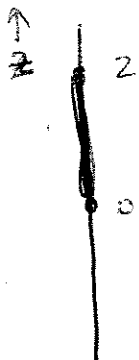


Don't waste time on questions you aren't sure of. Be clear and concise. A cluttered response, some of which is correct and some of which is incorrect, will not get full credit.

1. (7 pts) A non-conducting rod lies on the z-axis from (0, 0, 0) to (0, 0, 2a), where a is a constant. It has charge per unit length $\lambda = 8\alpha z^3$, where α is a constant. What units must a and α have? In terms of α and a, find the total charge Q on the rod, and the average charge per unit length $\bar{\lambda}$.



a is in meters (m)
 α is in coulombs/m⁴ (C/m⁴)

$$Q = \int dQ = \int \frac{dQ}{ds} ds = \int_0^{2a} \lambda dz = \int_0^{2a} 8\alpha z^3 dz = 8\alpha \cdot \frac{z^4}{4} \Big|_0^{2a}$$

$$= 8\alpha \frac{16a^4}{4} = 32\alpha a^4$$

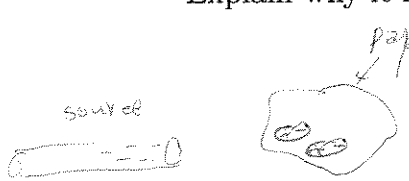
$$\bar{\lambda} = \frac{Q}{\int ds} = \frac{Q}{\int_0^{2a} dz} = \frac{Q}{2a} = 16\alpha a^3$$

2. (7 pts) Conducting globes A and B, on insulating bases, initially are neutral. An insulating charged rod with +5 units of charge is held near A, without contact or sparking, as in the figure. Next, B is made to contact A and then is withdrawn. Finally, the charged rod is removed. A second figure, with +6 units of charge on A and -4 units of charge on B, violates what physics principles?



1. A should have negative, not positive charge ^{by electrostatic induction}
2. ~~A~~ A should have charge of magnitude less than or equal to the magnitude of the source charge (5).
3. B should have positive, not negative charge, by electrostatic induction.
4. A and B should have equal and opposite charge, ^{by charge conservation.}

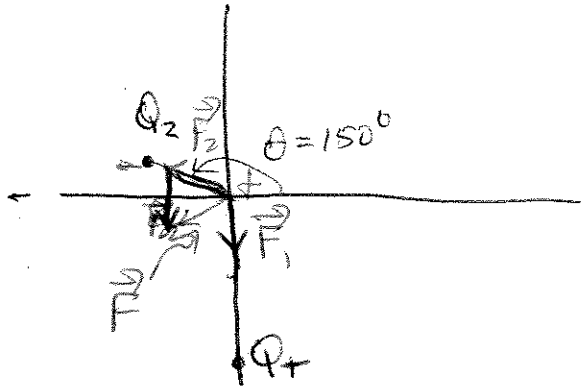
3. (6 pts) For the benefit of Bart Simpson's teacher, concisely describe the amber effect. Explain why it is attractive, using a simple figure with a negative source charge.



- The source polarizes the paper (neutral)
- In the amber effect, a charged object (source) attracts a neutral object (N.O.)
1. The source polarizes the N.O. (still neutral)
 2. The nearer ^(further) charge on the N.O. is attracted ^(is pulled by) to the source.
 3. The electric force falls off with distance.
 4. Hence, the attraction of the nearer charge dominates.

Key - Pink

4. (21 pts) A point charge $Q_1 = -8.0 \times 10^{-9}$ C is on the negative y -axis at $r_1 = 4$ cm from the origin. A point charge $Q_2 = -3.0 \times 10^{-9}$ C makes a counterclockwise angle $\theta = 150^\circ$ to the positive x -axis, at $r_2 = 2$ cm from the origin. A charge $Q = 3.0 \times 10^{-9}$ C is placed at the origin. Q_1 and Q_2 act on Q with forces \vec{F}_1 and \vec{F}_2 .



$$|\vec{F}_1| = \frac{k|Q_1Q|}{r_1^2} = \frac{9 \times 10^9 \frac{N \cdot m^2}{C^2} (3 \times 10^{-9}) (8 \times 10^{-9})}{(0.04 \text{ m})^2}$$

$$= 1.35 \times 10^{-4} \text{ N}$$

$$|\vec{F}_2| = \frac{k|Q_2Q|}{r_2^2} = \frac{(9 \times 10^9) (3 \times 10^{-9}) (3 \times 10^{-9})}{(0.02 \text{ m})^2}$$

$$= 2.025 \times 10^{-4} \text{ N}$$

- a. Find $|\vec{F}_1|$ and $|\vec{F}_2|$.

