

Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.

1. Two capacitors $C_1 = 36 \mu\text{F}$ and $C_2 = 18 \mu\text{F}$ have charges $Q_1 = 12 \mu\text{C}$ and $Q_2 = +27 \mu\text{C}$, plates a1 and a2 being +, b1 and b2 being -.

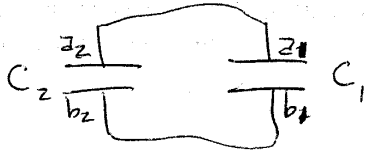
a. (8 pts) Find the voltage differences and electrical energy for each capacitor.

$$C_2 \begin{array}{|c|} \hline a_2 \\ \hline b_2 \\ \hline \end{array} \quad \begin{array}{|c|} \hline a_1 \\ \hline b_1 \\ \hline \end{array} C_1$$

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{12}{36} = 0.333 \text{ V}, \quad \Delta V_2 = \frac{Q_2}{C_2} = \frac{27}{18} = 1.5 \text{ V}$$

$$U_1 = \frac{1}{2} C_1 (\Delta V_1)^2 = 2 \mu\text{J}, \quad U_2 = \frac{1}{2} C_2 (\Delta V_2)^2 = 20.25 \mu\text{J}$$

- b. (12 pts) Now connect the plates, a1 to a2 and b1 to b2. Find $V_{a1} - V_{b1}$ and $V_{a2} - V_{b2}$, the charges on each plate, and the electrical energy for each capacitor.



C_1 & C_2 are in parallel, with $Q_{\text{total}} = Q_1 + Q_2 = 27 + 12 = 39 \mu\text{C}$

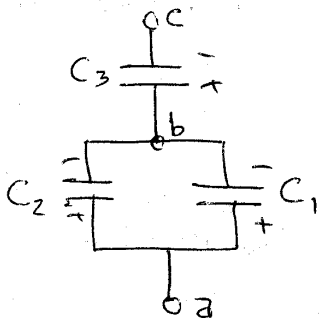
Then $\Delta V' = (V_{a1} - V_{b1}) = (V_{a2} - V_{b2}) = \frac{Q_{\text{total}}}{C_1 + C_2} = \frac{39}{54} = 0.722 \text{ V}$

$$U_1' = \frac{1}{2} C_1 (\Delta V')^2 = 10.25 \mu\text{J}, \quad U_2' = \frac{1}{2} C_2 (\Delta V')^2 = 4.69 \mu\text{J}$$

$$Q_2' = C_2 \Delta V' = 13 \mu\text{C}, \quad Q_1' = C_1 \Delta V' = 26 \mu\text{C}$$

Note: $U_1' + U_2' = 14.94 \mu\text{J} = U_{\text{final}}$; $U_1 + U_2 = 22.25 \mu\text{J} = U_{\text{initial}}$

2. (15 pts) Consider three capacitors. $C_1 = 35 \mu\text{F}$ and $C_2 = 55 \mu\text{F}$ are in parallel, and $C_3 = 45 \mu\text{F}$ is in series with them. $V_c = -30 \text{ V}$ and $V_a = -12 \text{ V}$. Find the charge and voltage difference for each capacitor. Find V_b .



$C' = C_1 + C_2 = 90 \mu\text{F}$

$C = (C_3^{-1} + C'^{-1})^{-1} = (45^{-1} + 90^{-1})^{-1} = 30 \mu\text{F}$

$Q = C \Delta V = (30 \mu\text{F})(18 \text{ V}) = 540 \mu\text{C}$

$Q_3 = Q$ (top is negative)

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{540 \mu\text{C}}{45 \mu\text{F}} = 12 \text{ V}$$

Thus $V_b = V_c + \Delta V_3 = -30 + 12 = -18 \text{ V}$

$$\Delta V_1 = \Delta V_2 = V_a - V_b = 6 \text{ V} \text{ (top are negative)}$$

$$Q_1 = C_1 \Delta V_1 = 210 \mu\text{C}$$

$$Q_2 = C_2 \Delta V_2 = 330 \mu\text{C}$$

Note: $Q_1 + Q_2 = 540 \mu\text{C} = Q$, as expected