Some Math Facts You Should Know

1. The Distributive Law and Approximations.

   Example: You want to buy 5 $0.98 items. (1) Estimate their cost. (2) Compute their cost exactly.

   (1) $0.98 is nearly $1.00, so 5 items will costly nearly $5.

   (2) $0.98=$1.00-$0.02, so 5 items will cost exactly

   5($0.98)=5($1.00-$0.02)=5($1.00)-5($0.02)=$5.00-$0.10=$4.90.

2. Scaling of areas. Areas involve the product of two lengths.

   Example: A 22x33 triangle has an area that we'll call A (this would take some effort to calculate, which we won't bother to do). Find the area of a 44x66 triangle, in terms of A.

   The larger triangle is larger by two in each direction (it doesn't matter how it is oriented). Hence, its area is larger by a factor of 4, so it has an area 4A.

3. Straight lines. $y = mx + b$ has slope m and passes through the origin.

   Example: Show that $y = y_0 + m(x - x_0)$ has slope m and passes through (x_0, y_0).

   $y = mx + b = m(x - x_0) + mx_0 + b = (mx_0 + b) + m(x - x_0) = y_0 + m(x - x_0)$, if we take $y_0 = mx_0 + b$.

4. Finding the local approximation to a straight line (the tangent line).

   Example: Find the tangent line to $y = 2x^2$ at $x = 3$.

   Use $y = y_0 + m(x - x_0)$ for $x_0 = 3$. Then $y_0 = 2x_0^2 = 2(3)^2 = 18$. To get the slope m, use $m = dy/dx$ evaluated at $x_0 = 3$. For $y = 2x^2$, $dy/dx = 4x$, so $m = 4x_0 = 4(3) = 12$. Thus $y = y_0 + m(x - x_0) = 18 + 12(x - 3)$, or $y = 12x - 18$.

5. Basic trig properties, for stupidity checks. sin(0)=0, sin(90)=1, cos(0)=1, cos(90)=0.

   Example: A student says that the x-component of a unit vector that makes an angle θ to the x-axis is sinθ. Can this be correct?

   Check this by taking $\theta = 0$, which corresponds to the unit vector being the x-axis, so the component should be 1. But sin(0)=0, so the student took the wrong component.

6. Why the radian is the natural variable for angle.

   Example: Find the arc-length s of an arc of radius a and angle θ, both in radians and in degrees.

   The arc-length s is proportional to both a and θ, since the doubling the radius doubles the arc-length, and doubling the angle doubles the arc-length. So write $s = Ca\theta$, where C is an unknown constant. (1) In radian measure, for a full circle, where $l = 2\pi a$, the angle is 2π, so $2\pi a = Ca(2\pi)$. Thus $C = 1$ for radian measure. (2) In degree measure, for a full circle, where $l = 2\pi a$, the angle is 360 degrees, so $2\pi a = Ca(360)$. Thus $C = 2\pi/360 = \pi/180$ for degrees measure.

7. The fundamental theorem of calculus simply says that the extra area dA under a curve $f(x)$ due to increasing x by dx is given by $dA = f(x)dx$. Thus the local value of the slope of the area $A(x)$ function is $f(x)$.

   Example: If $dA = 0.02$ cm^2 when $dx = 0.01$ cm and $x = 3$ cm, find $f(x)$ for $x = 3$ cm.

   $f(x) = dA/dx = 0.02/0.01 = 2$ cm at $x = 3$ cm. We can't get any information about other points.