2 - 1

- b. On the figure, draw \vec{F}_1 and \vec{F}_2 with their tails on Q , and in relative proportion.

- c. Find F_x , the x -component of the total force \vec{F} on Q .

$$F_x = |\vec{F}_2| \cos 150^\circ = -1.754 \times 10^{-4} \text{ N}$$

- d. Find F_y , the y -component of \vec{F} .

$$F_y = |\vec{F}_2| \sin 150^\circ - |\vec{F}_1| = -3.375 \times 10^{-5} \text{ N}$$

- e. Find the angle of \vec{F} with respect to the x -axis. Sketch the direction of \vec{F} .

θ is in the 3rd quadrant

$$\tan \theta = \frac{F_y}{F_x}, \quad \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) + 180^\circ = +10.89 + 180 = 190.89^\circ$$

- f. Find $|\vec{F}|$.

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 1.786 \times 10^{-4} \text{ N}$$

- g. Q_1 and Q_2 are rotated clockwise 25 degrees about the origin. Find the new F_x .

$$F_x = |\vec{F}| \cos(\theta - 25) = (1.786 \times 10^{-4} \text{ N}) \cos 165.89 = -1.732 \times 10^{-4} \text{ N}$$

5. Two charges Q are on the x -axis, A at $(a, 0)$ and B at $(-a, 0)$.

a. (3 pts) Find the electric field at the origin.



$$\vec{E}_A + \vec{E}_B = \frac{kq}{a^2} \hat{x} + \frac{kq}{a^2} \hat{x} = \vec{0}$$

b. (3 pts) Find the electrical potential at the origin.

$$V = V_A + V_B = \frac{kq}{a} + \frac{kq}{a} = \frac{2kq}{a}$$

6. Two line charges are normal to the page. A, with charge density 6λ , passes through the origin. B, with charge density 2λ , passes through $(6a, 0, 0)$.

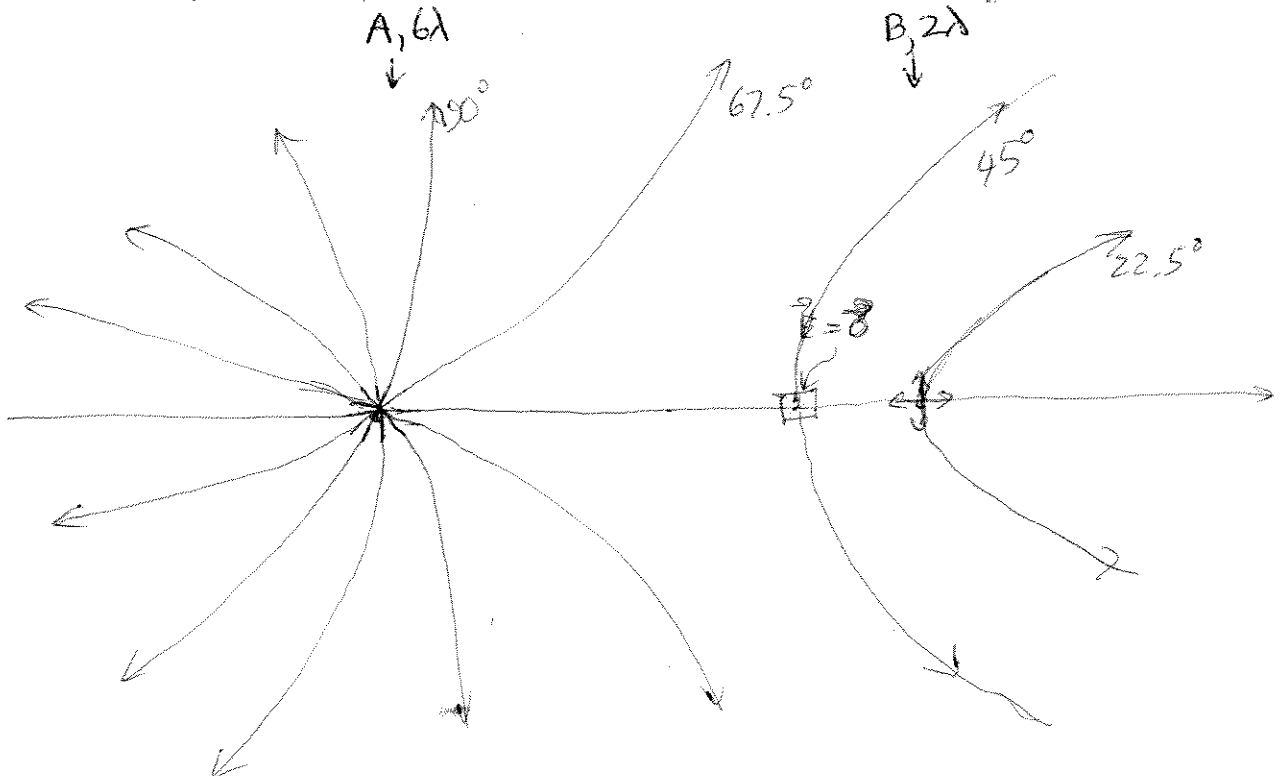
a. (4 pts) Find the position $(s, 0, 0)$ where the electric field is zero.

