3 = \frac{\mathcal{E}_3 - \Delta V}{r_3}$, $I = \frac{\Delta V}{R}$

(4) $\frac{\mathcal{E}_1 - \Delta V}{r_1} + \frac{\mathcal{E}_3 - \Delta V}{r_3} = \frac{\mathcal{E}_2 + \Delta V}{r_2} + \frac{\Delta V}{R}$, by combining (2) & (3)

Thus $\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_3}{r_3} - \frac{\mathcal{E}_2}{r_2} = \Delta V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{R}\right)$

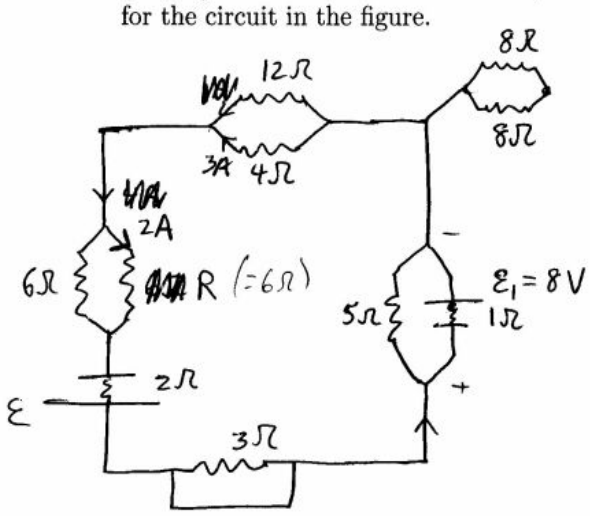
or $300 + 1200 - 1000 = \Delta V(50 + 100 + 33.3 + 25)$, so $\Delta V = -\frac{300}{208.3} = -1.440$ V

[Thus $\mathcal{E}_2$, with its low r_2, wins as a charge pump, relative to $\mathcal{E}_1$ and $\mathcal{E}_3$.]

(5) $I_1 = \frac{\mathcal{E}_1 - \Delta V}{r_1} = 372.0$ A $I_2 = \frac{\mathcal{E}_2 + \Delta V}{r_2} = 856$ A, $I_1 + I_3 = 820$ A ↺ check!

$I_3 = \frac{\mathcal{E}_3 - \Delta V}{r_3} = 448.0$ A $I = \frac{\Delta V}{R} = -36$ A, $I_2 + I = 820$ A

7. (15 pts) Find the unknown currents, the unknown resistance, and the unknown emf for the circuit in the figure.



(1) $I_{12}(12) = 3(4) = 12$

$\Rightarrow I_{12} = 1$ A

(2) Thus $I = 4$ A

(3) $(I - 2A) = 2$A, so 2A through 6Ω & R, in parallel, so $R = 6\Omega$

(4) $I_2 = 4$A

(5) $I_3 = 0$ (goes through wire)

(6) $I_8 = 0$

(7) As calculation on left shows, $\Delta V_5 = 10$, so $I_5 = \frac{\Delta V_5}{5} = \frac{10}{5} = 2$A, and thus $I_1 = 2$A.

(8) Voltage drops going around circuit are $5\times2 + 3\times4 + 6\times2 + 4\times2$, or $10 + 12 + 12 + 8 = 42$, so if $\mathcal{E}$ must give rise of 42V.

$\mathcal{E} = 42$V

$4 = I_5 + I_1$

$I_5 = \frac{\Delta V}{5}$

$I_1 = \frac{-8 + \Delta V}{1}$

$4 = \frac{\Delta V}{5} - 8 + \Delta V$

$12 = \frac{6}{5}\Delta V$

$\Delta V = 10$V

$10 + 12 + 12 + 8 = \mathcal{E}$

$\mathcal{E} = 42$V