

Selected Homework Solutions – Chapter R/P

R-2.1. The ground wire provides an alternate current path

[hot wire - drill - case - ground wire - ground],

and since it has lower resistance than the path

[hot wire - drill - case - person - ground],

it carries more current.

R-2.3. Let's draw a horizontal line with A on the left and B on the right, separated by the resistor $R = 5\Omega$:

A.....R.....B.

Ohm's Law says that current I flows from higher to lower voltage. Let's assume that $I > 0$ flows rightward. Then $V_A > V_B$, and $I = (V_A - V_B)/R$.

a. For $V_A=5$ V, $V_B=5$ V, we have $I = (5 - 5)/5 = 0/5 = 0$ A. $I^2R = 0$.

b. For $V_A=5$ V, $V_B=-5$ V, we have $I = (5 - (-5))/5 = 10/5 = 2$ A, rightward. $I^2R = 20$ W.

c. For $V_A=5$ V, $V_B=0$ V, we have $I = (5 - 0)/5 = 5/5 = 1$ A, rightward. $I^2R = 5$ W.

d. For $V_A=0$ V, $V_B=5$ V, we have $I = (0 - (5))/5 = -5/5 = -1$ A, leftward. $I^2R = 5$ W.

e. For $V_A=-5$ V, $V_B=5$ V, we have $I = (-5 - (5))/5 = -10/5 = -2$ A, leftward. $I^2R = 20$ W.

R-2.4. Power in watts is $\mathcal{P} = I(\Delta V)$, so $60 \text{ W} = I(120 \text{ V})$, giving $I=0.5$ A. Also, $I = \Delta V/R$, so $0.5 \text{ A} = 120 \text{ V}/R$, giving $R=240 \Omega$.

R-2.6. Since charge $Q = It$ for a constant current I during a time t , we have $Q=(120,000 \text{ A})(2 \times 10^{-4} \text{ s})=24 \text{ C}$.

1.7.1. We start with (A,B,C) at (1,2,3), with center-of-mass (CM) at 2.

(a) On shifting from (1,2,3) to (2,3,4), the CM shifts to 3.

(b) On shifting from (1,2,3) to (4,2,3), the CM shifts to 3.

(c) On shifting from (1,2,3) to (1,5,3), the CM shifts to 3. (This case was not stated in the handout.)

(d) In the first case there is a collective effect; in the other cases there is an individual effect. For electricity in conductors, the response of the charge-carriers usually is collective. For electricity in insulators, the response of the charge-carriers is usually individual.

R-8.1. Only the set $[F_x = ma_x, F_y = ma_y, F_z = ma_z]$ is completely correct. $F = ma$ doesn't contain any directional information.

R-10.8. $\vec{a} \equiv (3, -4, 2)$, $\vec{b} \equiv (2, 6, -1)$.

a. $|\vec{a}| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29} = 5.385$,

$$|\vec{b}| = \sqrt{2^2 + 6^2 + (-1)^2} = \sqrt{41} = 6.403,$$

$$\vec{a} \cdot \vec{b} = (3)(2) + (-4)(6) + (2)(-1) = -20.$$

Define $\vec{c} = \vec{a} \times \vec{b}$. Then

$$c_x = a_y b_z - a_z b_y = (-4)(-1) - (2)(6) = -8,$$

$$c_y = a_z b_x - a_x b_z = (2)(2) - (3)(-1) = 7,$$

$$c_z = a_x b_y - a_y b_x = (3)(6) - (-4)(2) = 26,$$

so $\vec{c} \equiv (-8, 7, 26)$. Thus $|\vec{c}| = \sqrt{789} = 28.1$.

b. $\hat{a} = \vec{a}/|\vec{a}| \equiv (.557, -.743, .371)$, $\hat{b} = \vec{b}/|\vec{b}| \equiv (.312, .937, -.156)$, $\hat{c} = \vec{c}/|\vec{c}| \equiv (-.285, .249, .926)$.

c. $\cos\theta_{ab} = (\vec{a} \cdot \vec{b})/|\vec{a}||\vec{b}| = -.580$. Then either $\theta_{ab} = 125.45^\circ \equiv -234.55^\circ$ (we can add or subtract 360 degrees) or $\theta_{ab} = -125.45^\circ \equiv 234.55^\circ$.

$|\sin\theta_{ab}| = |\vec{a} \times \vec{b}|/|\vec{a}||\vec{b}| = |\vec{c}|/|\vec{a}||\vec{b}| = 28.1/[(5.385)(6.403)] = .815$. Then $\theta_{ab} = \pm 54.55^\circ$ or $\pm 234.55^\circ$. Only the latter is consistent with the calculation based on the cosine.