It is zero somewhere between A and B, so $s < 6a$.
 At $(s, 0, 0)$, $|\vec{E}_A| = |\vec{E}_B|$, or $\frac{2k(6\lambda)}{s} = \frac{2k(2\lambda)}{6a-s}$, so $3(6a-s) = s$,
 or $18a = 4s$, so $s = \frac{9}{2}a$.

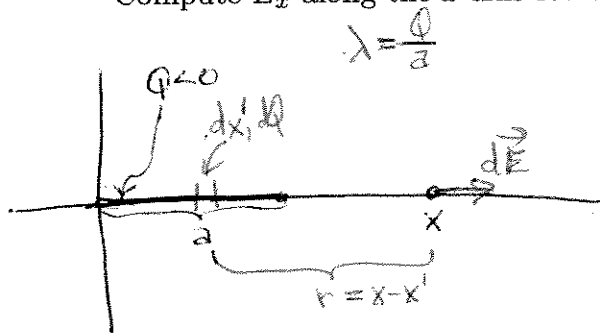
b. (4 pts) If λ is represented by two field lines, find the angle between the field lines as they originate from A. Repeat for B. Repeat for the angle between the field lines as viewed from far away.

For A, $6 \times 2 = 12$ lines, at $\frac{360^\circ}{12} = 30^\circ$ to one another
 For B, $2 \times 2 = 4$ " " " $\frac{360^\circ}{4} = 90^\circ$ " "
 From far away, 16 " " " $\frac{360^\circ}{16} = 22.5^\circ$ " "

c. (6 pts) Sketch the field lines for this geometry. Take one field line from A to go directly to the left, and take one field line from B to go directly to the right.



7. (6 pts) A negative charge ($Q < 0$) is uniformly distributed from the origin to $(a, 0)$. Compute E_x along the x -axis for $x > a$.



$$\lambda = \frac{Q}{a}$$

$$dE_x = |d\vec{E}| = \frac{k dQ}{r^2} = \frac{k \lambda dx'}{(x-x')^2}$$

$$= \frac{kQ}{a} \frac{dx'}{(x-x')^2}$$

$$E_x = \int dE_x = \frac{kQ}{a} \int_0^a \frac{dx'}{(x-x')^2} = \frac{kQ}{a} \int_0^a \frac{dx'}{(x'-x)^2}$$

$$= \frac{kQ}{a} \left(-\frac{1}{x-x'} \right) \Big|_0^a = \frac{kQ}{a} \left[-\left(\frac{1}{a-x} - \frac{1}{-x} \right) \right]$$

$$= \frac{kQ}{a} \left[\frac{1}{x-a} - \frac{1}{x} \right] = \frac{kQ}{ax(x-a)} [x - (x-a)] = \frac{kQ}{x(x-a)}$$

8. For a negatively-charged conductor, a surface element of area $dA = 4.5 \times 10^{-6} \text{ m}^2$ has its outward normal \hat{n} along $(3, 8, -4.5)$. For this element, $|\vec{E}| = 270 \text{ V/m}$.

- a. (3 pts) Find \hat{n} .

$$\hat{n} = \frac{(3, 8, -4.5)}{\sqrt{3^2 + 8^2 + (-4.5)^2}} = \frac{(3, 8, -4.5)}{9.179} = (0.327, 0.872, -0.490)$$

- b. (3 pts) Find the direction of \vec{E} , called \hat{E} .

\hat{E} is opposite \hat{n} , so $\hat{E} = -\hat{n} = (-0.327, -0.872, 0.490)$

(Surface is negatively charged, so field lines enter conductor)

- c. (3 pts) Find the flux $d\Phi_E$ through dA .

$$d\Phi_E = \frac{d\Phi_E}{dA} dA = \vec{E} \cdot \hat{n} dA = |\vec{E}| \hat{E} \cdot \hat{n} dA = |\vec{E}| (-\hat{n}) \cdot \hat{n} dA$$

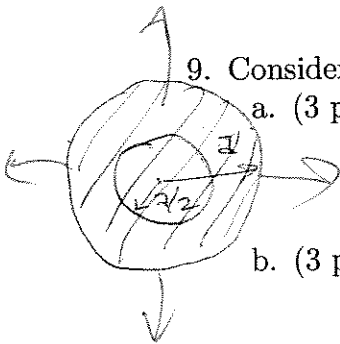
$$= -|\vec{E}| dA = -(270 \text{ V/m}) (4.5 \times 10^{-6} \text{ m}^2) = -1.215 \times 10^{-3} \text{ V}\cdot\text{m}$$

