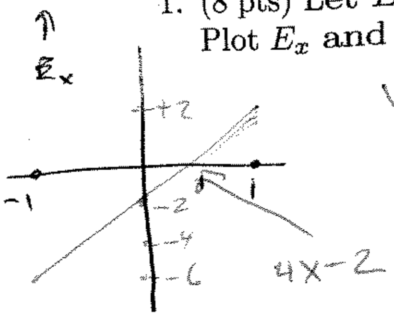
Exam 2, Physics 208 Name **Printed in Ink:** Yellow Key

Dr.Saslow, 7/02/04, Sects.301-305 **Section:**

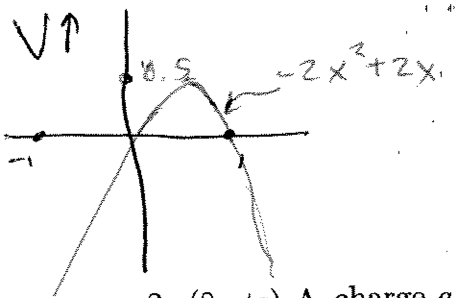
Don't waste time on problems you aren't sure of. Be clear and concise. A cluttered response will not get full credit.

1. (8 pts) Let $E_x = 4x - 2$, with E_x in volts/m and x in m. If $V = 0$ at $x = 0$, find $V(x)$. Plot E_x and $V(x)$ in the x-interval $(-1 \text{ m}, 1 \text{ m})$.

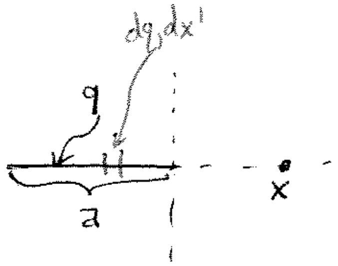


$$V(x) - V(0) = -\int_0^x E_x dx = -\int_0^x (4x-2)dx = -(2x^2 - 2x)\Big|_0^x = -(2x^2 - 2x)$$

$$V(x) = \cancel{V(0)} - 2x^2 + 2x = \boxed{-2x^2 + 2x} = -2(x - \tfrac{1}{2})^2 + \tfrac{1}{2}$$



2. (8 pts) A charge $q > 0$ is uniformly distributed over a rod of length a placed on the x-axis with its right end at the origin. Find the voltage at x on the positive x-axis.



$$dq = \lambda dx' = \frac{q}{a} dx'$$

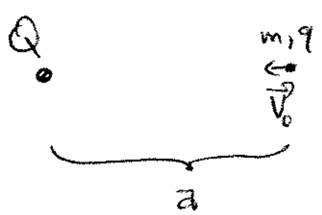
$$dV = \frac{k\,dq}{|x - x'|} = \frac{k\,dq}{x - x'} = \frac{kq}{a} \frac{dx'}{x - x'}$$

Let $r = x - x'$, so $dr = -dx'$. Then

$$dV = -\frac{kq}{a}\frac{dr}{r}, \quad V = -\frac{kq}{a}\int_{x+a}^{x} \frac{dr}{r} = -\frac{kq}{a} \ln r\Big|_{x+a}^{x} = \boxed{\frac{kq}{a} \ln\left(\frac{x+a}{x}\right)}$$

3. (8 pts) A fixed point charge Q is at the origin. At $t = 0$ a charge q with mass m is at $x = a$ with leftward velocity v_0 that satisfies $kQq/a = 3mv_0^2$. (a) Find b/a, where $b < a$ is the position where q turns around and starts to move rightward. (b) Find the velocity v_∞ of q at large distances from the origin, in the form $v_\infty / v_0 = \dots$

(a) $$\underbrace{\frac{1}{2}mv_0^2 + \frac{kQq}{a}}_{\frac{1}{6}\frac{kQq}{a} + \frac{kQq}{a}} = \underbrace{\text{const}}_{\text{Energy } E} = 0 + \frac{kQq}{b}$$ $v = 0$ at $r = b$.



