

Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.

1. Two capacitors  $C_1 = 21 \mu\text{F}$  and  $C_2 = 42 \mu\text{F}$  have charges  $Q_1 = 8 \mu\text{C}$  and  $Q_2 = +23 \mu\text{C}$ , plates a1 and a2 being +, b1 and b2 being -.
- a. (8 pts) Find the voltage differences and electrical energy for each capacitor.

$$C_1 \begin{array}{|c|} \hline a_1 \\ \hline b_1 \\ \hline \end{array} \quad \begin{array}{|c|} \hline a_2 \\ \hline b_2 \\ \hline \end{array} C_2$$

$$\Delta V_1 = V_{a_1} - V_{b_1} = \Delta V_1 = \frac{Q_1}{C_1} = \frac{8 \mu\text{C}}{21 \mu\text{F}} = 0.381 \text{ V}, \quad U_1 = \frac{1}{2} C_1 (\Delta V_1)^2 = 1.52 \mu\text{J}$$

$$\Delta V_2 = V_{a_2} - V_{b_2} = \Delta V_2 = \frac{Q_2}{C_2} = \frac{23 \mu\text{C}}{42 \mu\text{F}} = 0.547 \text{ V}, \quad U_2 = \frac{1}{2} C_2 (\Delta V_2)^2 = 6.30 \mu\text{J}$$

- b. (12 pts) Now connect the plates, a1 to a2 and b1 to b2. Find  $V_{a1} - V_{b1}$  and  $V_{a2} - V_{b2}$ , the charges on each plate, and the electrical energy for each capacitor.

$$C_1 \begin{array}{|c|} \hline a_1 \\ \hline b_1 \\ \hline \end{array} \quad \begin{array}{|c|} \hline a_2 \\ \hline b_2 \\ \hline \end{array} C_2$$

$$C_1 \text{ \& } C_2 \text{ are in parallel, with } Q = Q_{a_1} + Q_{a_2} = 8 + 23 = 31 \mu\text{C}$$

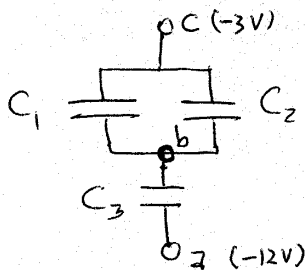
$$\Delta V' = V_{a_1} - V_{b_1} = V_{a_2} - V_{b_2} = \frac{Q}{C_1 + C_2} = \frac{31}{63} = 0.492 \text{ V}$$

$$U_1' = \frac{1}{2} C_1 (\Delta V')^2 = 2.54 \mu\text{J} \quad U_2' = \frac{1}{2} C_2 (\Delta V')^2 = 5.08 \mu\text{J}$$

$$Q_1' = C_1 \Delta V' = 21 \cdot 0.492 = 10.3 \quad Q_2' = C_2 \Delta V' = 42 \cdot 0.492 = 20.3$$

note:  $U_{\text{final}} = U_1' + U_2' = 7.62 < U_{\text{initial}} = U_1 + U_2 = 7.82$ , as expected.

2. (15 pts) Consider three capacitors.  $C_1 = 65 \mu\text{F}$  and  $C_2 = 25 \mu\text{F}$  are in parallel, and  $C_3 = 30 \mu\text{F}$  is in series with them.  $V_c = -3 \text{ V}$  and  $V_a = -12 \text{ V}$ . Find the charge and voltage difference for each capacitor. Find  $V_b$ .



$$C' = C_1 + C_2 = 90 \mu\text{F}$$

$$C = (C_3^{-1} + C'^{-1})^{-1} = (30^{-1} + 90^{-1})^{-1} = 22.5 \mu\text{F}$$

$$Q = C \cdot \Delta V = 22.5 \cdot 9 = 202.5 \mu\text{C}$$

$$Q_3 = Q \text{ (top positive)}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{202.5 \mu\text{C}}{30 \mu\text{F}} = 6.75 \text{ V}$$

$$\text{Thus } \Delta V_1 = \Delta V_2 = 9 - 6.75 = 2.25 \text{ V}$$

$$Q_1 = C_1 \Delta V_1 = 146 \mu\text{C} \quad (\text{note: } Q_1 + Q_2 = Q)$$

$$Q_2 = C_2 \Delta V_2 = 56.5 \mu\text{C}$$

$$V_b = V_a + \Delta V_3 = -12 + 6.75 = -5.25 \text{ V}$$