- d. (3 pts) Find the surface charge dQ_s .

$$d\Phi_E = 4\pi k dQ_s, \text{ so } dQ_s = \frac{d\Phi_E}{4\pi k} = 1.074 \times 10^{-14} \text{ C}$$

9. Consider a ball of charge of radius a with uniform charge density $\rho > 0$.

- a. (3 pts) Sketch the field lines.



radially outward

- b. (3 pts) For a concentric sphere with $r = a/2$, find the charge enclosed.

$$Q_{enc} = \rho \frac{4\pi}{3} r^3 = \rho \frac{4\pi}{3} \left(\frac{a}{2}\right)^3 = \frac{Q_{tot}}{8} = \frac{\pi}{6} \rho a^3$$

- c. (3 pts) Using Gauss's Law and symmetry, find $d\Phi_E/dA$ for $r = a/2$.

$$\frac{d\Phi_E}{dA} = \frac{\Phi_E}{A} = \frac{4\pi k Q_{enc}}{A} = \frac{4\pi k \rho \frac{4\pi}{3} \left(\frac{a}{2}\right)^3}{4\pi \left(\frac{a}{2}\right)^2} = \frac{kQ_{tot}}{2a^2} = \frac{2k\pi\rho a}{3}$$

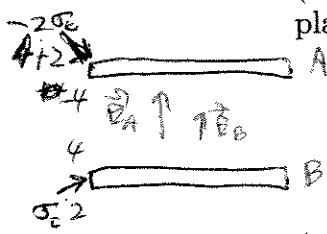
- d. (3 pts) Find E_r for $r = a/2$.

$$\vec{E} \cdot \hat{n} = \vec{E} \cdot \hat{r} = E_r = \frac{d\Phi_E}{dA} = \frac{kQ_{tot}}{2a^2} = \frac{2k\pi\rho a}{3}$$

Key - Pink

10. Assume that the charged conducting sheets in the figure are infinite in extent. The one on the top has total charge per unit area $-2\sigma_0$, and the one on the bottom has a total charge per unit area $6\sigma_0$, where $\sigma_0 > 0$.

a. (3 pts) Find the total electric field (in magnitude and direction) between the plates.



$$\vec{E} = \vec{E}_A + \vec{E}_B = 2\pi k (2\sigma_0) \hat{n} + 2\pi k (6\sigma_0) \hat{n} = 16\pi k \sigma_0 \hat{n}$$

b. (3 pts) Find the charge density on the top surface of the bottom sheet.

$$\vec{E} \cdot \hat{n} = 4\pi k \sigma_s, \text{ so } \sigma_s = \frac{\vec{E} \cdot \hat{n}}{4\pi k} = \frac{(16\pi k \sigma_0 \hat{n}) \cdot (\hat{n})}{4\pi k} = 4\sigma_0$$

$\hat{n} = \uparrow$ for top surface of bottom sheet

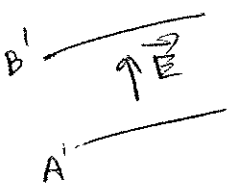
c. (3 pts) Which plate is at the higher voltage, and by how much (expressed in terms of σ)?

\vec{E} points from B to A, so $V_B > V_A$.

$$V_B - V_A = |\vec{E}| d = 16\pi k \sigma_0 d$$

11. Answer the following questions about voltage.

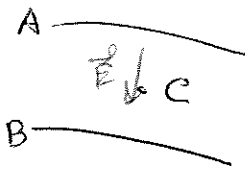
a. (3 pts) For two equipotentials A' and B' , 1.5 cm apart, the electric field between them is of magnitude 90 N/C, and points to B' . If $V_{A'} = -2.6$ V, estimate $V_{B'}$.



$$V_A > V_B, \Delta V \approx |\vec{E}| d = (90 \text{ N/C})(0.015 \text{ m}) = 1.35 \text{ V}$$

$$V_B = V_A - \Delta V = -2.6 \text{ V} - 1.35 \text{ V} = -3.95 \text{ V}$$

b. (3 pts) Equipotentials A and B have $V_A = 8.4$ V and $V_B = 8.1$ V, and a separation of 6 mm. For a point C midway between them, estimate the electric field (magnitude and direction).



\vec{E} is from A to B (see figure)

$$|\vec{E}| \approx \frac{\Delta V}{\Delta s} = \frac{0.3 \text{ V}}{6 \times 10^{-3} \text{ m}} = 50 \frac{\text{V}}{\text{m}}$$

c. (3 pts) On which equipotential in part b) will an electron have the lower energy, and by how much?

