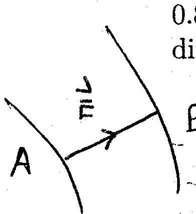
Jun Fu

10. Answer the following questions about voltage.

a. (5 pts) Equipotentials A and B, with $V_A = -5.04$ V and $V_B = -6.16$ V, are 0.8 mm apart. At their midway point C, estimate the field (magnitude and direction).



$$|\vec{E}| \sim \frac{\Delta V}{\Delta B} = \frac{(0.504) - (-6.16)}{0.8 \, mm} = 1400 \frac{V}{m}$$

$\vec{E}$ points from High V (A) to low V (B)

b. (5 pts) Let $V(z) = -z^5$, with V in volts and z in meters. From the voltages at $z = 0.9$ m and $z = 1.1$ m, estimate $\vec{E}$ at $z = 1.0$ m (magnitude and direction).

$V(1.1) = -1.1^5 = -1.61$ V $\quad V(0.9) = -0.9^5 = -0.59$ V

$$|\vec{E}| \sim \left|\frac{\Delta V}{\Delta B}\right| = \left|\frac{(-0.9^5) - (-1.1^5)}{0.2}\right| \frac{V}{m} = 5.10 \frac{V}{m}$$

$\vec{E}$ points from high V (at $z = 0.9$) to low V (at $z = 1.10$)

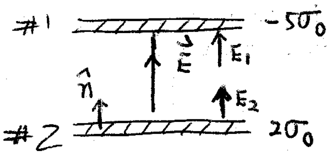
c. (5 pts) For $V(z) = -z^5$, with V in volts and z in m, find $\vec{E}$ at z=1.0 m exactly.

$$E_z = -\frac{dV}{dz} = +5z^4 \; ; \quad \text{at } z = 1, \quad E_z = 5 \frac{V}{m}$$

$\vec{E}$ points in $\hat{z}$ direction.

11. Consider two infinite conducting parallel plates, the top with total charge per unit area $-5\sigma_0$ and the bottom with total charge per unit area $2\sigma_0$ ($\sigma_0 > 0$).

a. (5 pts) Find the total field (magnitude and direction) between the plates.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 2\pi k \cdot (5\sigma_0) \uparrow + 2\pi k (2\sigma_0) \uparrow$$

$$= 14\pi k \sigma_0 \uparrow$$

b. (5 pts) Find the charge density on the top surface of the bottom plate.

since $\vec{E}_{out} \cdot \hat{n} = 4\pi k \sigma_s$, where $\hat{n} = \uparrow$, use (a)

$$\sigma_s = \frac{\vec{E}_{out} \cdot \hat{n}}{4\pi k} = \frac{14\pi k \sigma_0}{4\pi k} = \frac{7}{2} \sigma_0$$

12. Two spheres, with radii $r_A = 4$ cm and $r_B = 8$ cm, sit upon two distant insulating platforms. Their initial voltages relative to infinity are $V_A^0 = -60$ V and $V_B^0 = +15$ V.

a. (5 pts) Find their initial charges Q_A^0 and Q_B^0.

$$V_A = -60V = k\frac{Q_A^0}{r_A} \Rightarrow Q_A = \frac{V_A \cdot r_A}{k} = -2.67 \times 10^{-10} C$$

$$V_B = 15V = k\frac{Q_B^0}{r_B} \Rightarrow Q_B = \frac{V_B \cdot r_B}{k} = 1.33 \times 10^{-10} C$$

b. (10 pts) Find their final charges Q_A and Q_B and their final potentials V_A and V_B.

Total charge $Q_{total} = Q_A^{(0)} + Q_B^{(0)} = -1.34 \times 10^{-10} C = Q_A + Q_B$

in equilibrium, $V_A = V_B$, $\quad \frac{kQ_A}{r_A} = k\frac{Q_B}{r_B}$, so $\frac{Q_A}{Q_B} = \frac{r_A}{r_B} = \frac{1}{2}$

$Q_A + Q_B = -1.34 \times 10^{-10} C \Rightarrow \frac{3}{2} Q_B = -1.33 \times 10^{-10} C \Rightarrow Q_B = -0.886 \times 10^{-10} C$

$Q_A = \frac{1}{2} Q_B = -0.443 \times 10^{-10} C$

$\therefore V_A = V_B = \frac{kQ_A}{r_A} = 10 V$