Thus $\frac{7}{6}\frac{kQq}{a} = \frac{kQq}{b}$, so $\boxed{b = \frac{6}{7}a.}$

(b) $$\underbrace{\frac{1}{2}mv_0^2 + \frac{kQq}{a}}_{\frac{1}{2}mv_0^2 + 3mv_0^2} = \frac{1}{2}mv_\infty^2 + 0$$ $\frac{kQq}{r} \to 0$ as $r \to \infty$

Thus $\frac{7}{2}mv_0^2 = \frac{1}{2}mv_\infty^2$, so $\boxed{v_\infty = \sqrt{7}\, v_0.}$

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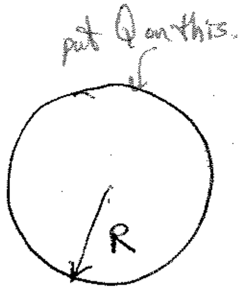
4. (8 pts) Derive the capacitance of a sphere of radius R. Explain your reasoning.



$$C = \frac{Q}{\Delta V} = \frac{Q}{V_{\text{at } R} - V_\infty}$$

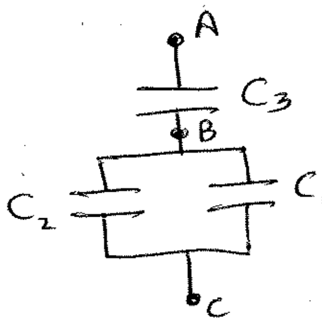
For a pt. charge and for a spherical charge, $\vec{E} = \frac{kQ}{r^2}\hat{r}$ outside, so $V = \frac{kQ}{r} + V_\infty$ in both cases.

Hence, with $V_\infty = 0$, at R we have $V = \frac{kQ}{R}$.

Thus $C = \frac{Q}{kQ/R} = \frac{R}{k}$.

5. Consider three capacitors. $C_1 = 10\ \mu$F and $C_2 = 20\ \mu$F are in parallel, and $C_3 = 20\ \mu$F is in series with them. $V_A = 10$ V and $V_C = -5$ V.

a. (8 pts) Find the charge and voltage difference for each capacitor. Find V_B.



$\Delta V = V_A - V_C = 15\,V$

$C_{eff} = \left(C_3^{-1} + (C_1 + C_2)^{-1}\right)^{-1} = \left(\frac{1}{20} + (30)^{-1}\right)^{-1} = \left(\frac{3}{60} + \frac{2}{60}\right)^{-1} = 12\,\mu F$

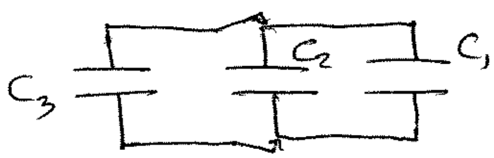
$Q = Q_3 = C_{eff} \cdot \Delta V = 180\,\mu C$

$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\,\mu C}{20\,\mu F} = 9\,V$, so $V_B = V_A - \Delta V_3 = 10 - 9 = 1\,V$

$\Delta V_1 = \Delta V_2 = V_B - V_C = 1 - (-5) = 6\,V$, so $Q_1 = C_1 \Delta V_1 = 10(6) = 60\,\mu C$

$Q_2 = C_2 \Delta V_2 = 20(6) = 120\,\mu C$

b. (8 pts) If, using insulating gloves, C_3 is disconnected and then placed in parallel with C_1 and C_2, find the new charge and voltage difference for each capacitor.



$Q^{(0)} = Q_1^{(0)} + Q_2^{(0)} + Q_3^{(0)} = 60 + 120 + 180 = 360\,\mu C$

$C'_{eff} = C_1 + C_2 + C_3 = 10 + 20 + 20 = 50\,\mu F$

$\Delta V'_1 = \Delta V'_2 = \Delta V'_3 = \frac{Q^{(0)}}{C'_{eff}} = \frac{360}{50} = 7.2\,V$

$Q'_1 = C_1 \Delta V'_1 = 10(7.2) = 72\,\mu C$

$Q'_2 = C_2 \Delta V'_2 = 20(7.2) = 144\,\mu C$

$Q'_3 = C_3 \Delta V'_3 = 20(7.2) = 144\,\mu C$

$Q'_1 + Q'_2 + Q'_3 = 360\,\mu C$

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6. A parallel plate capacitor has electrical energy 7.2×10^{-5} ergs when connected to a 6 V battery. It is now disconnected from the battery. A slab of dielectric constant $\kappa = 4$ and nearly the same thickness as the capacitor is slid into the capacitor.
   a. (2 pts) What is the voltage difference now?

$$\Delta V = \frac{\Delta V_0}{4} = \frac{6}{4} = 1.5\ \text{V}$$

   b. (2 pts) What is the electrical energy now?