Lower potential energy on more positive equipotential (A)

$$|\Delta U| = |e| |\Delta V| = (1.6 \times 10^{-19} \text{ C})(0.3 \text{ V}) = 0.48 \times 10^{-19} \text{ J}$$

d. (3 pts) If the electron starts at rest from the equipotential giving it the lower energy, find its speed when it strikes the other equipotential.

$$\frac{1}{2} m v_B^2 + (-e)V_B = \frac{1}{2} m v_A^2 + (-e)V_A$$

$v_B = 0$

$$\frac{1}{2} m v_A^2 = (-e)(V_B - V_A)$$

$$v_A = \sqrt{\frac{2e}{m} (V_A - V_B)} = 3.248 \times 10^5 \text{ m/s}$$

incorrect statement
starts at higher potential energy

$e = 1.6 \times 10^{-19} \text{ C}, m = 9.1 \times 10^{-31} \text{ kg}, V_A - V_B = 0.3 \text{ V}$

Don't waste time on questions you aren't sure of. Be clear and concise. A cluttered response, some of which is correct and some of which is incorrect, will not get full credit.

1. (7 pts) A non-conducting rod lies on the z -axis from $(0, 0, 0)$ to $(0, 0, b)$, where b is a constant. It has charge per unit length $\lambda = 6\alpha z^2$, where α is a constant. What units must b and α have? In terms of α and b , find the total charge Q on the rod, and the average charge per unit length $\bar{\lambda}$.



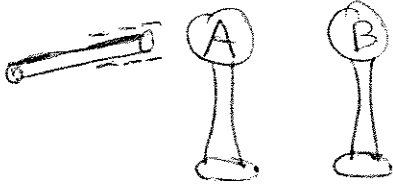
The unit of b is meters (m)

The unit of α is coulombs/m³ (C/m³)

$$Q = \int dQ = \int_0^b \lambda dz = \int_0^b 6\alpha z^2 dz = 6\alpha \left(\frac{1}{3} z^3 \right) \Big|_0^b = 2\alpha z^3 \Big|_0^b = 2\alpha b^3$$

$$\bar{\lambda} = \frac{Q}{b} = \frac{2\alpha b^3}{b} = 2\alpha b^2$$

2. (7 pts) Conducting globes A and B, on insulating bases, initially are neutral. An insulating charged rod with -6 units of charge is held near A, without contact or sparking, as in the figure. Next, B is made to contact A and then is withdrawn. Finally, the charged rod is removed. A second figure, with -5 units of charge on A and $+7$ units of charge on B, violates what physics principles?

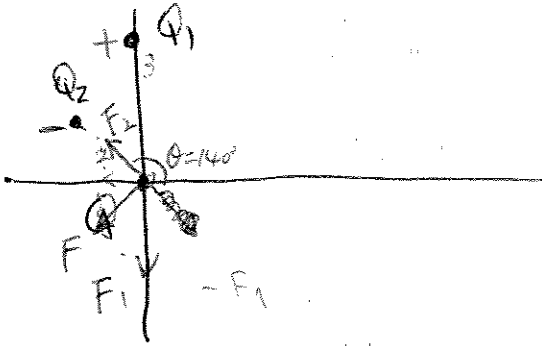


1. A should have positive charge by electrostatic induction
2. B should have charge magnitude less than or equal to the magnitude of the source charge (6)
3. B should have negative charge by electrostatic induction
4. A and B should have equal and opposite charge

3. (6 pts) For the benefit of Bart Simpson's teacher, concisely describe the amber effect. Explain why it is attractive, using a simple figure with a negative source charge.

The same as for pink solution.

4. (21 pts) A point charge $Q_1 = 6.0 \times 10^{-9} \text{ C}$ is on the positive y -axis at $r_1 = 3 \text{ cm}$ from the origin. A point charge $Q_2 = -3.0 \times 10^{-9} \text{ C}$ makes a counterclockwise angle $\theta = 140^\circ$ to the positive x -axis, at $r_2 = 2 \text{ cm}$ from the origin. A charge $Q = 5.0 \times 10^{-9} \text{ C}$ is placed at the origin. Q_1 and Q_2 act on Q with forces \vec{F}_1 and \vec{F}_2 .



- a. Find $|\vec{F}_1|$ and $|\vec{F}_2|$.

$$|\vec{F}_1| = \frac{k|Q_1Q|}{r_1^2} = \frac{9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} |6.0 \times 10^{-9} \text{ C} \times 5.0 \times 10^{-9} \text{ C}|}{(0.03 \text{ m})^2} = 3 \times 10^{-4} \text{ N}$$

$$|\vec{F}_2| = \frac{k|Q_2Q|}{r_2^2} = \frac{9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} |-3.0 \times 10^{-9} \text{ C} \times 5.0 \times 10^{-9} \text{ C}|}{(0.02 \text{ m})^2} = 3.375 \times 10^{-4} \text{ N}$$

- b. On the figure, draw \vec{F}_1 and \vec{F}_2 with their tails on Q , and in relative proportion.

