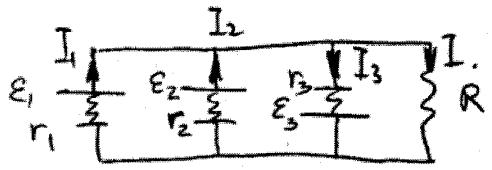


Key yellow

6. (20 pts) For the circuit below, take $\mathcal{E}_1 = 8 \text{ V}$, $\mathcal{E}_2 = 10 \text{ V}$, $\mathcal{E}_3 = 14 \text{ V}$, $r_1 = 0.02 \Omega$, $r_2 = 0.01 \Omega$, $r_3 = 0.03 \Omega$, $R = 0.05 \Omega$. Analyze the circuit using Kirchoff's rules. Solve for the voltage across R . Find the current through R and the currents provided by each of the batteries.



① Draw sign conventions and definitions for currents and voltage across R .

② Current conservation gives $I_1 + I_2 = I_3 + I$.

③ $I_1 = \frac{\mathcal{E}_1 - \Delta V}{r_1}$, $I_2 = \frac{\mathcal{E}_2 - \Delta V}{r_2}$, $I_3 = \frac{\mathcal{E}_3 + \Delta V}{r_3}$, $I = \frac{\Delta V}{R}$.

④ $\frac{\mathcal{E}_1 - \Delta V}{r_1} + \frac{\mathcal{E}_2 - \Delta V}{r_2} = \frac{\mathcal{E}_3 + \Delta V}{r_3} + \frac{\Delta V}{R}$ by combining ② & ③.

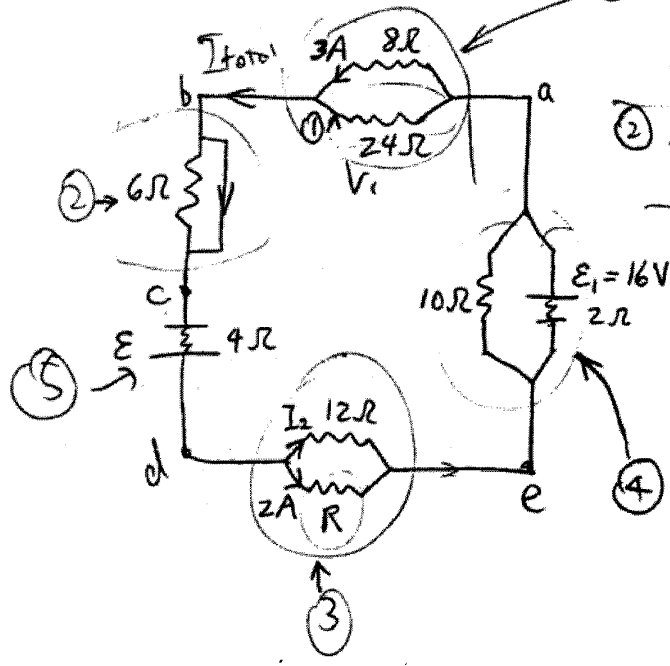
thus $\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2} - \frac{\mathcal{E}_3}{r_3} = \Delta V \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{R} \right)$

or $400 \text{ A} + 1000 \text{ A} - 466.7 \text{ A} = \Delta V \left(50 \frac{\text{A}}{\text{V}} + 100 \frac{\text{A}}{\text{V}} + 33.3 \frac{\text{A}}{\text{V}} + 20 \frac{\text{A}}{\text{V}} \right)$

$\therefore \Delta V = 4.59 \text{ V}$

⑤ $I_1 = \frac{\mathcal{E}_1 - \Delta V}{r_1} = 170.5 \text{ A}$, $I_2 = \frac{\mathcal{E}_2 - \Delta V}{r_2} = 541.0 \text{ A}$, $I_3 = \frac{\mathcal{E}_3 + \Delta V}{r_3} = 619.7 \text{ A}$.
 $I = \frac{\Delta V}{R} = 91.8 \text{ A}$

7. (15 pts) Find the unknown currents, the unknown resistance, and the unknown emf for the circuit in the figure.



① $V_1 = 3 \text{ A} \cdot 8 \Omega = 24 \text{ V} = V_{a \rightarrow b} = V_b - V_a$
 $\therefore I_1 = \frac{24 \text{ V}}{24 \Omega} = 1 \text{ A} \Rightarrow I_{\text{total}} = 1 \text{ A} + 3 \text{ A} = 4 \text{ A}$

② $I_2 = 0 \text{ A}$
 $I_{\text{wire}} = I_{\text{total}} = 4 \text{ A}$, $V_2 = 0 = V_b \rightarrow c$

③ $I_2 + 2 \text{ A} = I_{\text{total}} \Rightarrow I_2 = 2 \text{ A}$
 $I_2 \cdot 12 \Omega = 2 \text{ A} \cdot R \Rightarrow R = 12 \Omega$
 $V_3 = I_2 R = 24 \text{ V} = V_{d \rightarrow e}$

⑤ $\mathcal{E} - I R = \mathcal{E} - 16 \text{ V} = V_{c \rightarrow d}$

$\frac{V_{b \rightarrow a}}{10 \Omega} + \frac{V_{e \rightarrow a} + \mathcal{E}_1}{2 \Omega} = 4 \text{ A}$
 $\Rightarrow V_{e \rightarrow a} + 5 V_{e \rightarrow a} + 5 \mathcal{E}_1 = 40 \text{ A}$
 $V_{e \rightarrow a} = \frac{40 \text{ A} - 5 \mathcal{E}_1}{6} = \frac{40 - 80}{6} = -\frac{20 \text{ V}}{3}$
 $= V_{e \rightarrow a}$

⑤ $\mathcal{E} - 16 \text{ V} = 24 \text{ V} + \frac{20 \text{ V}}{3} = 24 \text{ V} = 0 \Rightarrow \mathcal{E} = 57 \frac{1}{3} \text{ V}$