

# PHYSICS 208 EXAM III: Fall 1999

## Formula/Information Sheet

• Basic constants:

Gravitational acceleration on Earth's surface	$g$	$=$	$9.8 \text{ m/sec}^2$
Permittivity of free space	$\epsilon_0$	$=$	$8.8542 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$ [ $k = 1/4\pi\epsilon_0 = 8.9875 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$ ]
Permeability of free space	$\mu_0$	$=$	$4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$ [ $k_m = \mu_0/4\pi = 10^{-7} \text{ Wb}/\text{A}\cdot\text{m}$ ]
Elementary charge	$e$	$=$	$1.60 \times 10^{-19} \text{ C}$
Unit of energy: electron volt	1 eV	$=$	$1.60 \times 10^{-19} \text{ J}$
Unit of energy: kilowatt-hour	1 kWh	$=$	$3.6 \times 10^6 \text{ J}$
Planck's Constant	$h$	$=$	$6.626 \times 10^{-34} \text{ J sec}$

• Properties of some particles:

Particle	Mass [kg]	Charge [C]
Proton	$1.67 \times 10^{-27}$	$+1.60 \times 10^{-19}$
Electron	$9.11 \times 10^{-31}$	$-1.60 \times 10^{-19}$
Neutron	$1.67 \times 10^{-27}$	0

• Some indefinite integrals:

$$\int \frac{dx}{x} = \ln x \quad \left| \int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx) \right.$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} \quad \left| \int \frac{x dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}} \right.$$

$$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}) \quad \left| \int \frac{x dx}{\sqrt{x^2\pm a^2}} = \sqrt{x^2 \pm a^2} \right.$$

Coulomb's law	( $q$ at $\vec{r}_1$ , $q'$ at $\vec{r}_2$ )	$ \vec{F}  = k \frac{ q_1 q_2 }{ \vec{r}_2 - \vec{r}_1 ^2}$
Electric field [N/C = V/m]	( $q$ at $\vec{r}'$ , observer at $\vec{r}$ , $\vec{R} \equiv \vec{r} - \vec{r}'$ )	$\vec{E}(\vec{r}) = k \frac{q}{R^2} \hat{R}$
	(group of charges)	$\vec{E} = \sum \vec{E}_i = k \sum \frac{q_i}{R_i^2} \hat{R}_i$
	(continuous charge distribution)	$\vec{E} = k \int \frac{dq}{R^2} \hat{R}$
line charge $\lambda$		$ \vec{E}  = \frac{2k \lambda }{r}$
sheet charge $\sigma$		$ \vec{E}  = 2\pi k \sigma $
Electric force [N]	(on $q$ subject to $\vec{E}$ )	$\vec{F} = q \vec{E}$

Electric flux	(small area $\Delta A_i$ , outward normal $\hat{n}_i$ )	$\Delta\Phi_i = \vec{E}_i \cdot \hat{n}_i \Delta A_i = E_i \Delta A_i \cos\theta_i$
	(through an entire surface area)	$\Phi_{surface} = \lim_{\Delta A \rightarrow 0} \sum \Delta\Phi_i = \int \vec{E} \cdot \hat{n} dA$
Gauss' law	(through a closed surface area)	$\Phi_{closed} \equiv \oint \vec{E} \cdot \hat{n} dA = 4\pi k Q_{enc}$
Conductor		$(\vec{E} \cdot \hat{n})_{out} = 4\pi k \sigma_S$

Electric potential [Volt = J/C]	(definition, step-length $ds$ along $\hat{s}$ )	$\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot \hat{s} ds$
	( $\vec{E} = \text{constant}$ )	$\Delta V = -\vec{E} \cdot (\vec{r}_B - \vec{r}_A)$
	(point charge $q$ at $\vec{r}'$ )	$V(\vec{r}) = k \frac{q}{ \vec{R} } \quad (V(\infty) = 0)$
	(group of charges, $\vec{R}_i \equiv \vec{r} - \vec{r}_i$ )	$V(\vec{r}) = \sum V_i( \vec{R}_i ) = k \sum \frac{q_i}{ \vec{R}_i } \quad (V_i(\infty) = 0)$
	(continuous charge distribution)	$V(\vec{r}) = k \int \frac{dq}{ \vec{R} } \quad (V(\infty) = 0)$
Electric potential energy [J]	(definition)	$\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot \hat{s} ds$
		$= q_0 (V_B - V_A)$
$\vec{E}$ from $V$		$\vec{E} = -\vec{\nabla} V$ ( $\vec{\nabla}$ = gradient operator)
Electric potential energy of two-charge system		$U_{12} = k \frac{q_1 q_2}{ \vec{r}_1 - \vec{r}_2 }$