- c. Find F_x , the x -component of the total force \vec{F} on Q .

$$F_x = |\vec{F}_2| \cos 140^\circ = 3.375 \times 10^{-4} \text{ N} \times \cos 140^\circ = -2.59 \times 10^{-4} \text{ N}$$

- d. Find F_y , the y -component of \vec{F} .

$$F_y = |\vec{F}_2| \sin 140^\circ - |\vec{F}_1| = 3.375 \times 10^{-4} \text{ N} \times \sin 140^\circ - 3 \times 10^{-4} \text{ N} = -8.3 \times 10^{-5} \text{ N}$$

- e. Find the angle of \vec{F} with respect to the x -axis. Sketch the direction of \vec{F} .

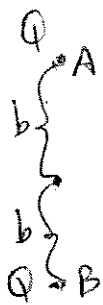
$$\tan \theta = \frac{F_y}{F_x} = 0.32, \quad \theta = \tan^{-1}(0.32) + 180^\circ = 17.8^\circ + 180^\circ = 197.8^\circ$$

- f. Find $|\vec{F}|$.

$$|\vec{F}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2} = \sqrt{(-2.59 \times 10^{-4} \text{ N})^2 + (-8.3 \times 10^{-5} \text{ N})^2} = 2.7 \times 10^{-4} \text{ N}$$

- g. Q_1 and Q_2 are rotated clockwise 35 degrees about the origin. Find the new F_x .

$$F_x = |\vec{F}| \cos(197.8^\circ - 35^\circ) = 2.7 \times 10^{-4} \text{ N} \times \cos(162.8^\circ) = -2.58 \times 10^{-4} \text{ N}$$



5. Two charges Q are on the y -axis, A at $(0, b)$ and B at $(0, -b)$.

a. (3 pts) Find the electric field at the origin.

$$\vec{E}_A + \vec{E}_B = \frac{kQ}{b^2} (-\hat{j}) + \frac{kQ}{b^2} \hat{j} = 0$$

b. (3 pts) Find the electrical potential at the origin.

$$V_A + V_B = \frac{kQ}{b} + \frac{kQ}{b} = \frac{2kQ}{b}$$

6. Two line charges are normal to the page. A, with charge density -4λ , passes through the origin. B, with charge density 2λ , passes through $(4a, 0, 0)$.

a. (4 pts) Find the position $(s, 0, 0)$ where the electric field is zero.

$$\text{At } (s, 0, 0), \quad \frac{2k(2\lambda)}{s-4a} + \frac{2k(-4\lambda)}{s} = 0$$

$$s = 8a$$

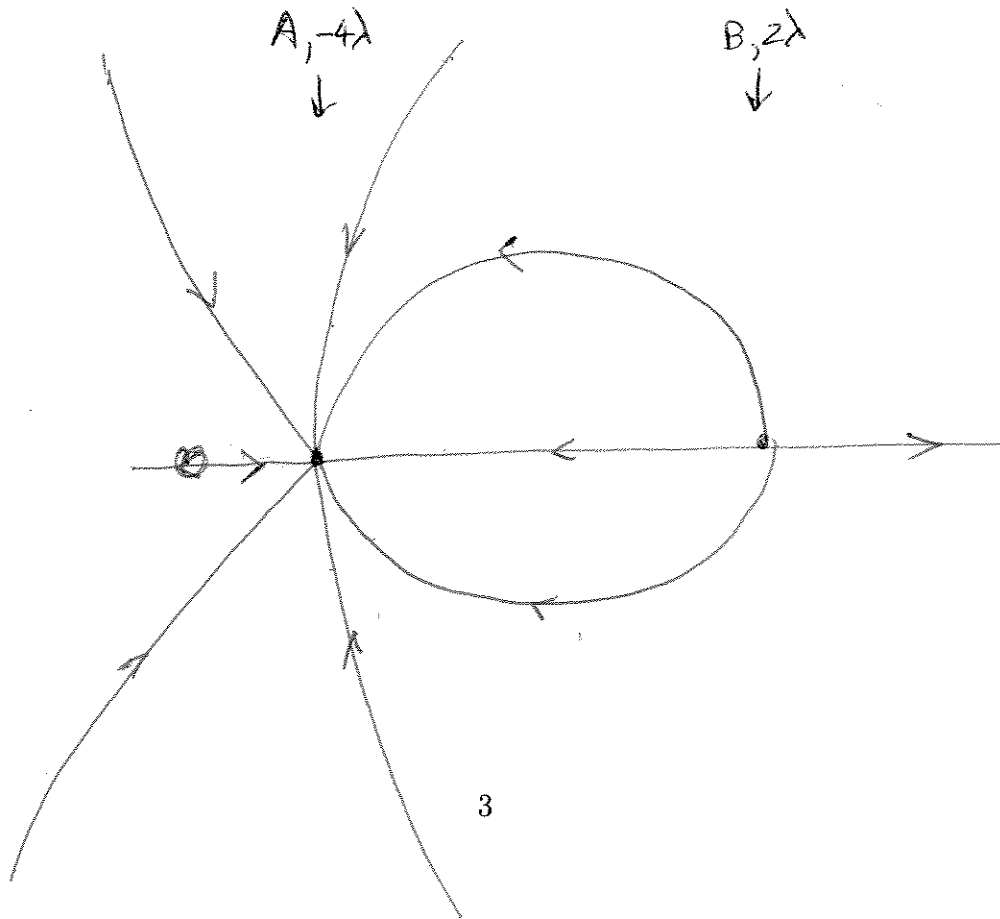
b. (4 pts) If λ is represented by two field lines, find the angle between the field lines as they originate from A. Repeat for B. Repeat for the angle between the field lines as viewed from far away.