R-10.9. (a) We now rotate $\vec{a} \equiv (3, -4, 2)$ to $\vec{a}' \equiv (5, 0, 2)$. Because the z -component doesn't change, this is a rotation about the z -axis. It is a counterclockwise by 53.1° , because it is a 3-4-5 right triangle. (We could also obtain the rotation angle by taking the dot product of $\vec{a}_{in-plane}$ and $\vec{a}'_{in-plane}$.) (Note: The angle between \vec{a} and \vec{a}' is irrelevant; it is associated with $\vec{a} \times \vec{a}'$, not z .)

(b) To find the rotated \vec{b} , which we'll call \vec{b}' , we note that the in-plane part of \vec{b} satisfies $\vec{b}_{in-plane} \equiv (2, 6)$, putting it in the first quadrant. It has length $|\vec{b}_{in-plane}| = \sqrt{40} = 6.325$, and angle $\text{atan}(6/2) = \text{atan}(3) = 71.6^\circ$. Thus

$$(2, 6) \equiv \vec{b}_{in-plane} \equiv |\vec{b}_{in-plane}|(\cos 71.6, \sin 71.6).$$

To find \vec{b} under 53.1° counterclockwise rotation, replace 71.6° by $71.6 + 53.1 = 124.7^\circ$, so

$$\vec{b}'_{in-plane} \equiv |\vec{b}_{in-plane}|(\cos 124.7, \sin 124.7) = (-3.60, 5.20).$$

$$\text{Thus } \vec{b}' \equiv (-3.60, 5.20, -1).$$

(c) To find the rotated $\vec{a} \times \vec{b}$, which we'll call $(\vec{a} \times \vec{b})'$, we note that the in-plane part of $\vec{a} \times \vec{b}$ satisfies $(\vec{a} \times \vec{b})_{in-plane} \equiv (-8, 7)$, putting it in the second quadrant. It has length $|(\vec{a} \times \vec{b})_{in-plane}| = \sqrt{113} = 10.63$, and angle $\text{atan}(-7/8) = -41.2 + 180 = 138.8^\circ$. (We add the 180 degrees to put it in the second quadrant. Calculators are stupid about this sort of thing.) Thus

$$(-8, 7) \equiv (\vec{a} \times \vec{b})_{in-plane} \equiv |(\vec{a} \times \vec{b})_{in-plane}|(\cos 138.8, \sin 138.8),$$

To find $\vec{a} \times \vec{b}$ under 53.1° counterclockwise rotation, replace 138.8° by $138.8 + 53.1 = 191.9^\circ$, so

$$(\vec{a} \times \vec{b})'_{in-plane} \equiv |(\vec{a} \times \vec{b})_{in-plane}|(\cos 191.9, \sin 191.9) = (-10.40, -2.20).$$

$$\text{Thus } (\vec{a} \times \vec{b})' \equiv (-10.40, -2.20, 26).$$

(d) An explicit computation of $\vec{a}' \times \vec{b}'$ gives

$$a'_y b'_z - a'_z b'_y = (0)(-1) - (2)(5.2) = -10.40,$$

$$a'_z b'_x - a'_x b'_z = (2)(-3.6) - (5)(-1) = -2.20,$$

$$a'_x b'_y - a'_y b'_x = (5)(5.2) - (0)(-3.6) = 26,$$

so $(\vec{a} \times \vec{b})' = \vec{a}' \times \vec{b}' = (-10.40, -2.20, 26)$, and $|(\vec{a} \times \vec{b})'| = 28.1$.

(e) It is easily verified that $|\vec{a}| = |\vec{a}'| = 5.385$. It can be verified easily that $|\vec{b}| = |\vec{b}'| = 6.403$. It also can be verified easily that $\vec{a} \cdot \vec{b} = \vec{a}' \cdot \vec{b}' = -20$. It is easily verified that $|\vec{a} \times \vec{b}| = |(\vec{a} \times \vec{b})'| = 28.1$.

(f) Both the scalar product and the vector product correspond to an angle of $\pm 54.55^\circ$. Because the scalar product $\vec{a} \cdot \vec{b}$ and the magnitudes of both versions of the vector product for $\vec{a} \times \vec{b}$ do not change under rotation, the angle between \vec{a}' and \vec{b}' is the same as between \vec{a} and \vec{b} .

(g) Yes, the vector lengths and angles transform as expected.