Capacitance [Farad = C/V]	(definition)	$C = \frac{Q}{\Delta V}$
Sphere of radius $R$		$C = \frac{4\pi\epsilon_0 R}{k}$
Parallel plates of area $A$ and separation $d$		$C = \frac{4\pi\epsilon_0 A}{kd}$
Effect of dielectric on electric field		$E = E_0/\kappa$
Electrostatic potential energy [J] stored in capacitance		$U_E(t) = \frac{1}{2} \frac{Q(t)^2}{C}$
Electric energy per unit volume		$u_{electric} = \epsilon_0 E^2/2 = E^2/8\pi k$
Electric dipole moment ( $2a$ = separation between two charges)		$ \vec{p}  = 2aq$
Torque on electric dipole moment		$\vec{\tau} = \vec{p} \times \vec{E}$
Potential energy of an electric dipole moment		$U = -\vec{p} \cdot \vec{E}$
Current [A]	(definition) with motion of charges	$I \equiv \frac{dQ(t)}{dt}$ $I = nqv_d A$
Current density [A/m <sup>2</sup> ]		$J = \frac{I}{A}$ (where $I = \int \vec{J} \cdot \hat{n} dA$ )
Resistivity [ $\Omega \cdot m$ ]		$\rho = \frac{ \vec{E} }{ \vec{J} }$
Resistance [ $\Omega$ ]	(definition) for uniform cross-sectional area $A$	$R \equiv \frac{\Delta V}{I}$ $R = \rho \frac{\ell}{A}$
Current provided by Emf $\mathcal{E}$ [V]	internal resistance $r$	$I = (\mathcal{E} - \Delta V)/r$
Power provided by emf $\mathcal{E}$ [J/s]		$\mathcal{P} = \mathcal{E}I$
Energy loss rate by $R$ [J/s]		$\mathcal{P} = I^2 R = (\Delta V)^2/R = I(\Delta V)$
Time constant in $RC$ circuit [s]		$\tau_{RC} = RC$
Magnetic force [N]	on pole $q_m$	$\vec{F} = q_m \vec{B}$
Magnetic field [N]	due to pole $Q_m$	$ \vec{B}  = \frac{k_m  Q_m }{R^2}$
Magnetic moment [A·m <sup>2</sup> or J/T]	poles $\pm q_m$ separated by $l$	$ \vec{\mu}  = q_m l$
Torque [N·m]	on a magnet	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Energy [N·m]	of a magnet	$U = -\vec{\mu} \cdot \vec{B}$
sheet charge $\sigma_m = q_m/A$		$ \vec{B}  = 2\pi k_m  \sigma_m $
Magnetic force [N]	on charge $q$	$\vec{F} = q \vec{v} \times \vec{B}$
	on current-carrying conductor	$\vec{F} = \int Id\vec{s} \times \vec{B}$
cyclotron frequency [radians/sec]	charge $q$ , mass $m$	$\omega =  q  \vec{B} /m$
Magnetic moment [A·m <sup>2</sup> or J/T]		$\vec{\mu} = I\hat{n}A$
Torque [N·m]	on a current loop	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Ampere's law		$\oint \vec{B} \cdot d\vec{s} = \mu_0 I = 4\pi k_m I$
Biot-Savart law		$d\vec{B} = k_m \frac{I d\vec{s} \times \hat{R}}{R^2}$
Magnetic field [T]	a long straight wire	$ \vec{B}  = \mu_0 I/(2\pi a)$
	inside a toroid	$ \vec{B}  = \mu_0 NI/(2\pi r)$
	inside a solenoid	$ \vec{B}  = \mu_0 NI/\ell$
	a straight wire segment	$ \vec{B}  = k_m I(\cos\theta_1 - \cos\theta_2)/a$
	a circular arc (radius $R$ )	$ \vec{B}  = k_m I\theta/R$
Displacement current [A]	(definition)	$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot \hat{n} dA$
Ampere-Maxwell law		$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d)$
Faraday's Law		$\mathcal{E} = -\frac{d\Phi_B}{dt}$
Mutual Inductance [H]	(definition)	$M_{21} = N_2 \Phi_{21}/I_1$
Electromotive force induced by mutual induction [V]		$\mathcal{E}_2 = -M_{21} \frac{dI_1}{dt}$
Self Inductance [H]	(definition)	$L = N\Phi_B/I$
Self Induced electromotive force [V]		$\mathcal{E} = -L \frac{dI}{dt}$
Magnetic energy per unit volume		$u_{magnetic} = B^2/2\mu_0 = B^2/8\pi k_m$
Magnetic energy stored in L [J]		$U_B(t) = \frac{1}{2} LI(t)^2$
Time constant in LR circuits [s]		$\tau_{LR} = L/R$
Angular frequency of LC circuit [rad/sec]		$\omega = \sqrt{\frac{1}{LC}}$