$$\text{For A, } 4 \times 2 = 8, \quad \frac{360^\circ}{8} = 45^\circ$$

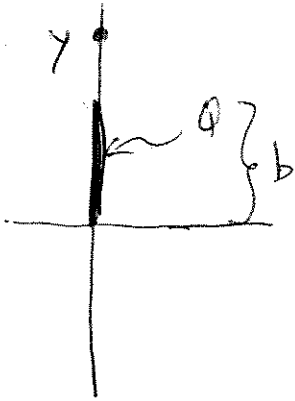
$$\text{For B, } 2 \times 2 = 4, \quad \frac{360^\circ}{4} = 90^\circ$$

$$\text{From far away, } 8 - 4 = 4, \quad \frac{360^\circ}{4} = 90^\circ$$

c. (6 pts) Sketch the field lines for this geometry. Take one field line from A to go directly to the left, and take one field line from B to go directly to the right.



7. (6 pts) A positive charge ($Q > 0$) is uniformly distributed from the origin to $(0, b)$. Compute E_y along the y -axis for $y > b$.



$$dE_y = \frac{k dQ}{r^2} = \frac{k \lambda dy'}{(y-y')^2}$$

$$E_y = \int_0^b dE_y = \int_0^b \frac{k \lambda dy'}{(y-y')^2} = k \lambda \int_0^b \frac{dy'}{(y-y')^2}$$

$$= k \lambda \left(-\frac{1}{y-y'} \right) \Big|_0^b = k \lambda \left(-\frac{1}{b-y} + \frac{1}{0-y} \right)$$

$$= k \lambda \frac{b}{y(y-b)} = k \frac{Q}{b} \frac{b}{y(y-b)} = \frac{kQ}{y(y-b)}$$

8. For a negatively-charged conductor, a surface element of area $dA = 3.8 \times 10^{-6} \text{ m}^2$ has its outward normal \hat{n} along $(7, 2, -4.5)$. For this element, $|\vec{E}| = 160 \text{ V/m}$.

- a. (3 pts) Find \hat{n} .

$$\hat{n} = \frac{(7, 2, -4.5)}{\sqrt{7^2 + 2^2 + (-4.5)^2}} = \frac{(7, 2, -4.5)}{8.56} = (0.82, 0.23, -0.53)$$

- b. (3 pts) Find the direction of \vec{E} , called \hat{E} .

$$\vec{E} \text{ is opposite to } \hat{n}, \quad \hat{E} = -\hat{n} = (-0.82, -0.23, 0.53)$$

- c. (3 pts) Find the flux $d\Phi_E$ through dA .

$$d\vec{\Phi}_E = \frac{d\vec{\Phi}_E}{dA} dA = \vec{E} \cdot \hat{n} dA = |\vec{E}| \hat{E} \cdot \hat{n} dA = |\vec{E}| (-\hat{n}) \cdot (\hat{n}) dA$$

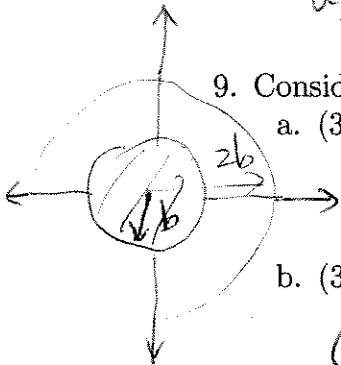
$$= -|\vec{E}| dA = -(160 \text{ V/m}) (3.8 \times 10^{-6} \text{ m}^2) = -6.08 \times 10^{-4} \text{ V}\cdot\text{m}$$

- d. (3 pts) Find the surface charge dQ_s .

$$d\vec{\Phi}_E = 4\pi k dQ_s, \quad dQ_s = \frac{d\vec{\Phi}_E}{4\pi k} = \frac{-6.08 \times 10^{-4} \text{ V}\cdot\text{m}}{4\pi \times 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = -5.3 \times 10^{-15} \text{ C}$$

9. Consider a ball of charge of radius b with uniform charge density $\rho > 0$.

- a. (3 pts) Sketch the field lines.



