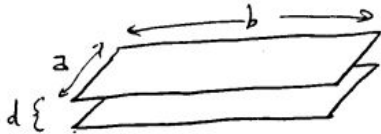


Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.

1. (10 pts) A parallel plate capacitor has plates that are aligned rectangles of sides  $a$  and  $b$ , separated by an air gap  $d$ , where  $d \ll a$ . Derive the capacitance.

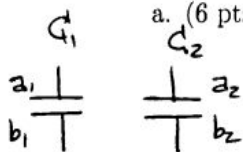


$$C = \frac{Q}{\Delta V}. \quad A = ab \text{ is plate area.}$$

$$\Delta V = E \cdot d = \frac{\sigma}{\epsilon_0} d = 4\pi k \left( \frac{Q}{A} \right) d = \frac{Q}{A \epsilon_0} d$$

$$C = \frac{Q}{4\pi k \frac{Qd}{A}} = \epsilon_0 \frac{A}{d} = \epsilon_0 \frac{ab}{d} = \frac{1}{4\pi k} \frac{ab}{d}$$

2. Two capacitors  $C_1 = 4 \mu\text{F}$  and  $C_2 = 12 \mu\text{F}$  are separately connected to a 12 V battery, plates  $a_1$  and  $a_2$  being +,  $b_1$  and  $b_2$  being -. After charging up they are disconnected from the battery.

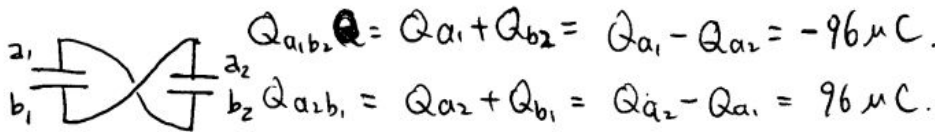


- a. (6 pts) Give the charges on each plate.

$$Q_1 = C_1 V = 4 \mu\text{F} \cdot 12\text{V} = 48 \mu\text{C} = Q_{a_1} = -Q_{b_1}$$

$$Q_2 = C_2 V = 12 \mu\text{F} \cdot 12\text{V} = 144 \mu\text{C} = Q_{a_2} = -Q_{b_2}$$

- b. (9 pts) Now the plates are connected,  $a_1$  to  $b_2$  and  $a_2$  to  $b_1$ . Hint: This is not a series connection. Find  $V_{a_1} - V_{b_1}$  and  $V_{a_2} - V_{b_2}$ , and the charge on each plate.



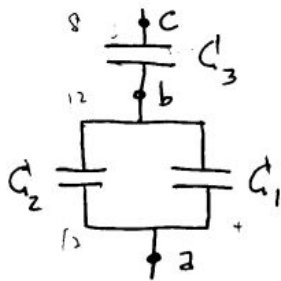
$$Q_{a_1 b_2} = Q_{a_1} + Q_{b_2} = Q_{a_1} - Q_{a_2} = -96 \mu\text{C}$$

$$Q_{a_2 b_1} = Q_{a_2} + Q_{b_1} = Q_{a_2} - Q_{a_1} = 96 \mu\text{C}$$

$$V_{a_1} = V_{b_2}, \quad V_{a_2} = V_{b_1} \quad \therefore V_{a_1} - V_{b_1} = -(V_{a_2} - V_{b_2}) = \frac{Q}{C_1 + C_2}$$

$$= \frac{96 \mu\text{C}}{16 \mu\text{F}} = 6\text{V}$$

3. (15 pts) Consider three capacitors.  $C_1 = 4 \mu\text{F}$  and  $C_2 = 12 \mu\text{F}$  are in parallel, and  $C_3 = 8 \mu\text{F}$  is in series with them.  $V_c = 8\text{V}$  and  $V_b = 12\text{V}$ . Find the charge and voltage differences for each capacitor. Find  $V_a$ .



$$C_{12} = C_1 + C_2 = 4 \mu\text{F} + 12 \mu\text{F} = 16 \mu\text{F} \quad - 1$$

$$V_{bc} = \Delta V_3 = 12\text{V} - 8\text{V} = 4\text{V} \quad - 2$$

$$Q_3 = C_3 \Delta V_3 = 8 \mu\text{F} \cdot 4\text{V} = 32 \mu\text{C} \quad - 2$$

$$Q_{12} = Q_3 = C_{12} \Delta V_{12} \Rightarrow \Delta V_{12} = V_{ab} = \frac{Q_{12}}{C_{12}}$$

$$= \frac{32 \mu\text{C}}{16 \mu\text{F}} = 2\text{V} \quad - 3$$

$$\therefore V_a = V_b + V_{ab} = 14\text{V} \quad - 1$$

