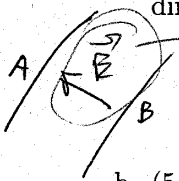


Pink

10. Answer the following questions about voltage.

a. (5 pts) Equipotentials A and B, with  $V_A = -6.84$  V and  $V_B = -6.18$  V, are 0.6 mm apart. At their midway point C, estimate the field (magnitude and direction).



added in solution

$$|\vec{E}_A| \approx \frac{\Delta V}{\Delta s} = \frac{0.66 \text{ V}}{.6 \text{ mm}} = 1.1 \times 10^3 \frac{\text{V}}{\text{m}}$$

$\vec{E}$  points from high V (B) to low V (A).

b. (5 pts) Let  $V(z) = -z^4$ , with V in volts and z in meters. From the voltages at  $z = 0.9$  m and  $z = 1.1$  m, estimate  $\vec{E}$  at  $z = 1.0$  m (magnitude and direction).

$$V(1.1) = -1.4641, \quad V(0.9) = -0.6561 \leftarrow \text{(higher voltage at } 0.9)$$

$\vec{E}$  points from 0.9 to 1.1, or  $\rightarrow$

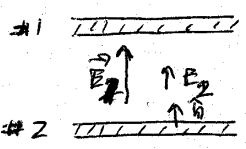
$$|\vec{E}| \approx \frac{|\Delta V|}{|\Delta s|} = \frac{0.808 \text{ V}}{.2 \text{ m}} = 4.04 \frac{\text{V}}{\text{m}}$$

c. (5 pts) For  $V(z) = -z^4$ , with V in volts and z in m, find  $\vec{E}$  at  $z=1.0$  m exactly.

$$E_z = -\frac{dV}{dz} = +4z^3. \quad \text{At } z=1, \quad E_z = +4 \frac{\text{V}}{\text{m}}$$

11. Consider two infinite conducting parallel plates, the top with total charge per unit area  $-3\sigma_0$  and the bottom with total charge per unit area  $+1\sigma_0$  ( $\sigma_0 > 0$ ).

a. (5 pts) Find the total field (magnitude and direction) between the plates.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2\pi k(3\sigma_0)\hat{a} + 2\pi k(\sigma_0)\hat{a} = 8\pi k\sigma_0 \hat{a}$$

b. (5 pts) Find the charge density on the top surface of the bottom plate.

$$\vec{E}_{\text{out}} \cdot \hat{n} = 4\pi k \sigma_s$$

Here  $\vec{E}_{\text{out}} = 8\pi k\sigma_0 \hat{a}$ ,  $\hat{n} = \hat{a}$ , so  $\sigma_s = \frac{\vec{E}_{\text{out}} \cdot \hat{n}}{4\pi k} = 2\sigma_0$ .

12. Two spheres, with radii  $r_A = 2$  cm and  $r_B = 6$  cm, sit upon two distant insulating platforms. Their initial voltages relative to infinity are  $V_A^0 = -50$  V and  $V_B^0 = +50$  V.

a. (5 pts) Find their initial charges  $Q_A^0$  and  $Q_B^0$ .



$$V_A^0 = -50 \text{ V} = \frac{kQ_A^0}{r_A}, \quad Q_A^0 = \frac{V_A^0 r_A}{k} = -1.111 \times 10^{-10} \text{ C}$$

$$Q_B^0 = \frac{V_B^0 r_B}{k} = 3.333 \times 10^{-10} \text{ C}$$

b. (10 pts) Find their final charges  $Q_A$  and  $Q_B$  and their final potentials  $V_A$  and  $V_B$ . They are connected by a thin insulating wire.

$$Q_{\text{tot}} = Q_A^0 + Q_B^0 = 2.222 \times 10^{-10} \text{ C} = Q_A + Q_B$$

In equilibrium,  $V_A = V_B$ , so  $\frac{kQ_A}{r_A} = \frac{kQ_B}{r_B}$ , so  $\frac{Q_A}{Q_B} = \frac{r_A}{r_B} = \frac{1}{3}$ .

$$\text{Thus } 2.222 \times 10^{-10} \text{ C} = Q_A + Q_B = \left(\frac{1}{3} + 1\right) Q_B = \frac{4}{3} Q_B$$

$$\text{so } Q_B = \frac{3}{4} \times 2.222 \times 10^{-10} \text{ C} = 1.667 \times 10^{-10} \text{ C}$$

$$Q_A = \frac{1}{3} Q_B = 0.555 \times 10^{-10} \text{ C}, \quad \text{and } V_A = V_B = \frac{kQ_A}{r_A} = 25 \text{ V}.$$