- b. (3 pts) For a concentric sphere with $r = 2b$, find the charge enclosed.

$$Q_{en} = \frac{4}{3} \pi b^3 \rho$$

- c. (3 pts) Using Gauss's Law and symmetry, find $d\Phi_E/dA$ for $r = 2b$.

$$\frac{d\vec{\Phi}_E}{dA} = \frac{\vec{\Phi}_E}{A} = \frac{4\pi k Q_{en}}{4\pi (2b)^2} = \frac{k Q_{en}}{4b^2} = \frac{k}{4b^2} \cdot \frac{4}{3} \pi b^3 \rho = \frac{1}{3} \pi k b \rho$$

- d. (3 pts) Find E_r for $r = 2b$.

$$E_r = \frac{d\vec{\Phi}_E}{dA} = \frac{1}{3} \pi k b \rho$$

10. Assume that the charged conducting sheets in the figure are infinite in extent. The one on the top has total charge per unit area $2\sigma_0$, and the one on the bottom has a total charge per unit area $4\sigma_0$, where $\sigma_0 > 0$.

- a. (3 pts) Find the total electric field (in magnitude and direction) between the plates.



$$\vec{E} = \vec{E}_A + \vec{E}_B = 2\pi k(2\sigma_0)\hat{j} + 2\pi k(4\sigma_0)\hat{j} = 4\pi k6_0\hat{j}$$



- b. (3 pts) Find the charge density on the top surface of the bottom sheet.

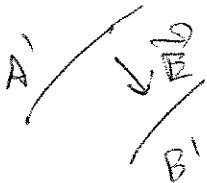
$$\vec{E} \cdot \hat{n} = 4\pi k6_0, \text{ so } \sigma_s = \frac{\vec{E} \cdot \hat{n}}{4\pi k} = \frac{(4\pi k6_0\hat{j}) \cdot (\hat{j})}{4\pi k} = 6_0$$

- c. (3 pts) Which plate is at the higher voltage, and by how much (expressed in terms of σ)? The bottom one has higher voltage.

$$\Delta V = |\vec{E}| \cdot d = 4\pi k6_0 d$$

11. Answer the following questions about voltage.

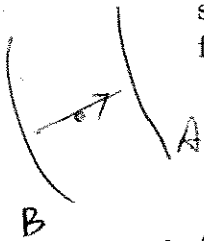
- a. (3 pts) For two equipotentials A' and B' , 1.8 cm apart, the electric field between them is of magnitude 72 N/C, and points to B' . If $V_{A'} = -1.4$ V, estimate $V_{B'}$.



$$\Delta V = |\vec{E}| d = 72 \text{ N/C} \times 1.8 \times 10^{-2} \text{ m} = 1.296 \text{ V}$$

$$V_{B'} = V_{A'} - \Delta V = -1.4 \text{ V} - 1.296 \text{ V} = -2.696 \text{ V}$$

- b. (3 pts) Equipotentials A and B have $V_A = -8.4$ V and $V_B = -8.1$ V, and a separation of 12 mm. For a point C midway between them, estimate the electric field (magnitude and direction).



$$|\vec{E}| = \frac{|V_A - V_B|}{12 \times 10^{-3} \text{ m}} = \frac{0.3 \text{ V}}{12 \times 10^{-3} \text{ m}} = 25 \text{ V/m}$$

From B to A.

- c. (3 pts) On which equipotential in part b) will an electron have the lower energy, and by how much?

B

$$|\Delta U| = |e| |\Delta V| = 1.6 \times 10^{-19} \text{ C} \times 0.3 \text{ V} = 0.48 \times 10^{-19} \text{ J}$$

- d. (3 pts) If the electron starts at rest from the equipotential giving it the lower energy, find its speed when it strikes the other equipotential.

$$\frac{1}{2} m v^2 = |\Delta U|$$

$$v = \sqrt{\frac{2|\Delta U|}{m}} = \sqrt{\frac{2 \times 0.48 \times 10^{-19} \text{ J}}{9.1 \times 10^{-31} \text{ kg}}} = 3.248 \times 10^5 \text{ m/s}$$