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(\kappa C_0)\left(\frac{\Delta V_0}{\kappa}\right)^2 = \frac{1}{2}\frac{C_0(\Delta V_0)^2}{\kappa} = \frac{U_0}{\kappa} = 1.6\times 10^{-5}\ \text{ergs}$$

   c. (4 pts) Was the dielectric attracted, repelled, or did it feel no force when it was part way in the capacitor, and why? (No reason, no credit.)

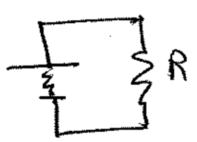
Attracted, since the final energy is lower.
(It gets polarized and is attracted by the amber effect)

7. (8 pts) You are given a voltaic cell with internal resistance of 4 Ω. When shorted, it briefly produces a current of 0.15 A.
   a. (4 pts) Find its emf and the rate at which it energy discharged.

$$I_{short} = \frac{\mathcal{E}}{r} \Rightarrow \mathcal{E} = I_{short}(r) = 0.15 \times 4 = 0.6\ \text{V}$$

$$P = I_{short}^2\, r = (0.15)^2(4) = 0.09\ \text{W}$$

   b. (4 pts) There is a certain load resistance for which this voltaic cell will provide maximum power to load. Find that resistance and that maximum power.



$R_L = r$ (impedance matching)

$$R_L = 4\,\Omega. \quad P_R = I^2 R_L = \left(\frac{\mathcal{E}}{r+R_L}\right)^2 R_L = \left(\frac{\mathcal{E}}{2R_L}\right)^2 R_L = \frac{\mathcal{E}^2}{4R_L} = \frac{(.6)^2}{4(4)} = 2.25\times 10^{-2}\ \text{W}$$

   c. (4 pts) A 100% efficient flashlight bulb produces 0.9 W when used with the AA cell. If its resistance R is much larger than the cell's internal resistance, find R and the efficiency at which the battery produces useful power.

Impossible problem! Bad choice of numbers. Can't produce more power than under shorting.

But, if $P_{bulb} = 0.009$ W (very low), then $.009 \approx \left(\frac{\mathcal{E}}{\cancel{r}+R_L}\right)^2 R_L \approx \frac{\mathcal{E}^2}{R_L}$,

$$R_L \approx \frac{\mathcal{E}^2}{.009} = \frac{(0.6)^2}{.009} = 40\,\Omega\ (\gg r = 4\,\Omega)$$

$$\text{efficiency} = \frac{R_L}{r+R_L} = \frac{40}{4+40} = 89\%$$

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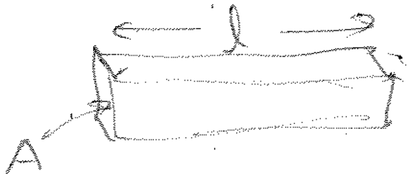
8. (6 pts) A sluiceway has length 80 m and cross-section 5 m^2. Fish move through it with average velocity 0.4 m/s, with each taking up a volume of 2 m^3. Find the rate at which the fish pass through the exit. If they each carry a charge of 10^{-6} C, find the electric current passing through the exit.

$$\frac{dN}{dt} = \frac{N}{t} = \frac{n(A\ell)}{\ell/v} = nAv = \left(\frac{1}{2\,m^3}\right)(5\,m^2)\left(.4\,\frac{m}{s}\right) = \frac{1\text{ fish}}{sec}$$

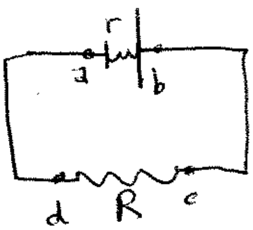


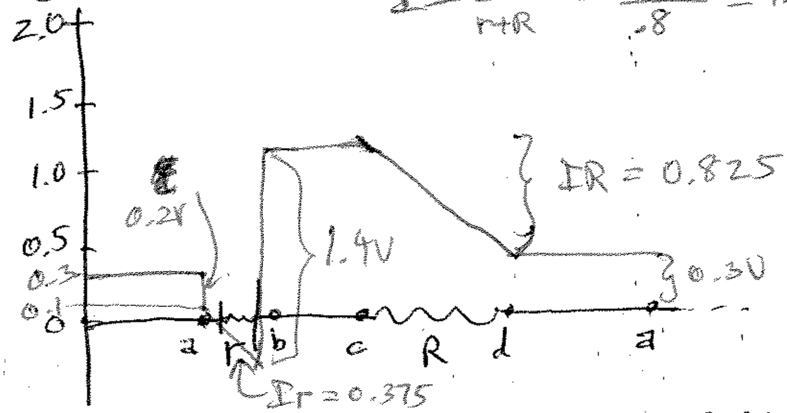
$$\frac{dQ}{dt} = Q\frac{dN}{dt} = 10^{-6}\frac{C}{sec} = 10^{-6}A = 1\mu A$$

9. A voltaic cell has internal resistance $r = 0.25\ \Omega$ and open circuit voltages across the left and right electrodes of 0.2 V and 1.4 V, for a net emf of $\mathcal{E} = 1.2$ V. It is in series with a resistor $R = 0.55\ \Omega$. Let $V_a = 0.3$ V. The connecting wires have zero resistance.

a. (8 pts) Find the current, the voltage drops across the resistances, and sketch the voltage around the circuit.

$$I = \frac{\mathcal{E}}{r+R} = \frac{1.2}{.8} = 1.5A,\quad IR = (1.5)(.55) = .825V$$
$$Ir = (1.5)(.25) = .375V$$
$$1.2V$$

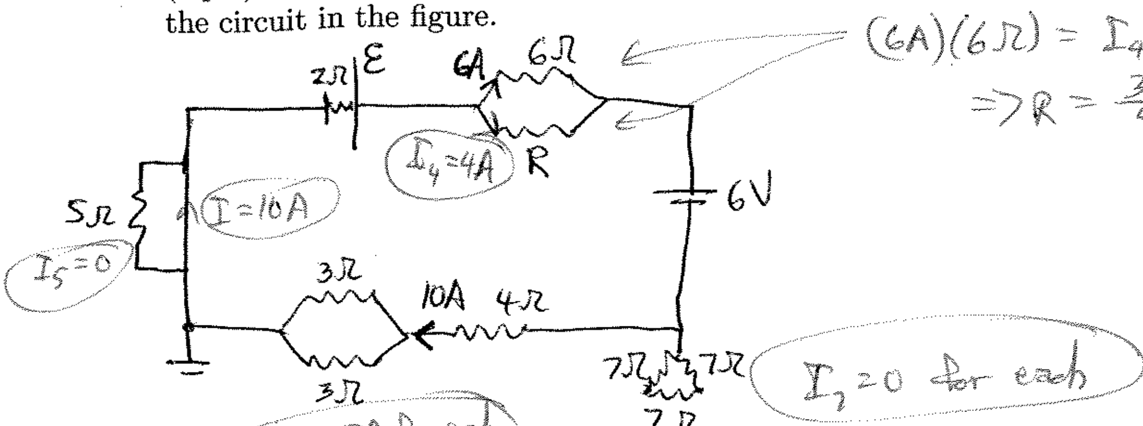




b. (2 pts) If the voltaic cell discharges in 50 minutes, find its initial "charge" and its initial energy.

10. (8 pts) Find the unknown currents, the unknown resistance, and the unknown emf for the circuit in the figure.

$$(6A)(6\Omega) = I_4 R = 4(R)$$
$$\Rightarrow R = \frac{36}{4} = 9\Omega$$



To get $\mathcal{E}$, start at the ground and go counterclockwise until you get to $\mathcal{E}$.

The voltage rises by $3\times5 + 4\times10 + 0 + 6 + 6(6) = 15 + 40 + 6 + 36 = 97V$

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either 3Ω resistor; the 4Ω resistor; the 7Ω resistors; the 6V cell; the 6Ω resistor

Starting at ground and going clockwise gives $-2(10) + \mathcal{E} = -20 + \mathcal{E}$.

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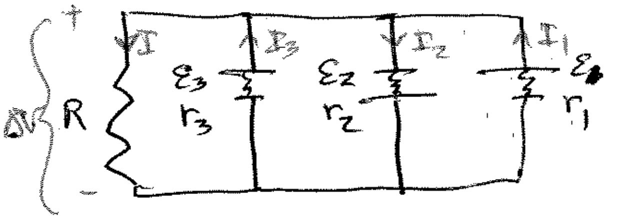
11. (12 pts) For the circuit below, take $\mathcal{E}_1 = 8$ V, $\mathcal{E}_2 = 11$ V, $\mathcal{E}_3 = 14$ V, $r_1 = 0.01\ \Omega$, $r_2 = 0.04\ \Omega$, $r_3 = 0.02\ \Omega$, $R = 0.03\ \Omega$. (1) Indicate and label the directions of positive currents and indicate the positive side of the voltage ΔV across R. (2) Analyze the circuit using Kirchoff's rules. (3) Solve for the voltage across R. (4) Find the current through R and the currents provided by each of the batteries.



(1) See Figure.

(2) $I_1 + I_3 = I + I_2$ note sign!

$$\frac{\mathcal{E}_1 - \Delta V}{r_1} + \frac{\mathcal{E}_3 - \Delta V}{r_3} = \frac{\Delta V}{R} + \frac{\mathcal{E}_2 + \Delta V}{r_2}$$

(3) $$\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_3}{r_3} - \frac{\mathcal{E}_2}{r_2} = \Delta V\left(\frac{1}{R} + \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

$$800 + 700 - 275 = \Delta V\,(33.3 + 100 + 25 + 50)$$

$$1225 = \Delta V\,(208.3) \Rightarrow \Delta V = 5.88\text{ V}$$

(4) $$I = \frac{\Delta V}{R} = \frac{5.88}{.03} = 196\text{A}$$

$$I_1 = \frac{\mathcal{E}_1 - \Delta V}{r_1} = \frac{8 - 5.88}{.01} = 212\text{A}$$

$$I_2 = \frac{\mathcal{E}_2 + \Delta V}{r_2} = \frac{11 + 5.88}{.04} = 422\text{A}$$

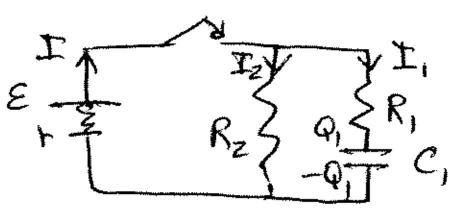
$$I_3 = \frac{\mathcal{E}_3 - \Delta V}{r_3} = \frac{14 - 5.88}{.02} = 406\text{A}$$

$I_1 + I_3 = 616$ A

$I + I_2 = 616$ A

12. The capacitor is uncharged initially. The switch is then closed at $t = 0$. Let $\mathcal{E}$=8 V, $r = 2\ \Omega$, $R_1 = 12\ \Omega$, $R_2 = 6\ \Omega$, $C_1 = 4\mu$ F.

a. (8 pts) Find I, Q_1, I_1, and I_2 just after the switch is closed. Explain.



$\boxed{Q_1 = 0}$ at $t=0$, so C_1 has $\Delta V = 0$, so it can be neglected.

Then $R_{eff} = r + (R_1^{-1} + R_2^{-1})^{-1} = 2 + \left(\frac{1}{12} + \frac{1}{6}\right)^{-1} = 2 + 4 = 6\,\Omega$

Thus $\boxed{I = \frac{\mathcal{E}}{R_{eff}} = \frac{8}{6} = \frac{4}{3}\text{ A}}$

$$\Delta V_1 = \Delta V_2 = \mathcal{E} - Ir = 8 - \frac{4}{3}\cdot 2 = \frac{16}{3}\text{ V}$$

$$I_1 = \frac{\Delta V_1}{R_1} = \frac{16/3}{12} = \boxed{\frac{4}{9}\text{A}} \quad I_2 = \frac{\Delta V_2}{R_2} = \frac{16/3}{6} = \boxed{\frac{8}{9}\text{A}} \quad \left(I_1 + I_2 = \frac{4}{3}\text{A} = I\right)$$

b. (6pts) Find I, Q_1, I_1, and I_2 a long time after the switch is closed. Explain.

When C_1 has charged up, $\boxed{I_1 = 0.}$

Then $\boxed{I = I_2 = \frac{\mathcal{E}}{r + R_2} = \frac{8}{2+6} = 1\text{ A}}$

For C_1, $\Delta V_1 = \Delta V_2 = I_2 R_2 = (1)(6) = 6\text{ V}$,

so $Q_1 = C_1 \Delta V_1 = (4\mu\text{F})(6\text{V}) = \boxed{24\mu\text{C}}$